## Rupert L. Frank

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PERSONAL DATA Born August 29, 1978 in Munich, Germany

Citizenship: German

EDUCATION Royal Institute of Technology Stockholm, Sweden

Ph.D. in Mathematics, May 2007

• Dissertation Topic: Hardy-Lieb-Thirring inequalities for eigenvalues of Schrödinger operators

• Advisor: Ari Laptev

Ludwig-Maximilians-Universität Munich, Germany

Diplom (MS) in Mathematics, October 2003

• Grade: very good (passed with distinction)

• Diplomarbeit (Master Thesis) Topic: On the spectral analysis and the scattering theory of the Laplacian on the halfplane with a periodic perturbation on the boundary

• Advisors: Heinz Siedentop and Mikhail Sh. Birman

Gymnasium Ottobrunn, Germany

Abitur (A-levels), June 1997

• Grade: 1.0 (highest possible grade)

POSITIONS HELD Princeton University, USA

Assistant Professor, as of Sept. 2010

Instructor, 2009 - 2010

Postdoctoral Research Fellow, 2007 – 2009

Prizes and Grants Young Scientist Prize in Mathematical Physics of the International Union of Pure and Applied Physics, 2009

Scholarship of the Deutsche Forschungsgemeinschaft (German Research Foundation), 2008 – 2009

Scholarship of the Deutscher Akademischer Auslandsdienst (German Academic Exchange Service), 2007 - 2008

Marie Curie Fellowship, 2004

Scholarship of the Studienstiftung des Deutschen Volkes (German National Merit Foundation), 1999-2003

Research grant of the Studienstiftung des Deutschen Volkes, 2002

Scholarship of the Deutscher Akademischer Auslandsdienst (German Academic Exchange Service), 2000-2001

# Rupert L. Frank

# RESEARCH INTERESTS

Analysis, Mathematical Physics and Spectral Theory, in particular

- Spectral estimates for differential operators
- Large Coulomb systems and the problem of stability of matter
- Sobolev and Hardy inequalities
- Ginzburg-Landau equations and superconductivity
- Non-linear equations involving fractional powers of the Laplacian

#### RESEARCH EXPERIENCE

Research stay at Princeton University (five months in 2006)

Advisor: Elliott H. Lieb

Research stay at Université Paris Sud XI (four months in 2004 and 2006)

Advisor: Bernard Helffer

Research stay at Mittag-Leffler Institute and KTH Stockholm (four months

in 2002)

Advisors: Ari Laptev and Mikhail Sh. Birman

In addition several short term visits at Vienna (2006, 2007, 2008, 2009, 2010 and 2011), Caltech (2008 and 2009), Atlanta (2006 and 2007), Stuttgart (2006, 2007, 2009 and 2010), Munich (2005, 2006, 2007, 2008, 2009 and 2011), Prague (2005).

#### TEACHING EXPERIENCE

Visiting lecturer at LMU Munich, July 2011

 $\label{lem:course} \mbox{Graduate course on } \mbox{\it Uncertainty principles and Sobolev inequalities in Mathematical Physics}$ 

Instructor at Princeton University, as of September 2009

Course on Calculus and Advanced Calculus

Teaching assistant at KTH Stockholm, 2004 - 2007

Courses on Calculus, Linear Algebra and Ordinary Differential Equations

Teaching assistant at LMU Munich, 1999 – 2003

Courses on Calculus, Analysis, Ordinary Differential Equations and Sobolev Spaces and Distributions

# FURTHER MATHEMATICAL ACTIVITY

Referee for Annals of Mathematics, Archiv der Mathematik, Arkiv för Matematik, Canadian Journal of Mathematics, Communications in Mathematical Physics, Communications on Pure and Applied Mathematics, Discrete and Continuous Dynamical Systems, Documenta Mathematica, Duke Mathematical Journal, Journal of Functional Analysis, Journal of Mathematical Physics, Letters in Mathematical Physics, Mathematical Research Letters, Proceedings of the Amer. Math. Soc., Proceedings of the London Math. Soc., Reviews in Mathematical Physics and Revista Matemática Complutense.

Reviewer for Mathematical Reviews

Co-organizer of the Seminar 'Spectral Theory' at KTH Stockholm, spring  $2007\,$ 

Publications

- [1] R. L. Frank, M. Loss, *Hardy-Sobolev inequalities for arbitrary domains*. Submitted.
- [2] R. L. Frank, C. Hainzl, R. Seiringer, J.-P. Solovej, *Microscopic derivation of Ginzburg-Landau theory*. Submitted.
- [3] R. L. Frank, M. Lewin, E.H. Lieb, R. Seiringer, How much energy does it cost to make a hole in the Fermi sea? Submitted.
- [4] R. L. Frank, R. Olofsson, Eigenvalue bounds for Schrödinger operators with a homogeneous magnetic field. Submitted.
- [5] R. L. Frank, D. Lenz, D. Wingert, Intrinsic metrics for non-local symmetric Dirichlet forms and applications to spectral theory. Submitted.
- [6] R. L. Frank, E. H. Lieb, A new, rearrangement-free proof of the sharp Hardy-Littlewood-Sobolev inequality. In: Spectral Theory, Function Spaces and Inequalities, B. M. Brown et al. (eds.), Oper. Theory Adv. Appl., Birkhäuser, Basel, to appear.
- [7] R. L. Frank, E. H. Lieb, R. Seiringer, L. E. Thomas, *Binding, stability and non-binding of multi-polaron systems*. Submitted.
- [8] R. L. Frank, E. Lenzmann, Uniqueness and nondegeneracy of ground states for  $(-\Delta)^s Q + Q Q^{\alpha+1} = 0$  in  $\mathbb{R}$ . Submitted.
- [9] R. L. Frank, E. H. Lieb, Sharp constants in several inequalities on the Heisenberg group. Submitted.
- [10] R. L. Frank, Eigenvalue bounds for Schrödinger operators with complex potentials. Bull. London Math. Soc., to appear.
- [11] R. L. Frank, E. H. Lieb, R. Seiringer, L. E. Thomas, *Stability and absence of binding for multi-polaron systems*. Publ. Math. IHES, to appear.
- [12] R. L. Frank, E. H. Lieb, R. Seiringer, L. E. Thomas, *Bi-polaron and N-polaron binding energies*. Phys. Rev. Lett. **104** (2010), 210402.
- [13] R. L. Frank, E. H. Lieb, Spherical reflection positivity and the Hardy-Littlewood-Sobolev inequality. Contemp. Math., to appear.
- [14] R. L. Frank, B. Simon, Critical Lieb-Thirring Bounds in Gaps and the Generalized Nevai Conjecture for Finite Gap Jacobi Matrices. Duke Math. J., to appear
- [15] R. L. Frank, E. H. Lieb, R. Seiringer, Equivalence of Sobolev inequalities and Lieb-Thirring inequalities. In: XVIth International Congress on Mathematical Physics, Proceedings of the ICMP held in Prague, August 3-8, 2009, P. Exner (ed.), 523–535, World Scientific, Singapore, 2010.

- [16] R. L. Frank, A. Laptev, Inequalities between Dirichlet and Neumann eigenvalues on the Heisenberg group. Int. Math. Res. Not. 15 (2010), 2889–2902.
- [17] R. L. Frank, R. Seiringer, *Sharp fractional Hardy inequalities in half-spaces*. In: Around the research of Vladimir Maz'ya, A. Laptev (ed.), 161–167, International Mathematical Series **11** (2010).
- [18] R. L. Frank, E. H. Lieb, *Inversion positivity and the sharp Hardy-Littlewood-Sobolev inequality*. Calc. Var. Partial Differential Equations **39** (2010), no. 1 2, 85–99.
- [19] R. L. Frank, S. Morozov, S. Vugalter, Weakly coupled bound states of Pauli operators. Calc. Var. Partial Differential Equations 40 (2010), no. 1 2, 253–271.
- [20] R. L. Frank, A. Laptev, R. Seiringer, A sharp bound on eigenvalues of Schrödinger operators on the halfline with complex-valued potentials. In: Spectral Theory and Analysis, J. Janas et al. (eds.), 39 44, Oper. Theory Adv. Appl. 214, Birkhäuser, Basel, 2011.
- [21] R. L. Frank, Remarks on eigenvalue estimates and semigroup domination. In: Spectral and Scattering Theory for Quantum Magnetic Systems, P. Briet, et al. (eds.), 63–86, Contemp. Math. 500, Amer. Math. Soc., Providence, RI, 2009.
- [22] R. L. Frank, A note on low energy scattering for homogeneous long range potentials. Ann. Henri Poincaré 10 (2009), no. 3, 573–575.
- [23] R. L. Frank, A simple proof of Hardy-Lieb-Thirring inequalities. Comm. Math. Phys. **290** (2009), no. 2, 789–800.
- [24] J. Breuer, R. L. Frank, Singular spectrum for radial trees. Rev. Math. Phys. **21** (2009), no. 7, 1–17.
- [25] R. L. Frank, H. Siedentop, S. Warzel, *The energy of heavy atoms according to Brown and Ravenhall: the Scott correction.* Doc. Math. **14** (2009), 463–516.
- [26] R. L. Frank, R. Seiringer, Non-linear ground state representations and sharp Hardy inequalities. J. Funct. Anal. **255** (2008), 3407–3430.
- [27] R. L. Frank, A. Laptev, Spectral inequalities for Schrödinger operators with surface potentials. In: Spectral theory of differential operators, T. Suslina and D. Yafaev (eds.), 91–102, Amer. Math. Soc. Transl. Ser. 2, 225 (2008).
- [28] T. Ekholm, R. L. Frank, H. Kovarik, *Remarks about Hardy inequalities on metric trees*. In: Analysis on graphs and its applications, P. Exner, et al. (eds.), 369–379, Proc. Sympos. Pure Math. **77** (2008).

- [29] T. Ekholm, R. L. Frank, H. Kovarik, Eigenvalue estimates for Schrödinger operators on metric trees. Adv. in Math., to appear.
- [30] R. L. Frank, E. H. Lieb, R. Seiringer, Number of bound states of Schrödinger operators with matrix-valued potentials. Lett. Math. Phys. 82 (2007), 107–116.
- [31] R. L. Frank, M. Loss, T. Weidl, *Pólya's conjecture in the presence of a constant magnetic field.* J. Eur. Math. Soc. **11** (2009), 1365–1383.
- [32] R. L. Frank, B. Simon, T. Weidl, Eigenvalue bounds for perturbations of Schrödinger operators and Jacobi matrices with regular ground states. Comm. Math. Phys. **282** (2008), no. 1, 199–208.
- [33] R. L. Frank, A. Laptev, S. Molchanov, Eigenvalue estimates for magnetic Schrödinger operators in domains. Proc. Amer. Math. Soc. 136 (2008), no. 12, 4245–4255.
- [34] R. L. Frank, E. H. Lieb, R. Seiringer and H. Siedentop, Müller's exchange-correlation energy in density-matrix-functional theory. Phys. Rev. A **76** (2007), 052517.
- [35] R. D. Benguria, R. L. Frank, M. Loss, The sharp constant in the Hardy-Sobolev-Maz'ya inequality in the three dimensional upper half-space. Math. Res. Lett. **15** (2008), no. 4, 613–622.
- [36] R. L. Frank, C. Hainzl, S. Naboko and R. Seiringer, *The critical temperature in the BCS model at weak coupling*. J. Geom. Anal. **17** (2007), no. 4, 559–568.
- [37] R. L. Frank, H. Siedentop, S. Warzel, *The ground state energy of heavy atoms: relativistic lowering of the leading energy correction.* Comm. Math. Phys. **278** (2008), no. 2, 549–566.
- [38] R. L. Frank, A. M. Hansson Eigenvalue estimates for the Aharonov-Bohm operator in a domain. In: Methods of Spectral Analysis in Mathematical Physics, J. Janas, et al. (eds.), 35–50, Oper. Theory Adv. Appl. 186, Birkhäuser, Basel, 2008.
- [39] T. Ekholm, R. L. Frank, *Lieb-Thirring inequalities on the half-line with critical exponent*. J. Eur. Math. Soc. **10** (2008), no. 3, 739–755.
- [40] R. L. Frank, E. H. Lieb, R. Seiringer, Stability of relativistic matter with magnetic fields for nuclear charges up to the critical value. Comm. Math. Phys. **275** (2007), no. 2, 479–489.
- [41] R. L. Frank, E. H. Lieb, R. Seiringer, *Hardy-Lieb-Thirring inequalities* for fractional Schrödinger operators. J. Amer. Math. Soc. **21** (2008), no. 4, 925–950.

- [42] R. L. Frank, A. Laptev, E. H. Lieb, R. Seiringer, *Lieb-Thirring inequalities for Schrödinger operators with complex-valued potentials*. Lett. Math. Phys. **77** (2006), 309–316.
- [43] R. L. Frank, On the asymptotic number of edge states for magnetic Schrödinger operators. Proc. London Math. Soc. (3) **95** (2007), no. 1, 1–19.
- [44] T. Ekholm, R. L. Frank, On Lieb-Thirring inequalities for Schrödinger operators with virtual level. Comm. Math. Phys. **264** (2006), no. 3, 725–740.
- [45] P. Exner, R. L. Frank, Absolute continuity of the spectrum for periodically modulated leaky wires in  $\mathbb{R}^3$ . Ann. Henri Poincaré 8 (2007), no. 2, 241–263.
- [46] R. L. Frank, O. Safronov, Absolutely continuous spectrum of a class of random non-ergodic Schrödinger operators, Int. Math. Res. Not. (2005), no. 42, 2559–2577.
- [47] R. L. Frank, On the tunneling effect for magnetic Schrödinger operators in antidot lattices. Asymptot. Anal. 48 (2006), no. 1-2, 91–120.
- [48] R. L. Frank, R. G. Shterenberg, On the spectrum of partially periodic operators. In: Operator Theory, Analysis and Mathematical Physics, J. Janas, et al. (eds.), 35–50, Oper. Theory Adv. Appl. 174, Birkhäuser, Basel, 2007.
- [49] R. L. Frank, On the Laplacian in the halfspace with a periodic boundary condition. Ark. Mat. 44 (2006), no. 2, 277–298.
- [50] R. L. Frank, R. G. Shterenberg, On the scattering theory of the Laplacian with a periodic boundary condition. II. Additional channels of scattering, Doc. Math. 9 (2004), 57–77.
- [51] R. L. Frank, On the scattering theory of the Laplacian with a periodic boundary condition. I. Existence of wave operators. Doc. Math. 8 (2003), 547–565.

## INVITED CONFERENCE CONTRIBUTIONS

Nodal domain bounds for fractional Schrödinger operators and applications. Workshop 'Selected Topics in Spectral Theory', Vienna, ESI, Jan. 2011

Sharp constants in inequalities on the Heisenberg group, Conference 'Non-linear Phenomena', Taipei, Jan. 2011

Sharp constants in inequalities on the Heisenberg group, QMath 11, Hradec Kralove, Sept. 2010

Lieb-Thirring inequalities in gaps, Conference 'Spectral Theory', St. Petersburg, July 2010

Stability and absence of binding for multi-polaron systems, Conference 'The isoperimetric problem of Queen Dido', Carthage, May 2010

Reflection positivity and the sharp Hardy-Littlewood-Sobolev inequality, Workshop 'Optimal Constants in the Theory of Sobolev Spaces and PDEs', Oberwolfach, Feb. 2010

*Hardy-Lieb-Thirring inequalities*, International Congress on Mathematical Physics, Prague, Aug. 2009

Inversion positivity and sharp the sharp Hardy-Littlewood-Sobolev inequality, Workshop 'Selected Topics in Spectral Theory', Vienna, July 2009

The magnetic Pólya conjecture, Workshop 'Low Eigenvalues of Laplace and Schrödinger Operators', Oberwolfach, Feb. 2009

Lieb-Thirring and Hardy-Sobolev inequalities, European Congress of Mathematics, Amsterdam, July 2008

Semi-classical spectral estimates for magnetic Schrödinger operators, Conference 'Spectral and Scattering Theory for Quantum Magnetic Systems', Luminy, July 2008

Non-linear ground state representations and sharp Hardy inequalities, Conference 'Operator Theory, Analysis and Mathematical Physics' (OTAMP 2008), Bedlewo, June 2008

The magnetic Pólya conjecture, Workshop 'Spectral Theory and Partial Differential Equations', Vienna, Dec. 2007.

Hardy-Lieb-Thirring inequalities and the stability of relativistic matter, Danish Symposium on Applied Analysis, Copenhagen, Aug. 2007

Absolute continuity of the spectrum for periodically modulated leaky wires, Conference 'Waves 2007', Reading, July 2007

The relativistic Scott correction, 'Enigma Conference on Mathematical Physics', Stockholm, June 2007

The sharp constant in the Hardy-Littlewood-Maz'ya inequality in the three-dimensional upper half-space, Workshop 'Geometric inequalities', Banff, June 2007

Eigenvalue estimates for Schrödinger operators on regular metric trees, Workshop 'Quantum Graphs, their spectra and applications', Cambridge, April 2007

Hardy-Lieb-Thirring inequalities and the stability of relativistic matter, DMV Minisymposium 'Mathematical models of complex quantum systems', Berlin, March 2007

## Rupert L. Frank

On Lieb-Thirring-Hardy inequalities, Workshop 'Spectral Theory and its Applications', Cambridge, July 2006

On Lieb-Thirring-Hardy inequalities, Conference 'Operator Theory, Analysis and Mathematical Physics' (OTAMP 2006), Lund, June 2006

On Lieb-Thirring-Hardy inequalities, Workshop 'Low eigenvalues of Laplace and Schrödinger operators', Palo Alto, May 2006

On Lieb-Thirring inequalities for Schrödinger operators with virtual level, Network meeting 'Analysis and Quantum', Vienna, March 2006

On Lieb-Thirring inequalities for Schrödinger operators with virtual level, Workshop 'Spectral Theory of PDE', Stockholm, Sep. 2005

On the Laplacian on the halfplane with a periodic boundary condition, Workshop 'Spectral analysis of partial differential equations', Oberwolfach, Dec. 2004

On the Laplacian on the halfplane with a periodic boundary condition, Conference 'Operator Theory, Analysis and Mathematical Physics' (OTAMP 2004), Bedlewo, July 2004

On the Laplacian on the halfplane with a periodic boundary condition, Network meeting 'Analysis and Quantum', Vienna, Dec. 2003

# INVITED SEMINAR TALKS

Sharp constants in inequalities on the Heisenberg group, Rutgers, March 2011

Critical Lieb-Thirring inequalities in gaps, Munich, LMU, Dec. 2010

Stability and absence of binding for multi-polaron systems, Munich, LMU, July 2010

Lieb-Thirring inequalities in gaps, Munich, LMU, July 2010

Stability and absence of binding for multi-polaron systems, Jerusalem, July 2010

Stability and absence of binding for multi-polaron systems, Leipzig, MPI, June  $2010\,$ 

Reflection positivity and the sharp Hardy-Littlewood-Sobolev inequality, Pasadena, Caltech, Jan. 2010

Reflection positivity and the sharp Hardy-Littlewood-Sobolev inequality, Munich, LMU, Dec. 2009

Uniqueness and symmetry of ground states for the  $L^2$ -critical boson star equation, Princeton, Oct. 2009

Hardy-Lieb-Thirring inequalities and the stability of matter, Munich, TU, June 2009

Sobolev and Hardy inequalities, Stuttgart, June 2009

Sobolev and Hardy inequalities, Munich, LMU, Feb. 2009

Non-linear ground state representations and sharp Hardy inequalities, Paris, Dauphine, Jan. 2009

Hardy-Lieb-Thirring inequalities, Munich, LMU, Dez. 2008

Non-linear ground state representations and sharp Hardy inequalities, Munich, TU, Dez. 2008

The sharp constant in the Hardy-Sobolev-Maz'ya inequality, Houston, Rice University, Okt. 2008

Hardy-Sobolev and Lieb-Thirring inequalities, Houston, Rice University, Okt. 2008

The sharp constant in the Hardy-Sobolev-Maz'ya inequality, Cambridge, MA, MIT, Oct. 2008

*Hardy-Sobolev and Lieb-Thirring inequalities*, New Brunswick, Rutgers, Sep. 2008

Non-linear ground state representations and sharp Hardy inequalities, Munich, June 2008

Non-linear ground state representations and sharp Hardy inequalities, Pasadena, Caltech, March 2008

The magnetic Pólya conjecture, Munich, Dec. 2007

Stability of matter and Lieb-Thirring inequalities in the magnetic case, New York, Courant Institute, Dec. 2007

Hardy-Sobolev and Lieb-Thirring inequalities, Princeton, Oct. 2007

The Pólya's conjecture in the presence of a magnetic field, Stuttgart, July 2007

Hardy-Sobolev and Lieb-Thirring inequalities, Munich, May 2007

Hardy-Sobolev and Lieb-Thirring inequalities, Birmingham, March 2007

The relativistic Scott correction, Birmingham, March 2007

Hardy-Sobolev and Lieb-Thirring inequalities, Atlanta, Feb. 2007

 $Semi\mbox{-}classical\ spectral\ estimates\ for\ magnetic\ Laplacians,\ Stockholm,\ Jan.\ 2007$ 

 $Semi\mbox{-}classical\ spectral\ estimates\ for\ magnetic\ Laplacians,\ Grenoble,\ Jan.\ 2007$ 

On Lieb-Thirring-Hardy inequalities and the stability of relativistic matter, Davis, Nov. 2006

On Lieb-Thirring-Hardy inequalities and the stability of relativistic matter, Atlanta, Nov. 2006

On Lieb-Thirring inequalities for surface potentials, Stuttgart, Sep. 2006

On Lieb-Thirring-Hardy inequalities for relativistic Schrödinger operators, Munich, July 2006

On the asymptotic number of edge states for magnetic Schrödinger operators, Paris, Feb. 2006

On Lieb-Thirring inequalities for Schrödinger operators with virtual level, Munich, Dec. 2005

On the asymptotic number of edge states for magnetic Schrödinger operators, Stockholm, Oct. 2005

Absolutely continuous spectrum for non-ergodic random operators, Stockholm, June 2005

On Lieb-Thirring inequalities for Schrödinger operators with virtual level, Prague, April 2005

On the absolute continuity of the spectrum of partially periodic operators, Prague, April 2005

On the magnetic Schrödinger operator in an antidot lattice and applications to superconductivity, Paris, Nov. 2004

On the tunneling effect for magnetic Schrödinger operators in antidot lattices, Paris, June 2004

On the Laplacian on the halfplane with a periodic boundary condition, Paris, May 2004

On the Laplacian on the halfplane with a periodic boundary condition, Munich, July 2003