Mathematisches Institut
der Universität München
Prof. Otto Forster

SoSe 2024
April 24, 2024

Selected Topics from Number Theory

Problem sheet #2

Problem 5 Let $x = \mathrm{cfrac}(a_0, a_1, a_2, a_3, \ldots)$ be the CF expansion of an irrational number x and let

$$\frac{p_n}{q_n} = \mathrm{cfrac}(a_0, a_1, a_2, \ldots, a_n), \quad n \geqslant 0$$

be its convergents. Prove

$$\left| x - \frac{p_n}{q_n} \right| > \frac{1}{q_n(q_n + q_{n+1})}.$$

Problem 6 Let $\mathrm{cfrac}(a_0, a_1, a_2, \ldots)$ be an infinite continued fraction with $a_\nu \in \mathbb{R}$ (not necessarily integers), $a_\nu > 0$ for $\nu \geqslant 1$. Prove that the continued fraction converges iff

$$\sum_{\nu=1}^{\infty} a_\nu = \infty.$$

Problem 7 Let A be the set of all irrational numbers $0 < x < 1$ whose decimal expansion

$$x = \sum_{\nu=1}^{\infty} c_\nu 10^{-\nu}$$

satisfies $c_\nu \neq 9$ for all ν. Prove that A has Lebesgue measure 0.

Problem 8 Let M be the set of all positive integers m whose decimal representation

$$m = \sum_{\nu=0}^{N} c_\nu 10^{\nu}$$

satisfies $c_\nu \neq 9$ for all ν. Prove that

$$\sum_{m \in M} \frac{1}{m} < \infty.$$

These problems will be discussed Wednesday, May 8, 2024, 16-18 h