

Selected Topics from Number Theory
Problem sheet #1

Problem 1 Calculate the CF (= continued fraction) expansion of the numbers

$$x_1 := \sqrt{7}, \quad x_2 := \frac{\sqrt{7}}{2}, \quad x_3 := \frac{\sqrt{7}}{3}.$$

Problem 2 Let k be a natural number. Derive formulas for the CF expansions of the numbers

- a) $\sqrt{k^2 + 1}, \quad (k \geq 1)$
- b) $\sqrt{k^2 - 1}, \quad (k \geq 2),$
- c) $\sqrt{k^2 + 2}, \quad (k \geq 1),$
- d) $\sqrt{k^2 - 2}, \quad (k \geq 3).$

Problem 3 Let k be a natural number. Derive formulas for the CF expansions of the numbers

- a) $\sqrt{k^2 + 4}, \quad (k \geq 2),$
- b) $\sqrt{k^2 - 4}, \quad (k \geq 5).$

Hint. Distinguish the cases k even and k odd.

Problem 4 Let

$$x = a_0 + \cfrac{1}{a_1} + \cfrac{1}{a_2} + \cfrac{1}{a_3} + \dots =: \text{cfac}(a_0, a_1, a_2, a_3, \dots)$$

be the CF expansion of an irrational real number x . Determine the CF expansion of $-x$.

These problems will be discussed Wednesday, April 24, 2024, 16-18 h