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Riemann Surfaces

Problem sheet #5

Problem 17

Let $X_1 := \mathbb{C}/\Lambda_1$ and $X_2 := \mathbb{C}/\Lambda_2$ be two tori and $f\colon X_1 \to X_2$ be a non-constant holomorphic map with $f(0) = 0$.

a) Prove that there exists a constant $\alpha \in \mathbb{C}^*$ such that $\alpha\Lambda_1 \subset \Lambda_2$ and

$$f(z \bmod \Lambda_1) = \alpha z \bmod \Lambda_2 \quad \text{for all } z \in \mathbb{C}.$$

b) Show that f is an unbranched covering map and the number of sheets equals the index $[\Lambda_2 : \alpha\Lambda_1]$.

Problem 18

a) For the torus $E_i := \mathbb{C}/(\mathbb{Z} + \mathbb{Z}i)$ construct a two-sheetet holomorphic covering map $p\colon E_i \to E_i$ of the torus onto itself.

b) Does there exist a two-sheeted holomorphic covering map $f : E_\varrho \to E_\varrho$ for the torus $E_\varrho = \mathbb{C}/(\mathbb{Z} + \mathbb{Z}\varrho)$ where $\varrho = e^{2\pi i/3}$?

Problem 19

Let $p : X_3 \to \mathbb{P}_1$ be the Riemann surface of $\sqrt[3]{1 - z^3}$, i.e. of the algebraic function defined by the polynomial

$$w^3 + z^3 - 1 \in \mathcal{M}(\mathbb{P}_1)[w], \qquad \mathcal{M}(\mathbb{P}_1) \cong \mathbb{C}(z).$$

Determine all branch points and critical values of p and show that there are no branch points above $\infty \in \mathbb{P}_1$.

Problem 20

On the compact Riemann surface X_3 defined in Problem 19 consider the function $f := w+z$.

a) Determine the poles and zeros of f.

b) Calculate the elementary symmetric functions of f with respect to $p : X_3 \to \mathbb{P}_1$.
