Mathematisches Institut
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Riemann Surfaces

Problem sheet #11

Problem 41

On a Riemann surface X let $\mathfrak{D}$ be the sheaf of divisors, i.e., for $U \subset X$ open, $\mathfrak{D}(U)$ consists of all maps $D : U \to \mathbb{Z}$ such that for every compact set $K \subset U$ there are only finitely many $x \in K$ with $D(x) \neq 0$.

a) Show that $\mathfrak{D}$, together with the natural restriction morphisms, is actually a sheaf and there is a short exact sequence of sheaves

$$0 \to \mathcal{O}^* \to \mathcal{M}^* \to \mathfrak{D} \to 0.$$

b) Prove that

$$H^1(X, \mathfrak{D}) = 0.$$

Problem 42

Let X be a compact Riemann surface of genus $g > 0$. Prove that for every point $a \in X$ there exists a holomorphic 1-form $\omega \in \Omega(X)$ with $\omega(a) \neq 0$.

Problem 43

Let X be a compact Riemann surface of genus two. Let (ω_1, ω_2) be a basis of $H^0(X, \Omega)$ and define $f \in \mathcal{M}(X)$ by $\omega_1 = f\omega_2$. Show that $f : X \to \mathbb{P}^1$ is a 2-sheeted (branched) covering map.

Problem 44

Let $p : X \to \mathbb{P}^1$ be the Riemann surface of $\sqrt[n]{1 - z^n}$, i.e., of the algebraic function defined by the polynomial

$$w^n + z^n - 1 \in \mathcal{M}(\mathbb{P}^1)[w], \qquad \mathcal{M}(\mathbb{P}^1) \cong \mathbb{C}(z).$$

Prove that the genus of X is

$$g = \frac{(n-1)(n-2)}{2}.$$
