Mathematisches Institut
der Universität München
Prof. Otto Forster

WS 2012/13
November 23, 2012

Riemann Surfaces

Problem sheet #6

Problem 21

a) Prove that the differential form $\omega := \dfrac{dz}{1+z^2}$, which is holomorphic on $\mathbb{C} \smallsetminus \{\pm i\}$, can be holomorphically extended to $\mathbb{P}_1 \smallsetminus \{\pm i\}$.

b) Let $p := \tan : \mathbb{C} \to \mathbb{P}_1 \smallsetminus \{\pm i\}$ be the covering map of Problem 10b). Find $p^*\omega$.

Problem 22

Let $F : X \to S$ be a non-constant holomorphic mapping of Riemann surfaces, $b \in X$ a branch point of multiplicity k and $a := F(b)$. Let ω be a meromorphic 1-form on S. Prove that

$$\operatorname{Res}_b(F^*\omega) = k \operatorname{Res}_a(\omega).$$

Problem 23

Let $\Lambda = \mathbb{Z}\omega_1 + \mathbb{Z}\omega_2 \subset \mathbb{C}$ be a lattice (ω_1,ω_2 linearly independent over $\mathbb{R}$) and $X := \mathbb{C}/\Lambda$. It is well-known that the fundamental group $\pi_1(X)$ is a free abelian group with two generators which may be represented by the closed curves $t \mapsto p(t\omega_\nu)$, $0 \leqslant t \leqslant 1$, $\nu = 1, 2$, where $p : \mathbb{C} \to \mathbb{C}/\Lambda$ is the canonical projection.

Prove that for every homomorphism $a : \pi_1(X) \to \mathbb{C}$ there exists a closed differential form $\sigma \in \mathcal{E}^{(1)}(X)$ (not necessarily holomorphic) whose period homomorphism is equal to a.

Problem 24

a) Show that on $\mathbb{P}_1$ every holomorphic differential 1-form is identically zero.

b) Let $X = \mathbb{C}/\Lambda$ be a torus. Prove that every holomorphic map $F : \mathbb{P}_1 \to X$ is constant.
Hint. Use part a)

Due: Wednesday, December 5, 2012, 15 h