Mathematisches Institut
der Universität München
Prof. Otto Forster

WS 2012/13
November 9, 2012

Riemann Surfaces

Problem sheet #4

Problem 13

Determine all branch points of the map $\cos\colon \mathbb{C} \to \mathbb{C}$ and prove that

$$\cos : \mathbb{C} \setminus \mathbb{Z}\pi \longrightarrow \mathbb{C} \setminus \{\pm 1\}$$

is an unbranched covering map.

Problem 14 (Continuation of Problem 13)

Define the closed curves $\alpha, \beta : [0,1] \to \mathbb{C} \setminus \{\pm 1\}$ by

$$\alpha(t) := 1 - e^{2\pi i t}, \quad \beta(t) := -1 + e^{2\pi i t}.$$

a) Let $\widehat{\alpha}, \widehat{\beta} : [0,1] \to \mathbb{C} \setminus \mathbb{Z}\pi$ be the liftings of α, β with initial point $\widehat{\alpha}(0) = \widehat{\beta}(0) = \pi/2$. Determine the end points $a := \widehat{\alpha}(1)$ and $b := \widehat{\beta}(1)$.

b) Let $\widehat{\alpha}_1 : [0,1] \to \mathbb{C} \setminus \mathbb{Z}\pi$ be the lifting of α with $\widehat{\alpha}_1(0) = b$ and $\widehat{\beta}_1 : [0,1] \to \mathbb{C} \setminus \mathbb{Z}\pi$ be the lifting of β with $\widehat{\beta}_1(0) = a$. Determine the end points $\widehat{\alpha}_1(1)$ and $\widehat{\beta}_1(1)$.

Problem 15

Let X be a compact Riemann surface and $p_1, \ldots, p_n \in X$. Set

$$X' := X \setminus \{p_1, \ldots, p_n\}.$$

Show that every automorphism of X' (i.e. biholomorphic map onto itself) extends to an automorphism of X.

Problem 16

a) Determine all automorphisms of $\mathbb{C}^*$.

b) Determine all automorphisms of $X := \mathbb{C} \setminus \{0,1\}$ and show that they form a group isomorphic to the symmetric group S_3 (group of permutations of three elements).

c) Let $X_\lambda := \mathbb{C} \setminus \{0, 1, \lambda\}$, where $\lambda \in \mathbb{C}$, $\lambda \neq 0, 1$. Determine the group of automorphisms of X_λ (as a function of λ).

Hint. Use problem 15.

Due: Wednesday, November 21, 2012, 15 h