MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
Prof. Otto Forster

WS 2004/2005
Nov. 25, 2004

Algebraic Number Theory

Problem Sheet #6

Problem 21

Calculate the values of the following periodic continued fractions:

$$\begin{aligned} x &= \operatorname{cfrac}(1, \overline{1}\,), \\ y &= \operatorname{cfrac}(0, \overline{1, 2, 3, 4}\,). \end{aligned}$$

Hint: x satisfies the equation $x = 1 + \frac{1}{x}$. Find a similar equation for y.

Problem 22

Calculate the continued fraction expansions of $\sqrt{n^2+1}$ and $\sqrt{n^2-1}$, where n is a positive integer ($n > 1$ in the second case).

Problem 23

Let u_k/v_k and u_{k+1}/v_{k+1} be two consecutive convergents of the continued fraction expansion of an irrational number $\theta \in \mathbb{R} \setminus \mathbb{Q}$. Show that at least one of them satisfies the estimate

$$\left|\theta - \frac{u_i}{v_i}\right| < \frac{1}{2v_i^2}, \qquad i = k \text{ or } k+1.$$

Problem 24

Let $\mathfrak{o}_K$ be the ring of integers in a real quadratic number field K.

a) Complete the prove given in the course for the structure of the group of units in $\mathfrak{o}_K$ by showing:

There exists a $\delta > 0$ such that for every unit $u \in \mathfrak{o}_K^*$ with $u > 1$ and $N(u) = -1$ one has $u \geq 1 + \delta$.

b) Suppose there exists a unit u with $N(u) = -1$. Show that also the fundamental unit ε satisfies $N(\varepsilon) = -1$.

Due: Tuesday, December 7, 2004, 14:10 h