MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
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WS 2004/2005
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Algebraic Number Theory

Problem Sheet #1

Problem 1 Let p, q be two different prime numbers. Show that

$$\mathbb{Q}(\sqrt{p}, \sqrt{q}) = \mathbb{Q}(\sqrt{p} + \sqrt{q})$$

and express $\sqrt{p}$ as a polynomial in $x := \sqrt{p} + \sqrt{q}$ with rational coefficients.

Problem 2

a) Show that

$$x := 2\cos\frac{2\pi}{5}$$

is an algebraic integer and determine its minimal polynomial.

Hint: Use $x = e^{2\pi i/5} + e^{-2\pi i/5}$.

b) Deduce a construction of the regular pentagon with ruler and compass.

Problem 3

a) Show that

$$z := 2\cos\frac{2\pi}{7}$$

is an algebraic integer and determine its minimal polynomial.

b) Express the numbers

$$z_1 := \frac{1}{z}, \quad z_2 := 2\cos\frac{4\pi}{7} \quad \text{and} \quad z_3 := 2\cos\frac{6\pi}{7}$$

as polynomials in z with rational coefficients.

Problem 4

a) Calculate the greatest common divisor of 13 and $8 + i$ in the ring $\mathbb{Z}[i]$.

b) Show that this greatest common divisor is a prime element of $\mathbb{Z}[i]$.

Due: Thursday, October 28, 2004