Bohmian Mechanics and the Meaning of the Wave Function

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1 Introduction

Despite its extraordinary predictive successes, quantum mechanics has, since its inception some seventy years ago, been plagued by conceptual difficulties. Few physicists have done more than Abner Shimony to remind us of this somewhat unpleasant fact. The most commonly cited of these difficulties is the measurement problem, or, what amounts to more or less the same thing, the paradox of Schrödinger's cat. Indeed, for many physicists the measurement problem is not merely one of the conceptual difficulties of quantum mechanics; it is *the* conceptual difficulty.

While we have a good deal of sympathy for this view, we believe that the measurement problem is merely a manifestation of a more fundamental conceptual inadequacy: It is far from clear just what it is that quantum mechanics is about. What, in fact, does quantum mechanics describe? Many physicists pay lip service to the Copenhagen interpretation, and in particular to the notion that quantum mechanics is about results of measurement. But hardly anybody truly believes this anymore—and it is hard to believe anyone really ever did. It seems clear now to any student of the subject that quantum mechanics is fundamentally about atoms and electrons, quarks and strings, and not primarily about those particular macroscopic regularities associated with what we call measurements.

It is, however, generally agreed that any quantum mechanical system—whether of atoms or electrons or quarks or strings—is completely described by its wave function, so that it is also widely accepted that quantum mechanics is fundamentally about the behavior of wave functions. The measurement problem provides a dramatic demonstration of the severe difficulty one faces in attempting to maintain this view.

We have argued elsewhere [6] that if one focuses directly on the question as to what quantum mechanics is about, one is naturally led to the view that quantum mechanics is fundamentally about the behavior of particles, described by their positions—or fields, described by field configurations, or strings, described by string configurations—and only secondarily about the behavior of wave functions. We are led to the view that the wave function does not in fact provide a complete description or representation of a quantum system and that the complete description of the system is provided by the configuration Q defined by the positions $\mathbf{Q}_k$ of its particles together with its wave function. We are led in fact, for a nonrelativistic system of particles, to Bohmian mechanics, for which the *state* of the system is (Q, ψ), which evolves according to the equations of motion

$$\frac{dQ}{dt} = \mathrm{Im}\frac{\nabla\psi}{\psi}(Q), \tag{1}$$

where ∇ is a configuration-space gradient, and

$$i\frac{\partial\psi}{\partial t} = H\psi, \tag{2}$$

where H is the Schrödinger Hamiltonian. This deterministic theory of particles in motion, with trivial modifications to deal with spin, completely accounts for all the phenomena of nonrelativistic quantum mechanics, from spectral lines to interference effects, and it does so in a completely ordinary manner. It was first presented, in a somewhat more complicated but completely equivalent form, by David Bohm more than forty years ago [3]. Moreover, a preliminary version of this theory was presented by de Broglie almost at the inception of quantum mechanics. Its principal advocate for the past three decades was John Bell [1].

We will here outline how Bohmian mechanics works: how it deals with various issues in the foundations of quantum mechanics and how it is related to the usual quantum formalism. We will then turn to some objections to Bohmian mechanics, raised perhaps most forcefully by Abner Shimony. These objections will lead us to our main concern: a more careful consideration of the meaning of the wave function in quantum mechanics as suggested by a Bohmian perspective. We wish now to emphasize, however, that a grasp of the meaning of the wave function as a representation of a quantum system is crucial to achieving a genuine understanding of quantum mechanics from any perspective.

2 The Measurement Problem

Suppose that we analyze the process of measurement in quantum mechanical terms. The after-measurement wave function for system and apparatus arising from Schrödinger's equation for the composite system typically involves a superposition over terms corresponding to what we would like to regard as the various possible results of the measurement—e.g., different pointer orientations. Since it seems rather important that the actual result of the measurement be a part of the description of the after-measurement situation, it is difficult to see how this wave function could be the complete description of this situation. By contrast, with a theory or interpretation like Bohmian mechanics in which the description of the after-measurement situation includes, in addition to the wave function, at least the values of the variables that register the result, the measurement problem vanishes.

The remaining problem of then justifying the use of the "collapsed" wave function—corresponding to the actual result—in place of the original one is often confused with the measurement problem. The justification for this replacement is nowadays frequently expressed in terms of decoherence. One of the best descriptions of the mechanisms of decoherence, though not the word itself, was given by Bohm in 1952 [3] as part of his explanation of why from the perspective of Bohmian mechanics this replacement is justified as a practical matter. (See also [6].)

Moreover, if we focus on what should be regarded as the wave function, not of the composite of system and apparatus, which strictly speaking remains a superposition if the composite is treated as closed during the measurement process, but of the system itself, we find that for Bohmian mechanics this does indeed collapse, precisely as described by the quantum formalism. The key element here is the notion of the conditional wave function of a subsystem of a larger system, described briefly in section 7 below, and discussed in some detail, together with the related notion of the effective wave function, in [6].

3 The Two-Slit Experiment

Bohmian mechanics resolves the dilemma of the appearance, in one and the same phenomenon, of both particle and wave properties in a rather trivial manner: Bohmian mechanics is a theory of motion describing a particle (or particles) guided by a wave. For example, in Figure 1 we have a family of Bohmian trajectories for the two-slit experiment. Notice that while each trajectory passes through but one of the slits, the wave passes through both, and the interference profile that therefore develops in the wave generates a similar pattern in the trajectories guided by this wave.

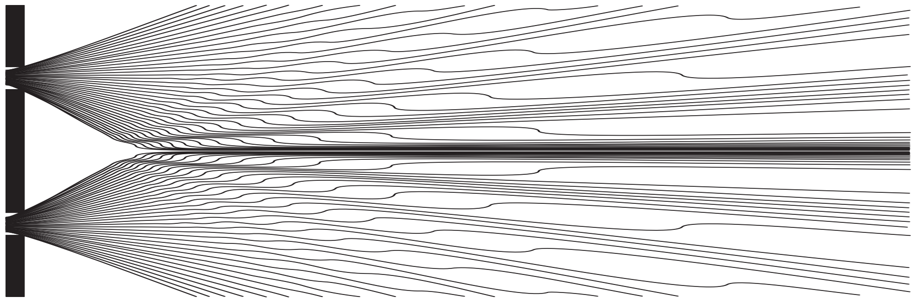

Figure 1: An ensemble of trajectories for the two-slit experiment, uniform in the slits. (Drawn by G. Bauer from [8].)

4 The Detailed Equations and Nonlocality

We have given, in (1) and (2), the equations of Bohmian mechanics in a somewhat schematic form, without explicitly exhibiting the parameters required for a detailed specification of the theory. Less schematically, the equations defining Bohmian mechanics for an N-particle universe of spinless particles with masses m_k interacting via the potential energy function $V = V(q)$ are

$$\frac{d\mathbf{Q}_k}{dt} = \mathbf{v}_k^{\psi}(\mathbf{Q}_1, \dots, \mathbf{Q}_N) \equiv \frac{\hbar}{m_k}\mathrm{Im}\frac{\nabla_{\mathbf{q}_k}\psi}{\psi}(\mathbf{Q_1}, \dots, \mathbf{Q_N}) \tag{3}$$

$$i\hbar\frac{\partial\psi}{\partial t} = -\sum_{k=1}^{N}\frac{\hbar^2}{2m_k}\nabla_{\mathbf{q}_k}^2\psi + V\psi \tag{4}$$

We have given these more detailed equations here in order to emphasize two points. First of all, Bohmian mechanics is manifestly nonlocal, since the velocity of any one of the particles, as expressed in (3), will typically depend upon the positions of the other particles. Thus does Bohmian mechanics make manifest that most dramatic effect of quantum theory, quantum nonlocality, that Abner Shimony has so effectively expounded. As John Bell [1, page 115] has stressed,

> That the guiding wave, in the general case, propagates not in ordinary three-space but in a multidimensional-configuration space is the origin of the notorious 'nonlocality' of quantum mechanics. It is a merit of the de Broglie-Bohm version to bring this out so explicitly that it cannot be ignored. (*Bell* 1980)

Second, we wish to emphasize that a Bohmian universe with potential V is completely specified by these two equations. Whatever is true of such a universe must be so merely by virtue of these equations, without the addition of further postulates such as, for example, an axiom governing the results of momentum measurements. And thus it is with regard to probability.

5 Probability

According to the quantum formalism, the probability density for finding a system whose wave function is ψ at the configuration q is $|\psi(q)|^2$. To the extent that the results of measurement are registered configurationally, at least potentially, it follows that the predictions of Bohmian mechanics for the results of measurement must agree with those of orthodox quantum theory (assuming the same Schrödinger equation for both) provided that it is somehow true for Bohmian mechanics that configurations are random, with distribution given by the *quantum equilibrium* distribution $|\psi|^2$. Now the status and justification of this quantum equilibrium hypothesis is a rather delicate matter, one that we have explored in considerable detail elsewhere [6]. We would like to mention here but a few relevant points.

It is nowadays a rather familiar fact that dynamical systems quite generally give rise to behavior of a statistical character, with the statistics given by the (or a) stationary probability distribution for the dynamics. So it is with Bohmian mechanics, except that for the Bohmian system stationarity is not quite the right concept, and it is rather the notion of *equivariance* that is relevant. We say that a probability distribution ρ^ψ on configuration space, depending upon the wave function ψ, is equivariant if

$$\left(\rho^\psi\right)_t = \rho^{\psi_t} \tag{5}$$

where the dependence on t on the right arises from Schrödinger's equation and on the left from the evolution on probability densities arising from the flow (1). Thus equivariance expresses the mutual compatibility, relative to ρ^ψ, of the Schrödinger evolution (2) and the Bohmian motion (1).

Now the crucial point is that $\rho^\psi = |\psi|^2$ is equivariant, a more or less immediate consequence of the elementary fact that the quantum probability current $J^\psi = \rho^\psi v^\psi$, where v^ψ is the r.h.s. of (1). We thus have that

$$\rho_{t_0}(q) = |\psi_{t_0}(q)|^2 \text{ at some time } t_0 \Longrightarrow \\ \rho_t(q) = |\psi_t(q)|^2 \text{ for all } t$$

It is perhaps helpful, in trying to understand the status in Bohmian mechanics of the quantum equilibrium distribution, to think of

$$\textit{quantum equilibrium} \qquad \rho = |\psi|^2 \tag{6}$$

as roughly analogous to (classical)

$$\textit{thermodynamic equilibrium} \qquad \rho \sim e^{-\beta H_{\text{class}}} \tag{7}$$

6 Operators as Observables

It would appear that inasmuch as orthodox quantum theory supplies us with probabilities not merely for positions but for a huge class of quantum observables, it is a much richer theory than Bohmian mechanics, which seems exclusively concerned with positions. Appearances would, however, be misleading. In this regard, as with so much else in the foundations of quantum mechanics, the crucial observation has been made by Bell [1, page 166]:

> ...in physics the only observations we must consider are position observations, if only the positions of instrument pointers. It is a great merit of the de Broglie-Bohm picture to force us to consider this fact. If you make axioms, rather than definitions and theorems, about the 'measurement' of anything else, then you commit redundancy and risk inconsistency. (*Bell* 1982)

Now when it comes to "definitions and theorems" we find [4] that Bohmian mechanics leads to a natural association between an experiment $\mathcal{E}$ and a "generalized observable" defined by a Positive-Operator-Valued measure or POV [5] $O(dz)$ (on the value space for the result of the experiment)

$$\mathcal{E} \mapsto O(dz) \tag{8}$$

This association is such that the probability distribution $\mu_Z^\psi(dz)$ of the result Z of the experiment, when performed upon a system with wave function ψ, is given by

$$\mu_Z^\psi(dz) = \langle \psi, O(dz)\psi \rangle \tag{9}$$

The simplest instance of a POV is a standard quantum observable, corresponding to a self-adjoint operator A on the Hilbert space of "states." We find that more or less every "measurement-like" experiment $\mathcal{M}$ is associated with this special kind of POV

$$\mathcal{E} = \mathcal{M} \mapsto A \tag{10}$$

and we thus recover the familiar measurement axiom that the statistics for the result of the "measurement of the observable A" are given by the spectral measure for A relative to ψ.

Moreover, the conclusion (8) is basically an immediate consequence of the very meaning of an experiment from a Bohmian perspective: a coupling of system to apparatus leading after a time t to a result $Z = F(Q_t)$ that is a function of the final configuration

Q_t of system and apparatus, e.g., the orientation of a pointer on the apparatus. It follows that the experiment $\mathcal{E}$ defines the following sequence of maps

$$\psi \mapsto \Psi = \psi \otimes \Phi_0 \mapsto \Psi_t = e^{-iHt}\Psi \mapsto \mu(dq) = \Psi_t^* \Psi_t dq \mapsto \mu_Z(dz) := \mu(F^{-1}(dz)),$$

from the initial wave function of the system, to the initial wave function of system and apparatus, to the final wave function of system and apparatus, to the distribution of the final configuration of the system and apparatus, to the distribution of the result. Thus the map

$$\psi \mapsto \mu_Z^{\psi} \tag{11}$$

is bilinear, since each of the maps in the sequence is linear except for the map to the quantum equilibrium distribution, which is bilinear. Such a bilinear map (11) is equivalent to a POV.

7 The Wave Function of a Subsystem

The existence of configurations in Bohmian mechanics as part of the reality leads, naturally enough, to many advantages over the orthodox view that the wave function provides us with a complete description of a physical system. One of these advantages is that it permits a clear and natural notion for the wave function of a subsystem of a larger system, say the universe, a notion that from an orthodox perspective is surprisingly problematical. Indeed, if we insist that the wave function is everything, it is not at all clear what, in fact, is to be meant by the wave function of anything that is directly of interest.

Let Ψ_t be the wave function of the universe (at time t), and decompose the configuration of the universe $Q = (X, Y)$ into the configuration X of the system of interest, the x-system, and the configuration Y of the environment of the x-system, i.e., the configuration of the rest of the universe. Then we define the *conditional wave function* of the x-system at time t by

$$\psi_t(x) = \Psi_t(x, Y). \tag{12}$$

This turns out to be just the right notion for the wave function of a subsystem. Moreover, under appropriate conditions it satisfies Schrödinger's equation for the x-system and is indeed the *effective wave function* of the x-system. See [6] for details.

8 The Role of the Wave Function

In this brief section we wish to emphasize one simple point about the structure of Bohmian mechanics: that this theory of motion is a *first-order theory*, in which it is the first derivative of the configuration with respect to time, rather than the second, that the theory directly specifies. And the role of the wave function in this theory, expressed by the association

$$\Psi \mapsto v^{\Psi}, \tag{13}$$

is to generate the vector field, given by the right hand side of (3), that defines the motion.

9 Quantum Cosmology

Quantum cosmology is an embarrassment for the orthodox interpretation of quantum mechanics as concerning merely the results of measurement—by an external observer. When it is the entire universe with which we are concerned, there would seem to be no room for such an observer. For Bohmian mechanics, by contrast, there is no difficulty whatsoever on this score.

Moreover, there is another difficulty in quantum cosmology that Bohmian mechanics greatly alleviates. The wave function Ψ of the universe, as given by a solution of the Wheeler-de Witt equation, which we may schematically represent by

$$\mathcal{H}\Psi = 0, \tag{14}$$

is stationary, and one must thus address the problem of accounting for the emergence of change in a universe whose wave function is timeless. Now for Bohmian mechanics we have no such difficulty, since a timeless wave function can easily generate a nontrivial dynamics.

It is true that for Bohmian mechanics as defined by (1) and (2), the ground state wave function, because it may be taken to be real, generates the trivial motion. However, this will not be true for the generic stationary state. More important, when we contemplate a Bohmian mechanics for quantum cosmology, we do not have in mind any particular form for the right hand side of (1) and in particular it need not be the case for a Bohmian mechanics understood in this general sense—what we have called elsewhere a Bohmian theory [7]—that a ground state wave function generates the trivial motion.

10 Important Criticisms

The most serious problem with Bohmian mechanics, (3) and (4), is that it manifestly fails to be Lorentz invariant. We have little to say about this very important issue here, beyond reminding our readers that nonlocality is an established fact that poses a challenge, not just for a Bohmian theory, but for any precise version of quantum theory. (For some steps in the direction of the formulation of a Lorentz invariant Bohmian theory, as well as some reflections on the problem of Lorentz invariance, see [2].)

We wish to focus here upon two objections. First of all, as has been emphasized by Abner Shimony, Bohmian mechanics violates the action-reaction principle that is central to all of modern physics, both classical and (non-Bohmian) quantum: There is no back action of the configuration upon the wave function, which evolves, autonomously, according to Schrödinger's equation,

$$\Psi \longrightarrow Q \qquad \text{but} \qquad Q \stackrel{\text{not}}{\longrightarrow} \Psi \tag{15}$$

Second of all, the wave function

$$\Psi = \Psi(\mathbf{q_1}, \ldots, \mathbf{q_N}), \tag{16}$$

which is part of the state description of—and hence presumably part of the reality comprising—a Bohmian universe, is not the usual sort of physical field on physical space to which we are accustomed, but a field on the abstract space of all possible configurations, a space of enormous dimension, a space constructed, it would seem, by physicists as a matter of convenience.

11 Some Responses

Perhaps the simplest response we might make is: So what? That's just the way it is, the way world works. Bohmian mechanics is well defined, and who are we—as Bohr once asked of Einstein, though for a slightly different purpose—to tell God what kinds of structures to use in creating a world.

We might also respond that in classical physics the action-reaction principle is more or less an expression of conservation of momentum, which is itself an expression of Galilean invariance (more precisely, of translation invariance). Most physicists would also say the same thing concerning quantum mechanics. However, in Bohmian mechanics, because it is a first-order theory, we are able to achieve Galilean invariance despite the no-back-action. In other words, Bohmian mechanics is based on a fundamentally different sort of structure than classical mechanics, one that does not require the action-reaction principle to achieve the desired underlying symmetry.

It might also be mentioned that the wave function of a subsystem, the conditional wave function (12), will in general be affected by the configuration, via its dependence upon the configuration of the environment.

However, we think that these responses don't go far enough. We think that the problems just mentioned suggest that we give more careful consideration to just what sort of entity the wave function is and how it should be regarded. Indeed, we think that both of the above objections point in the same direction for an answer: to the question of the meaning of the wave function.

12 The Wave Function as LAW

We propose that the reason, on the universal level, that there is no action of configurations upon wave functions, as there seems to be between all other elements of physical reality, is that the wave function of the universe is not an element of physical reality. We propose that the wave function belongs to an altogether different category of existence than that of substantive physical entities, and that its existence is nomological rather than material. We propose, in other words, that the wave function is a component of physical law rather than of the reality described by the law.

We note in this regard that nobody objects to classical mechanics because it involves a Hamiltonian $H_{\text{class}}(\mathbf{q}_1, \ldots, \mathbf{q}_N, \mathbf{p}_1, \ldots, \mathbf{p}_N) \equiv H_{\text{class}}(\xi)$ that is a function on a space, the phase space, that is of greater dimension and even more abstract than configuration space. This is because we think of the state in classical mechanics as given by the q's and p's, and we regard the Hamiltonian as the generator of the evolution of the state—i.e., as part of the law—and not as an object in whose behavior we are directly interested.

To pursue this analogy, between the wave function and the classical Hamiltonian, a bit further, let's compare

$$H_{\text{class}} \longleftrightarrow \log \Psi \tag{17}$$

and note that both of these generate motions in pretty much the same way

$$\frac{d\xi}{dt} = \text{Der} H_{\text{class}} \longleftrightarrow \frac{dQ}{dt} = \text{Der}(\log \Psi), \tag{18}$$

with Der a derivation. Moreover, when we proceed to the level of statistical mechanics, we find statistics of the more or less the same form

$$\rho_{\text{class}} \sim e^{\text{const.} H_{\text{class}}} \longleftrightarrow \rho_{\text{quant}} \sim |e^{\text{const.} \log \Psi}|, \tag{19}$$

(with the constant on the right equal to 2).

Now we do not think that this analogy should be taken too seriously or too literally; it's not a particularly good analogy—but it's better than it has any right to be. It does, however, have the virtue that it stimulates a new direction of thought concerning the meaning of the wave function, and that is a great virtue indeed.

Perhaps the most serious weakness in the analogy is that, unlike H_{class}, $\psi = \psi_t$ is time-dependent, and indeed is a solution of what we regard as the *fundamental* equation of *motion* for ψ,

$$i\frac{\partial\psi}{\partial t} = H\psi. \tag{20}$$

Moreover, for a particular choice of classical theory, with specified interactions, H_{class} is fixed; it is not free, not something to be chosen as an initial condition, like ψ.

But think now again of the Wheeler-de Witt equation for *the* wave function of the universe. This fundamental wave function Ψ, the universal wave function, is static, stationary, and, in the view of many physicists, unique. The fundamental equation for Ψ

$$\mathcal{H}\Psi = 0 \tag{21}$$

or more generally

$$\mathcal{H}\Psi = E\Psi \tag{22}$$

should be regarded as a sort of generalized Laplace equation that selects the central element Ψ of the law of motion

$$dQ/dt = v^{\Psi}(Q), \tag{23}$$

the object that generates the vector field v^{Ψ} defining the motion. Here Q is rather general—not merely particle positions, and certainly including the configuration of the gravitational field. Moreover, the form of v^{Ψ} should arise from the mathematical and geometrical character of the structure defined by Q, and should not be conceived of as being of any particular a priori form, such as given in the r.h.s. of (1).

The equation (23) is now the fundamental equation of motion, with Ψ the (natural) solution to the "Laplace equation," which defines the law of motion (23) through the selection of Ψ. We may regard this selection as analogous to that of the Coulomb interaction via the equation $\boldsymbol{\nabla}^2\phi = \delta$. (Note also that H_{class} for the Coulomb interaction satisfies something much like Poisson's equation on phase space, $(\boldsymbol{\nabla}_p^2 + \boldsymbol{\nabla}_q^2)H_{\text{class}} = \text{const} + \sum\delta$.) In particular Ψ, and hence (23), does not explicitly depend upon time t—since there is no t in (21) or (22).

13 The Schrödinger Evolution as Phenomenological

We wish to stress that we are now exploring the possibility that the time-dependent Schrödinger equation is not fundamental. We must thus address the question, not of how change is at all possible in a theory with a change-less wave function—since this is trivial when, in addition to the wave function, there is the configuration Q whose very motion it is the role of the wave function to specify—but rather why we should arrive, as we do, at a picture with time-dependent Schrödinger wave function when we start with a theory with a fixed timeless wave function that knows nothing of the time-dependent Schrödinger equation.

Now we already know that for Bohmian mechanics the Schrödinger evolution is hereditary, so that if the universal wave function Ψ satisfies Schrödinger's equation then subsystems will (in the usual situations and under the usual assumptions, see [6]) have their own wave functions, nontrivially evolving according to their own Schrödinger evolutions. Since a wave function satisfying (22) *does* define a solution to Schrödinger's (albeit a very special one), we should perhaps expect to find subsystems behaving as just described even for a theory in which the time-dependent Schrödinger evolution is not fundamental.

However, since it may not be clear how a stationary wave function *could* yield an evolution rich enough to generate genuinely evolving subsystem wave functions,[1] we wish to give a very simple example in which this occurs, as well as to tentatively propose a more general analysis.

Suppose that the configuration of the universe has a decomposition of the form

$$q = (x, y) \tag{24}$$

$$Q(t) = (X(t), Y(t)), \tag{25}$$

where X describes the degrees of freedom with which we are somehow most directly concerned and Y describes the remaining degrees of freedom. For example, X might be the configuration of all the degrees of freedom governed by standard quantum field theory, describing the fermionic matter fields as well as the bosonic force fields, while Y refers to the gravitational degrees of freedom. We wish to focus on the conditional wave function

$$\psi_t(x) = \Psi(x, Y(t)) \tag{26}$$

for the x-system and to ask whether $\psi_t(x)$ could be—and might, under suitable conditions, be expected to be—a solution to Schrödinger's equation for the x-system.

First, the simple example: Suppose our universe consists merely of two particles, with configurations x and y respectively, moving in a 1-dimensional space. Suppose further that the particles are noninteracting, so that the l.h.s. of (22) is just the free Hamiltonian ($\hbar = m_k = 1$)

$$H = -\frac{1}{2}\nabla^2 = -\frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) = H_x + H_y \tag{27}$$

Let

[1]Note that in the usual measurement theory picture, it is the *motion* of the composite system wave function that appears to be directly responsible for the motion of the "collapsed" wave function.

$$\Psi(x,y) = e^{i(x-y)}\cos(x+y). \tag{28}$$

This "wave function of the universe" satisfies (22) with $E = 2$

$$H\Psi = 2\Psi \tag{29}$$

[This wave function is of course best arrived at by rotating the obvious eigenfunction $e^{ikx}\cos ky$ ($k = \sqrt{2}$) by 45 degrees.]

It then follows immediately from (1) that

$$Y(t) = y_0 - t, \tag{30}$$

so that the conditional wave function

$$\psi_t(x) \sim e^{i(x+t)}\cos(x + y_0 - t) \equiv e^{2it}\hat{\psi}_t(x) \tag{31}$$

is clearly not stationary and moreover is (projectively and hence physically) equivalent to $\hat{\psi}_t$, which satisfies

$$i\frac{\partial\hat{\psi}}{\partial t} = H_x\hat{\psi}. \tag{32}$$

We will now present an argument suggesting that what we've just found in the example—a time-dependent conditional wave function obeying Schrödinger's equation emerging from a stationary universal wave function—should be expected to occur much more generally. Suppose we can write

$$\Psi(x,y) \simeq \sum_\alpha \psi_t^\alpha(x)\phi_t^\alpha(y) \tag{33}$$

where for each t, $\phi_t^\alpha(y)$ is a "narrow wave packet," centered around y_t^α [$\neq y_t^{\alpha'}$]. Suppose that the time-dependence in (33) is such that $\phi_t^\alpha(y)$ "follows" $Y(t)$, i.e., that $Y(t) \approx y_t^\alpha$ for all t, where α is such that $Y(0) \approx y_0^\alpha$. It then follows from (33) that for the conditional wave function of the x-system we have that $\psi_t(x) \approx \psi_t^\alpha(x)$.

Now we know what kind of time-dependence is such that $\phi_t^\alpha(y)$ keeps up with $Y(t)$. This occurs when $\phi_t^\alpha(y)$ is a solution of Schrödinger's equation (with Hamiltonian H_y). Since Ψ itself has no time-dependence in it, a natural way to arrive at (33) is to consider a single decomposition of the form (33), involving narrow and approximately disjoint y-wave packets, say for $t = 0$, and write

$$\begin{aligned}\Psi \sim e^{-iEt}\Psi = e^{-iHt}\Psi = &\quad e^{-i(H_x+H_y)t}\sum_\alpha \psi_0^\alpha(x)\phi_0^\alpha(y)\\ = &\quad \sum_\alpha \left(e^{-iH_xt}\psi_0^\alpha(x)\right)\left(e^{-iH_yt}\phi_0^\alpha(y)\right)\\ \equiv &\quad \sum_\alpha \psi_t^\alpha(x)\phi_t^\alpha(y),\end{aligned}$$

from which we see that $i\frac{\partial\psi_t^\alpha}{\partial t} = H_x\psi_t^\alpha$.[2] Now if, for example, we are dealing here with the semi-classical regime for the y-system, an initial collection of narrow and approximately disjoint wave packets $\phi_0^\alpha(y)$ should remain so under their evolution. Then the conditional wave function of the x-system will approximately satisfy

$$i\frac{\partial\psi}{\partial t} = H_x\psi.$$

It is perhaps worth noting that if Y describes the gravitational degrees of freedom, we might imagine that the evolution $Y(t) \approx y_t^\alpha$ describes the expansion of the universe.

We thus see how Schrödinger's (time-dependent) equation might indeed rather generally arise as a phenomenological equation that emerges when we look for a description of the behavior of subsystems of a universe governed by a timeless universal wave function that knows nothing about Schrödinger's equation.

14 Overview

We wish to underline the transitions in quantum ontology implied by our discussion, proceeding from what is arguably the ontology of Orthodox Quantum Theory, to that of Orthodox Bohmian Mechanics, and finally to the ontology of the Universal Bohmian Theory upon which we have just focused:

OQT	OBM	UBT
Ψ	(Ψ, Q)	Q

In conclusion, we note that Bohmian mechanics is profoundly unromantic. It tends to be a counterexample to lots of seductive notions about quantum mechanics, for example:

- many-worlds
- observer-created reality
- noncommutative epistemology
- quantum logic

There is, however, one element of quantum peculiarity that Bohmian mechanics is normally regarded as retaining and amplifying. Bell [1, page 128] has said that

[2]More generally, we might have considered $e^{-i\gamma Et}\Psi$, but our desire that $y_t^\alpha \approx Y(t)$ leads to the choice $\gamma = 1$.

> *No one can understand this theory until he is willing to think of Ψ as a real objective field rather than just a 'probability amplitude.' Even though it propagates not in 3-space but in $3N$-space.* (Bell 1981)

Concerning the notion that the wave function is fundamentally, if not *the* reality, at least a substantive part of reality, what we are suggesting here is that Bohmian mechanics may turn out to be a counterexample to this as well.

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