Radiative ionization: The link between radiative electron capture and bremsstrahlung

D. H. Jakubassa-Amundsen
Mathematics Institute, University of Munich,
Theresienstrasse 39, 80333 Munich, Germany

Abstract

This chapter is devoted to the theory of radiative ionization of target atoms in energetic collisions with highly stripped projectiles. It is set into context with the simultaneously occurring processes of nonradiative electron capture to continuum and radiative capture to bound projectile eigenstates. Among other processes linked by inverse kinematics, particular emphasis is laid on the relation between radiative ionization and electron-nucleus bremsstrahlung. Specific features of the electron and photon spectra and their angular distributions as well as the photon linear polarization are reviewed. In addition, new results are presented and are compared with experiments using relativistic uranium beams. The validity of the theoretical model is also inferred from a comparison with accurate partial-wave calculations for bremsstrahlung and radiative electron capture.

Contents

1 Introduction

The interaction of charged particles by means of the Coulomb field as well as their coupling to weak photon fields are basically well-understood processes. Nevertheless, the atomic collision physics has kept its fascination all over the years. A particularly intriguing aspect is the close relation between specific processes which usually are treated isolated from each other. This interrelation allows for experimental tests of the theory from a different point of view, maybe even at conditions which never could be met in the isolated process. On the other hand, previously unexplained experimental features suddenly become clear if viewed from another frame of reference.

There exist several possible relations between atomic processes in collision physics. The first is that the physical process induced by the interaction between the active particles is the same, but the initial or final states differ from each other. Restricting ourselves to radiation physics, an example is radiative recombination (where the electron is initially free) in relation to radiative electron capture (REC, where the electron is quasifree, i.e. in a loosely bound initial state) [1]. As another case we have REC (with a bound electronic final state) and radiative ionization (RI, sometimes also termed RECC, where the electron is released), respectively. A second type of interrelation concerns different possible processes under the restriction of the same initial and final states. Such processes usually occur simultaneously and often require coincidence experiments to separate them. As examples may serve the two processes RI and the nonradiative electron capture to continuum (ECC) [2, 3], but also REC and Coulomb capture to bound states (EC) [4], where either the Coulomb field or the radiation field induces the transition. Like REC and RI, also EC and ECC are linked by means of the continuity across the projectile's ionization threshold [5].

A very important type of interrelation is the one by inverse kinematics. In processes related in this way the initial and final states are interchanged; one might also view one process as the time-reversed second process. For instance we have photoionization (incoming photon, emitted electron) and radiative recombination (incoming electron, emitted photon), respectively [6, 7]. Relaxing the requirement that the initial state of one process be completely the same as the final state of the second process, one may also view photoionization (with a free outgoing electron) and REC (with a quasifree incoming electron) as processes linked by inverse kinematics [7]. Likewise, photoionization (with an initially bound electron) and bremsstrahlung (with a free outgoing electron) have been considered as inverse processes [8]. However, the phrase inverse kinematics is also applied to processes which are linked by a frame transformation. For these not only the physics is the same, but also their observation provided the reference frame is changed accordingly. The show-piece is target ionization and electron loss, respectively, in H + H collisions but, again relaxing the condition of identical initial states, RI and bremsstrahlung can be viewed as inverse processes too [9]. In particular, this is true for the short-wavelength limit of bremsstrahlung and the radiative electron capture to the continuum threshold which leads to cusp electrons, respectively [10, 11].

Once two reactions are identified as being linked by one of the above-mentioned relations, the same theoretical model can be applied to their description. If, for instance, a process initiated by a free electron is well described by some theory T, the impulse approximation (IA) will give an excellent description for the same process initiated by a quasifree electron, provided the collision is sufficiently energetic. In fact, the frame-transformed RI for relativistic collision velocities and a hydrogen target leads to results which are very close to those for bremsstrahlung under the condition that the underlying theory T is the same [12]. Similarly, the radiative recombination provides a very good approximation to the impulse approximation for REC at relativistic collision velocities [7]. There remains, however, a basic difference between processes initiated by a free and a quasifree electron if multiply differential cross sections are considered. In the quasifree case, the momentum provided by the parent nucleus adds to the momentum balance and changes a sharp photon line (in the case of a free electron) to a broad energy distribution governed by the Compton profile. That becomes evident in the REC spectra (see e.g. [13]).

This chapter is devoted to the relativistic formulation of the theory for radiative ionization, nonradiative electron capture to continuum and radiative electron capture to bound states using the formal scattering theory as a common starting point. The competition between RI and ECC in the electron spectra is reviewed in section 2, supplemented with the interpretation of new experimental results for the forward peak. The close relation between RI and the elementary process of bremsstrahlung is discussed in section 3. Particular emphasis is laid on the influence of the collision parameters, such as projectile and target nuclear charge as well as the collision velocity, on the angular distribution of the emitted photons and their degree of polarization. Furthermore, in section 4, the relativistic theory for REC is derived in some detail. This theory is compared to an existing REC theory that is based on the inverse photoeffect, and the calculated photon yield is contrasted to that from RI near the continuum threshold of the projectile. A summary of all results is given in section 5.

2 Radiative ionization: The model and its comparison with ECC

RI, interpreted as capture of a target electron into the continuum of an ionized, ideally bare, projectile with the simultaneous emission of a photon, is the dominant background process in photon spectra from fast, asymmetric collisions ($Z_P \gg Z_T$, Z_P and Z_T being the nuclear charges of the projectile and target, respectively) for photon energies $\hbar\omega$ below the threshold T_0. This threshold is related to the fact that an electron at rest in the target frame cannot radiate more than its complete kinetic energy relative to the projectile frame. One has $T_0 = (E_0' - mc^2)/[\gamma(1 - \frac{v}{c}\cos\vartheta_k)]$ where $\boldsymbol{v}$ is the collision velocity, mc^2 the electron's rest energy, $\gamma = (1 - v^2/c^2)^{-\frac{1}{2}}$, $E_0' = \sqrt{(\gamma mvc)^2 + m^2c^4}$ the electronic collision energy in the projectile reference frame, and ϑ_k the photon emission angle. As such RI was first identified by Kienle et al [14] and correctly interpreted soon afterwards [15, 16]. Later, RI became directly visible as a ridge near T_0 in the continuous photon spectra in very fast collisions ([17]; for recent work see e.g. [10]). The breakthrough for RI came, however, just now with a coincidence experiment where U^{88+} was collided with nitrogen and where the photon and electron momenta were recorded simultaneously [11]. There, a photon spectrum, due entirely to RI, was measured for the first time and interpreted by theory.

2.1 The impulse approximation for relativistic collisions

Within the semiclassical independent-particle approximation, the general expression for the transition amplitude from an initially bound target state to a projectile continuum state $\overline{\psi_{f,P}^{(\sigma_f)'}}(x') = \psi_{f,P}^{(\sigma_f)'+}(x')\,\gamma_0$ of momentum $\boldsymbol{k}_f'$ and spin σ_f is given by (in atomic units,

$\hbar = m = e = 1$)

$$a_{fi} \;=\; -\frac{i}{c} \int d^4x' \; \overline{\psi_{f,P}^{(\sigma_f)'}(x')} \; d_\lambda^+ \; \left(\hat{S}\, \mathbb{A}(x)\, \hat{S}^{-1} \right) \; \hat{S}\; \Psi_i^{(\sigma_i)}(x). \tag{2.1}$$

The RI and ECC processes differ only in the choice of the electromagnetic transition field $\mathbb{A}(x)$ and in the presence (RI) or absence (ECC) of the photon creation operator d_λ^+ in (2.1). Projectile-frame related quantities are denoted by a prime, and we have chosen this frame as our frame of reference. $\Psi_i^{(\sigma_i)}(x)$ is the exact electronic scattering state which relates asymptotically to a target eigenstate $\psi_{i,T}^{(\sigma_i)}(x)$ with spin σ_i and space-time vector $x = (ct, \boldsymbol{x})$. x is connected to x' by a Lorentz transformation, $x = \Gamma x' + b$, where b is the impact parameter. The scattering state as well as the field $\mathbb{A}(x)$ have to be transformed into the projectile frame by means of the Lorentz boost operator $\hat{S}$. If the z-axis (respectively the unit vector $\boldsymbol{e}_z$) is chosen along $\boldsymbol{v}$, then

$$\hat{S}(v) \;=\; \sqrt{\frac{1+\gamma}{2}} \left(1 \;-\; \frac{\gamma v/c}{1+\gamma}\, \alpha_z \right) \tag{2.2}$$

with its inverse $\hat{S}^{-1}(v) \;= \gamma_0 \hat{S}(v)\, \gamma_0 \;= \hat{S}(-v)$. $\alpha_z,\ \gamma_0$ are Dirac matrices [18].

For asymmetric collisions with $Z_T \ll Z_P$ one may expand the scattering state $\Psi_i^{(\sigma_i)}(x)$ in terms of the weak target field while retaining the correct asymptotics. The lowest-order term in this expansion leads to the strong potential Born approximation [19, 60],

$$\hat{S}\; \Psi_i^{(\sigma_i)}(x) \;=\; \frac{1}{c} \sum_{s=1}^{4} \int d\boldsymbol{q}\; dE\; \psi_{q,off}^{(s)'}(x') \; \left(q_s'(x'), \hat{S}\, \psi_{i,T}^{(\sigma_i)}(x) \right). \tag{2.3}$$

Here, $q_s'(x')$ is a relativistic plane wave of energy E characterized by the four-spinor $u_q^{(s)}$ [21]. $s = 1, 2$ denotes the spin directions of the particle states, and $s = 3, 4$ those of the antiparticle states. Furthermore, $\psi_{q,off}^{(s)'}(x')$ is an off-shell projectile continuum state with momentum $\boldsymbol{q}$. The deviation from the energy shell is determined by the binding energy of the initial target state. Therefore the target potential is included to some extent in the transition operator. The off-shell effects vanish for $Z_T = 0$.

For sufficiently high collision velocities ($v \gg Z_T/n_i$ with n_i the initial-state main quantum number) an on-shell approximation can be made. This replacement of $\psi_{q,off}^{(s)'}(x')$ by a projectile continuum eigenstate $\psi_{q,P}^{(s)'}(x')$ leads to the impulse approximation. Since the differences between the on-shell and off-shell approximation relates to the target field, they can be used as an indicator of validity of the strong potential Born theory itself. In the nonrelativistic case where the IA was tested against the strong potential Born theory, the difference between the two theories was on the level of 10 percent in the cusp region, getting smaller when v increases [20].

In the case of radiative ionization the interaction in (2.1) arises from the photon field, defined in the projectile frame, $\gamma_0\, \hat{S}\, \mathbb{A}(x)\, \hat{S}^{-1} \;= -\boldsymbol{\alpha} \boldsymbol{A}_\lambda' e^{ik'x'} d_\lambda^+$ with $\boldsymbol{A}_\lambda' = c\, \boldsymbol{e}_\lambda/(2\pi\omega'^{1/2})$ where $k' = (\omega'/c, -\boldsymbol{k}')$ is the 4-momentum of the photon and $\boldsymbol{e}_\lambda$ its polarization direction. $\boldsymbol{\alpha} = (\alpha_x, \alpha_y, \alpha_z)$ is the vector of the Dirac matrices. In the IA, the transition amplitude is thus governed by the radiation matrix element

$$\boldsymbol{W}_{rad}(\sigma_f, s, \boldsymbol{q}) \;=\; \int d\boldsymbol{x}' \; \psi_{f,P}^{(\sigma_f)'+}(\boldsymbol{x}')\; \boldsymbol{\alpha}\, e^{-i\boldsymbol{k}'\boldsymbol{x}'} \; \psi_{q,P}^{(s)'}(\boldsymbol{x}') \tag{2.4}$$

and is given by [22]

$$a_{fi,\lambda}^{RI} \;=\; \frac{2\pi i}{\gamma} \sqrt{\frac{1+\gamma}{2}}\; \boldsymbol{A}_\lambda' \sum_{s=1}^{4} \int d\boldsymbol{q}\; e^{i\boldsymbol{q}_\perp \boldsymbol{b}}\; \boldsymbol{W}_{rad}(\sigma_f, s, \boldsymbol{q})$$

$$\cdot \left[u_q^{(s)+} \left(1 \;-\; \frac{\gamma v/c}{1+\gamma}\, \alpha_z \right) \; \varphi_{i,T}^{(\sigma_i)}(\boldsymbol{q}_0) \right] \; \delta\left(E_f' + \omega' - E_i^T/\gamma + q_z v \right) \tag{2.5}$$

where $\varphi_{i,T}^{(\sigma_i)}(\boldsymbol{q}_0)$ is the initial-state wavefunction in momentum space, $\boldsymbol{q}_0 = (\boldsymbol{q}_\perp, q_{0z})$ with $\boldsymbol{q}_\perp$ perpendicular to $\boldsymbol{v}$ and $q_{0z} = E_i^T v/c^2 + q_z/\gamma$, E_i^T and E_f' being the electron energy in its initial and final state, respectively[1].

For the nonradiative capture, the interaction in (2.1) results from the target Coulomb potential $V_T(\boldsymbol{x})$, viz. $\gamma_0\,\hat{S}\,\mathbb{A}(x)\;\hat{S}^{-1} = \gamma\,V_T(\boldsymbol{x})\;\left(1 + \frac{v}{c}\,\alpha_z\right)$, which is conventionally decomposed into its Fourier components with weight factor $1/p^2$. Using the IA, we define the transition matrix element T_{00}/p^2 with

$$T_{00}(s,\boldsymbol{q},\boldsymbol{p}) \;=\; \int d\boldsymbol{x}'\;\psi_{f,P}^{(\sigma_f)'+}(\boldsymbol{x}')\;e^{i\boldsymbol{p}_\perp\boldsymbol{x}'_\perp + ip_z\gamma z'}\left(1 + \frac{v}{c}\,\alpha_z\right)\;\psi_{q,P}^{(s)'}(\boldsymbol{x}'). \tag{2.6}$$

Then the transition amplitude for ECC is given by [3]

$$a_{fi}^{ECC} \;=\; \frac{iZ_T}{\pi}\sqrt{\frac{1+\gamma}{2}}\sum_{s=1}^{4}\int d\boldsymbol{q}\int\frac{d\boldsymbol{p}}{p^2}\;e^{i(\boldsymbol{p}_\perp+\boldsymbol{q}_\perp)\boldsymbol{b}}\;T_{00}(s,\boldsymbol{q},\boldsymbol{p}) \tag{2.7}$$

$$\cdot\left[u_q^{(s)+}\left(1 - \frac{\gamma v/c}{1+\gamma}\,\alpha_z\right)\;\varphi_{i,T}^{(\sigma_i)}(\boldsymbol{q}_0)\right]\;\delta(E_f' + p_z\gamma v + q_z v - E_i^T/\gamma),$$

where $\boldsymbol{q}_0 = (\boldsymbol{q}_\perp, q_{0z})$ is defined as above.

The differential cross section for the emission of an electron with energy E_f into the solid angle $d\Omega_f$ is obtained by means of integrating over impact parameter and by averaging and summing, respectively, over the initial and final spin states. For ECC, one has

$$\frac{d^2\sigma^{ECC}}{dE_f d\Omega_f} \;=\; \frac{k_f E_f'}{2c^2}\sum_{\sigma_i,\sigma_f}\int d^2\boldsymbol{b}\;\left|a_{fi}^{ECC}\right|^2. \tag{2.8}$$

Concerning RI, this prescription leads to the fourfold differential cross section for a photon of frequency ω ejected into the solid angle $d\Omega_k$ with a fixed polarization direction $\boldsymbol{e}_\lambda$ and a simultaneously emitted electron,

$$\frac{d^4\sigma_\lambda^{RI}}{dE_f d\Omega_f d\omega\, d\Omega_k} \;=\; \frac{k_f E_f'\omega\omega'}{2c^5}\sum_{\sigma_i,\sigma_f}\int d^2\boldsymbol{b}\;\left|a_{fi,\lambda}^{RI}\right|^2. \tag{2.9}$$

For the comparison with ECC, (2.9) has to be summed over the two polarization directions and integrated over the photon momentum degrees of freedom,

$$\frac{d^2\sigma^{RI}}{dE_f d\Omega_f} \;=\; \int d\omega\; d\Omega_k\sum_\lambda\frac{d^4\sigma_\lambda^{RI}}{dE_f d\Omega_f d\omega d\Omega_k}. \tag{2.10}$$

For the numerical evaluation, a semirelativistic approximation is used for the wavefunctions in order to deal with analytic, closed expressions for the transition matrix elements (2.4) and (2.6). It involves Darwin functions for the bound states [23] and Sommerfeld-Maue functions for the continuum states [24]-[26] which are accurate up to first order in Z/c, where Z is the respective nuclear charge (for their explicit form, see section 4.1). They coincide with the exact Coulomb eigenstates in the nonrelativistic limit. Tests of their validity by comparing the results with those obtained from accurate relativistic wavefunctions are provided in sections 3 and 4.

In order to understand the features in the momentum distribution of the photons and electrons given below we will analyze the fourfold differential RI cross section in somewhat more detail. To do so, we apply for the sake of demonstration a peaking approximation. This approximation relies on the fact that the bound-state wavefunction $\varphi_{i,T}^{(\sigma_i)}(\boldsymbol{q}_0)$ is (for

[1] In previous work [3, 22] this spin sum was erroneously truncated at $s = 2$. However, the contribution of $s = 3, 4$ is negligibly small in the weakly relativistic regime considered.

s-states) strongly peaked at $q_0 = 0$. According to the δ-function in (2.5), the z-component q_{0z} vanishes if the photon energy takes the value

$$\omega_{peak}(\vartheta_k) \;=\; \frac{\omega'_{peak}}{\gamma(1-\frac{v}{c}\cos\vartheta_k)}, \qquad \omega'_{peak} = \gamma E_i^T - E'_f. \tag{2.11}$$

For photons with $\omega \approx \omega_{peak}$ we set $\boldsymbol{q} = (\boldsymbol{0}, -\gamma E_i^T v/c^2)$ corresponding to $q_0 = 0$ everywhere in the transition amplitude (2.5) except in $\varphi_{i,T}^{(\sigma_i)}(\boldsymbol{q}_0)$ and in the δ-function. Splitting the Darwin function into a scalar part $\tilde{\varphi}_{i,T}(\boldsymbol{q}_0)$ times a spinor function which can be taken at $q_0 = 0$ (see the expression above (4.2)), the differential cross section becomes proportional to

$$v \int d\boldsymbol{q}\; |\tilde{\varphi}_{i,T}(\boldsymbol{q}_0)|^2\; \delta(E'_f + \omega' - E_i^T/\gamma + q_z v) \;=\; J_i\left(E_i^T/v \,-\, \frac{1}{\gamma v}\,(E'_f + \omega')\right) \tag{2.12}$$

which is the target Compton profile. Hence one obtains a peak in $d^4\sigma_\lambda^{RI}/dE_f d\Omega_f d\omega d\Omega_k$ at $\omega = \omega_{peak}(\vartheta_k)$, shaped by this Compton profile.

While in the actual RI calculations no such peaking approximation is made we have, however, resorted to a transverse peaking approximation in the case of ECC. This approximation is applied to handle the multiple integral in the transition amplitude (2.7). For fast collisions it is well justified, the more so, the higher the collision velocity [3].

2.2 RI photon spectra and angular distributions in comparison with experiment

Fig.1 shows the photon spectrum from collisions of U^{90+} with N_2 at a fixed photon emission angle in comparison with theory. The experimental RI spectrum is obtained from the singles photon spectrum by subtracting the REC spectrum (which was recorded in coincidence with U^{89+} ejectiles) as well as the background radiation [10]. Its absolute scale results from a fit of the measured L-REC spectrum to an accurate REC theory [27]. In order to obtain the theoretical doubly differential RI cross section, (2.9) is summed over the polarization directions and integrated over the electron degrees of freedom. Here and in the following, if not stated otherwise, partly stripped projectiles are treated as bare projectiles with the ionic charge. Also, for the states of a multielectron target atom, an average over the subshells, a Slater-screened charge and experimental binding energies are used. The molecular character of N_2 is disregarded (i.e. the result for N is multiplied by 2). In the figure the one-electron contributions from the target K- and L-shell are shown separately.

The threshold ridge at $T_0 = 70.4$ keV is clearly identified in experiment and theory. While there is qualitative agreement, theory underestimates experiment by a global factor of 2 which we ascribe to the use of semirelativistic wavefunctions for the heavy uranium projectile (see Figs.15 and 16).

In Fig.2 the angular distribution of the photons is shown. The frequency $\omega = 65$ keV was chosen such that the threshold ridge is present ($T_0 = 65$ keV for $\vartheta_k = 150.1°$). In comparison with experiment the RI theory provides cross sections which are a factor of 3 too low, and the slope is also not well reproduced. The origin of this discrepancy remains unclear.

2.3 Characteristics of the RI and ECC electron spectra

For electrons emitted close to the beam direction there are three prominent structures in the electron spectra. One is the binary encounter peak at a kinetic electron energy of $E_{f,kin} = (2v^2\cos^2\vartheta_f - 2E_B)/(1-\frac{v^2}{c^2}\cos^2\vartheta_f)$ where $E_B = mc^2 - E_i^T$ is the binding energy of the target state [28, 29]. This energy is the maximum energy which can be transferred from the projectile nucleus to an electron at rest. Having a classical origin, the peak rises the higher above the background the larger v. Its shape is determined by the target Compton profile, the width increasing with γv. The second structure is the cusp-shaped forward peak near $E_{f,kin} = (\gamma - 1)mc^2$ which is the energy of the projectile continuum threshold measured in the target frame of reference. The forward peak was first observed by Rudd et al [30] and originates from the long-range Coulomb interaction between the ejected electron and the projectile which manifests itself in the normalization constant of the final-state electronic wavefunction. This normalization constant introduces a divergence $\sim 1/k_f'$ into the target-frame doubly differential cross section [31, 32]. If $\vartheta_f = 0$ and $k_f = v$, the shape of the measured peak is governed by the detector resolution (the consideration of which renders the cusp maximum finite). The underlying background with its discontinuity at $\boldsymbol{k}_f = \boldsymbol{v}$ (which is only correctly described in a higher-order theory with respect to the electron-projectile interaction [33, 34]) leads to an asymmetry of the forward peak. Finally, the third structure in the electron spectra is the increase of intensity towards $E_{f,kin} \to 0$, the so-called soft-electron peak. Such electrons with a small kinetic energy can be released in distant collisions and usually represent the main portion of the total ionization cross section.

In Fig.3a the spectra of the forward electrons ($\vartheta_f = 1.5°$) resulting from Ar^{18+}+ H collisions are shown. The cross sections for RI and ECC were calculated from (2.10) and (2.8), respectively. The collision velocity ($v = 33.85$ a.u., $\gamma = 1.03$) is on the border to the nonrelativistic regime. The three structures are clearly seen in both processes, the binary encounter peak at $E_{f,kin} = 66.3$ keV, the cusp near $E_{f,kin} = 16.3$ keV and the rise when $E_{f,kin} \to 0$. At the chosen collision energy ECC is dominant for the soft electrons and the binary encounter electrons, but the cusp intensity is larger for RI than for ECC. While the ECC cusp is skewed to the low-energy side and is rather weak, the RI cusp rises much higher above the background, and its high-energy wing is enhanced.

The reason for the different cusp intensities lies in the fact that ECC requires a high momentum transfer ($q_{min}^{ECC} = |\frac{1}{\gamma v}(E_f' - \gamma E_i^T) + p_z|$) to the target nucleus, while in RI the emitted photon carries away the excess energy ($q_{min}^{RI} = \frac{1}{\gamma v}|E_f' + \omega' - \gamma E_i^T|$). Therefore the dominant cusp contribution for RI comes from the maximum of the target Compton profile. On the other side, q_{min}^{ECC} increases with collision velocity (for $p_z = 0$) such that for ECC mainly the outer wing of the Compton profile contributes. As a consequence, ECC decreases much faster with v than RI. We remark that for ECC the matter is actually more complicated because the interaction potential V_T can help to supply the necessary momentum ($p_z \neq 0$). The inference given above remains correct, however.

The analysis given above may also help to explain the different cusp asymmetries of the two processes. For ECC, in the projectile frame of reference, the incoming electron moves close to the target nucleus because it has to exchange a large momentum. It is then dragged along with the target nucleus after colliding quasielastically with the projectile. Thus it is ejected predominantly along $-\boldsymbol{v}$. In contrast, in the RI process the electron is quasifree and thus only subject to the projectile field. The loss of nearly all its energy to the photon requires a deeply inelastic scattering in the vicinity of the nucleus. The electron is thereby guided around the projectile by the attractive potential and emerges in the direction of $\boldsymbol{v}$. Therefore, in the target frame of reference, the ECC and RI electrons tend to appear slightly below and above the cusp maximum, respectively.

Fig.3b depicts the (single-electron) spectra from Kr^{36+}+ He collisions at the same velocity but for a larger angle ($\vartheta_f = 15°$). At this angle the ECC cusp has disappeared while the RI forward peak has broadened but is still clearly visible above the background. For the heavier He target, ECC largely dominates the soft-electron and binary encounter peak (the latter having shifted to 61.6 keV). In the forward peak region, ECC and RI are now of comparable importance. The reason is that ECC from the target K-shell strongly increases with Z_T because the momentum distribution is broadened. On the other hand, the doubly differential RI cross section is approximately independent of Z_T since (2.10) involves an integration over the target Compton profile.

Fig.4 shows the RI and ECC spectra from U^{88+}+ N at $\vartheta_f = 3°$ for a higher collision velocity ($v = 56.24$ a.u., $\gamma = 1.1$). For this heavy projectile the cusp asymmetry is clearly visible in both processes since this asymmetry (as well as the peak intensity) increases with Z_P [3, 22]. Moreover, ECC is now also dominating in the cusp region because of the larger Z_T. The RI contribution from the nitrogen L-shell is approximately 5 times that for one K-shell electron while for ECC, the L-shell contribution is only about 5 percent of the K-shell yield at this velocity (due to the stronger fall-off of the bound-state momentum space wavefunction for the less tightly bound L-electrons). Therefore, the curves in Fig.4 were obtained by calculating the capture of an electron from the target K-shell and multiplying it with a factor of 7 and 2 for RI and ECC, respectively.

A comparison with (relative) experimental data for this collision system [11] is made in Fig.5. At an electron emission angle of 0°, the RI spectrum was directly recorded in coincidence with photons emitted at $\vartheta_k = 90°$. The ECC spectrum was obtained from the singles electron spectrum by subtracting the simultaneously measured electron loss to continuum (ELC, in coincidence with U^{89+} ejectiles) and RI spectra (the latter extrapolated to a 4π solid angle for the emitted photon [35]). In the calculations the target subshells were treated separately, using Hartree-Fock states (and experimental binding energies) in the case of RI. The difference to the results from Slater-screening is small, however. Theory is averaged over the spectrometer resolution (in order to facilitate the calculations, ECC is calculated for $\vartheta_f = 1°$ instead of averaging over the angular resolution of 1.9°, and is only averaged over the energy resolution). In the case of RI, the fourfold differential cross section is in addition integrated over the energy and angular acceptance of the photon detector. Experiment is normalized to theory in the peak maximum. It is seen that the theoretical peak positions and widths compare fairly well with these pioneer data.

As mentioned above the shape of the cusp is strongly influenced by the spectrometer resolution. In Fig.6 the RI cusp cross section (2.10) for one electron from the K-shell in U^{88+}+ N collisions at $\vartheta_f = 0°$ is shown for several different angular resolutions θ_0. There is a shift of the peak maximum to higher electron energies and the peak becomes broader, the more, the larger θ_0. A similar effect is observed when θ_0 is kept fixed at a small value whereas the energy resolution ΔE_f of the spectrometer is increased. The peak shift is exclusively due to the strong asymmetry of the cusp.

2.4 Crossing velocity of RI/ECC

For low collision energies ECC is strongly dominant since the coupling to the radiation field is suppressed because of the smallness of the fine structure constant. However, due to the slower decrease of RI with velocity as compared to ECC, RI eventually gains importance in the cusp region. Therefore there exists a velocity where the RI and ECC processes provide equal electron intensities in the cusp maximum. This crossing velocity v_{cr} marks the change of shape in the singles cusp spectra (for bare projectiles where ELC is not present) from left-hand skewed to right-hand skewed. Moreover, when $v \gg v_{cr}$, the singles cusp spectra will exclusively be due to RI. The crossing velocity was first estimated within a nonrelativistic approach by Shakeshaft and Spruch [2] for a hydrogen target. Scaling properties of v_{cr} were derived in [3] using the relativistic theory. The crossing velocity is independent of the electron emission angle (for ϑ_f in the forward region). Also, it is only weakly dependent on the projectile charge because RI and ECC increase with approximately the same power of Z_P (lying between 2 and 3 [3]).

In Fig.7 the doubly differential cross section in the peak maximum at $\vartheta_f = 3°$ for ECC respectively RI from U^{88+}+ N collisions is shown as a function of collision momentum γv. The calculations are done in the same way as described in the discussion of Fig.4. One obtains $v_{cr} = 72.64$ a.u. (from $\gamma v_{cr} = 85.67$ a.u.). Since at $\gamma v = 61.67$ a.u. the ratio of the ECC and RI peak intensities is known experimentally, we have in Fig.7 normalized the experimental RI peak intensity to theory and derived from this an experimental ECC peak intensity which lies a factor of 1.5 above the ECC theory. This is in good accord with the estimated accuracy of the present model: Taken into consideration that for uranium the RI theory is approximately a factor of 2 too low (see Fig.1 and Fig.16) ECC is underpredicted by a factor of 3. In fact, the Sommerfeld-Maue functions lack the relativistic spatial contraction such that the high momentum tails of the target bound state are underestimated more severely than the centroid of the Compton profile. Therefore, these functions are not so good for ECC than for RI.

The crossing velocity depends strongly on the target. This is displayed in Fig.8 for one-electron capture by Xe^{54+} from the target K-shell. The increase of γv_{cr} with Z_T is approximately linear in the weak-relativistic regime ($\gamma \lesssim 1.5$). A similar behaviour is found if the total capture from neutral targets is considered (in the figure, the RI $1s$-capture cross section is multiplied by the number N of target electrons, and the ECC $1s$-capture cross section by a factor of 2). The crossing velocity is, however, lower if all target electrons are considered. This is so because RI gains the factor $N/2$ as compared to ECC[2].

3 Radiative ionization in terms of inverse bremsstrahlung

The elementary process of bremsstrahlung occupies a very important place in physics [36]. Due to the simultaneous observation of the decelerated electron and the emitted photon, a stringent test of the underlying theory, describing the coupling of the radiation field with the field of electrons and nuclei, becomes possible. There exists a vast literature on the (electron-nucleus) bremsstrahlung theory in comparison with experiment, starting with the work of Bethe and Heitler [37] using the Born approximation, and followed later by Bess [25], Maximon and Bethe [26] and Elwert and Haug [38] who employed the Sommerfeld-Maue wavefunctions. Nowadays, calculations with accurate relativistic wavefunctions have become feasible, based on the work of Tseng and Pratt [39].

The measurements of the triply differential cross section $d^3\sigma^{brems'}/d\Omega_f' d\omega' d\Omega_k'$ for a free electron scattering from a (screened) nucleus, which we identify with the heavy projectile, are theoretically well understood. However, they do not cover the short-wavelength limit (SWL) of bremsstrahlung, $\omega' = (\gamma - 1)mc^2$, where the electron has given all its kinetic energy to the photon. The SWL is of particular interest since it requires a large momentum transfer to the nucleus which necessitates close collisions. This allows for a test of the electronic wavefunction in the vicinity of heavy nuclei. The detection of electrons with near-zero kinetic energy in the rest frame of the nucleus is a very difficult task. It can, however, be made feasible by performing the experiment in a moving reference frame. Here lies the importance of radiative electron capture to near-threshold continuum projectile states.

3.1 Radiative ionization in inverse kinematics

The change of reference frame for a given process proceeds in two steps. First we note that the particle momenta in a reference frame moving with a constant velocity $\boldsymbol{v}$ are subject to a Lorentz transformation. Switching from the (unprimed) target reference frame (where RI is observed) to the (primed) projectile reference frame (which is the natural choice for bremsstrahlung), the energies and polar angles of electron and photon are transformed according to

$$\begin{aligned} E_f' &= \gamma\,(E_f - v k_f \cos\vartheta_f), \qquad & \omega' &= \gamma\omega\,(1 - \frac{v}{c}\cos\vartheta_k) \\ k_f'\,\cos\vartheta_f' &= \gamma\left(-\frac{vE_f}{c^2} + k_f\cos\vartheta_f\right), \qquad & \cos\vartheta_k' &= \frac{\cos\vartheta_k - \frac{v}{c}}{1 - \frac{v}{c}\cos\vartheta_k}. \end{aligned} \tag{3.1}$$

[2]Due to an error in the RI code (only present for relativistic velocities) the numerical results given in the three earlier papers [3, 9, 22] are in part incorrect.

The inverse transformation (from primed to unprimed quantities) is obtained by replacing in (3.1) v with $-v$ throughout.

In the second step we have to account for the reversal of the direction of $\boldsymbol{v}$, i.e. of the z-axis, since a projectile moving with $\boldsymbol{v}$ corresponds to the target moving with $-\boldsymbol{v}$ when the reference frame is changed. The polar angles θ'_f and θ'_k of the decelerated electron and the emitted photon, respectively, in the projectile frame of reference are connected to ϑ'_f and ϑ'_k from the Lorentz transformation (3.1) by means of

$$\theta'_f \;=\; \pi - \vartheta'_f, \qquad \theta'_k \;=\; \pi - \vartheta'_k. \tag{3.2}$$

Finally, the cross section has to be transformed. We use the relativistic invariance of the phase space elements $c^2 d\boldsymbol{k}_f/E_f$ and $c^2 d\boldsymbol{k}/\omega$ of electron and photon, respectively [18, p.124], when changing between the primed and unprimed reference frames. Then the fourfold differential cross section (2.9) is transformed into the projectile frame of reference by means of

$$\frac{d^4\sigma_\lambda^{RI'}}{dE'_f d\Omega'_f d\omega' d\Omega'_k} \;=\; \frac{\omega' k'_f}{\omega k_f}\;\frac{d^4\sigma_\lambda^{RI}}{dE_f d\Omega_f d\omega d\Omega_k}. \tag{3.3}$$

3.2 The bremsstrahlung limit of RI

In the bremsstrahlung process the initial electron is free and therefore the energy conservation requires

$$E'_0 \;=\; \gamma c^2 \;=\; E'_f \,+\, \omega' \tag{3.4}$$

which fixes the electron energy E'_f once ω' and v are given. In contrast, one has for RI

$$E'_f \;=\; \gamma E_i^T \,-\, \omega' \,-\, q_{0z}\gamma v \tag{3.5}$$

with q_{0z} distributed according to the bound-state target Compton profile (2.12). Therefore, the comparison with bremsstrahlung necessitates the integration of (3.3) with respect to E'_f. This leads to the correspondence, valid for ω' below the SWL [12],

$$\frac{d^3\sigma_\lambda^{brems'}}{d\Omega'_f d\omega' d\Omega'_k} \;=\; \lim_{Z_T\to 0}\frac{d^3\sigma_\lambda^{RI'}}{d\Omega'_f d\omega' d\Omega'_k} \;=\; \lim_{Z_T\to 0}\int_{c^2}^{\infty} dE'_f\;\frac{d^4\sigma_\lambda^{RI'}}{dE'_f d\Omega'_f d\omega' d\Omega'_k} \tag{3.6}$$

and in this limit, the RI theory from section 2.1 agrees with the Elwert-Haug theory [38] for bremsstrahlung.

In the following we display the transition from RI to bremsstrahlung by varying the target nuclear charge. In Fig.9 the differential cross section for K-shell RI in $\mathrm{Xe}^{54+} + T$ collisions at $\tilde{E}_0 \equiv E'_0 - mc^2 = 100$ keV as a function of photon emission angle θ'_k is shown. The nuclear charge of the one-electron target T is varied from 18 to 0. If one is far from the SWL (e.g. at $\omega' = 60$ keV) the bremsstrahlung limit is approached monotonically when Z_T is decreased, and for a hydrogen target, RI is already indistinguishable from bremsstrahlung. At the SWL, however ($\omega' \approx 100$ keV), the integral on the r.h.s. of (3.6) for any small, but finite Z_T is a factor of 2 below the bremsstrahlung result. This is due to the fact that for non-zero Z_T only one half of the peak shaped by the Compton profile lies in the integration regime (i.e. above c^2) whereas for $Z_T = 0$ one has a δ-type singularity at c^2 in the fourfold differential RI cross section [12]. From Fig.9 it is also evident that the radiation is stronger when electron and photon are emitted to the same side of the beam axis ($0° < \theta'_k < 180°$), a feature which is generally true for bremsstrahlung [36].

Fig.10 provides the dependence of RI on the electron emission angle θ'_f for the same collision parameters and a (one-electron) hydrogen, carbon and argon target. The photon angle $\theta'_k = 30°$ is chosen close to the maximum of the photon distribution (which is located in the forward hemisphere because of the relativistic retardation). It is seen that for the lower frequencies, the electron distribution is also strongly peaked at small angles, and its shape is only weakly influenced by the target. This sharp peak for the lower frequencies is well-known from the bremsstrahlung experiments [40], whereas the angular variations are considerably weakened, but do not vanish, when the SWL is approached [36]. In fact, at the SWL the maximum for forward angles changes to a maximum at backward angles when Z_T is decreased from 18 to 1. We note that, by the inverse kinematics, the SWL backward-to-forward intensity ratio $A \equiv d^4\sigma'(\theta'_f = 180°)/d^4\sigma'(\theta'_f = 0°)$ determines the asymmetry of the RI cusp [12]. In contrast to the ratio calculated from the triply differential cross section as given in Fig.10, A is larger than one (at $\theta'_k = 30°$, translating to a cusp skewed to the high-energy side) and approximately target independent.

Let us now consider the case when the electron is not ejected into the collision plane spanned by $\boldsymbol{v}$ and the photon momentum, but forms an angle φ with the collision plane. In Fig.11 the triply differential RI cross section is shown for this noncoplanar geometry, using again the same collision parameters (and one-electron targets) as before. For the Xe projectile, the variation with φ is quite smooth and the strong target dependence for electrons emitted into the forward hemisphere persists. The φ-dependence of the photon intensity near $\varphi = 180°$ increases, however, considerably when lighter projectiles are used [12].

Figs.10 and 11 can help to understand the dependence of the doubly differential RI cross section, obtained by integrating over the electron angles, on the target nuclear charge (or on the target shells, respectively). The large difference between Ar and H at small angles θ'_f is suppressed by the weight factor $\sin\theta'_f$ when performing the integration. Well below the SWL these differences diminish at the larger θ'_f such that the doubly differential cross section is nearly independent of the initial target state (see Fig.1). In contrast, at the SWL, the intensity for the larger θ'_f is much lower for Ar than for H and therefore also the integrated cross section (see again Fig.1). This is due to the fact that the electrons corresponding to the SWL are very sensitive to the target Compton profile (the tip of which decreases with increasing binding energy).

3.3 Photon linear polarization

The most detailed observable quantity in the radiative electron capture to continuum process is the fourfold differential cross section including the polarization correlations. This corresponds to a coincidence experiment where in addition to the momentum distributions of electron and photon, also the spin polarization of the electron in its initial and final state as well as the photon polarization are measured. Such a so-called complete experiment provides the most stringent test of theory. In the case of electron-nucleus bremsstrahlung this goal has not entirely been achieved, although there exist measurements on the photon linear polarization induced by unpolarized electrons as well as on the photon emission asymmetry for polarized electron beams (for an overview, see [41]).

Here we will only consider electrons which are unpolarized in their initial state which is the usual situation for electron capture from neutral targets in their ground state. Then the emitted photons can be linearly (but not circularly) polarized [42, 43]. Taking the (x, z) plane as the collision plane, the photon momentum is given by $\boldsymbol{k}' = k'\,(\sin\vartheta'_k, 0, \cos\vartheta'_k)$. The two polarization directions of the photon, which have to be perpendicular to $\boldsymbol{k}'$, are chosen as

$$\boldsymbol{e}_{\lambda_1} \;=\; (0,1,0), \qquad \boldsymbol{e}_{\lambda_2} \;=\; (-\cos\vartheta'_k, 0, \sin\vartheta'_k). \tag{3.7}$$

$\boldsymbol{e}_{\lambda_2}$ lies in the collision plane while $\boldsymbol{e}_{\lambda_1}$ is perpendicular to it. For any multiply differential cross section, abbreviated by $d\sigma_\lambda$, the degree P of the photon linear polarization is defined by

$$P \;=\; \frac{d\sigma_{\lambda_2} \,-\, d\sigma_{\lambda_1}}{d\sigma_{\lambda_2} \,+\, d\sigma_{\lambda_1}} \tag{3.8}$$

which coincides with the Stokes parameter C_{03} [42]. In the bremsstrahlung literature, P is usually defined with a negative sign (see e.g. [36, 43, 44]). The definition (3.8) holds also for the projectile reference frame since the transformation (3.1) - (3.3) does not affect the polarization degree of freedom.

It is well-known that for the coplanar geometry (where $\boldsymbol{v}$, $\boldsymbol{k}'$ and $\boldsymbol{k}'_f$ lie in the (x, z) plane) the nonrelativistic bremsstrahlung theory predicts an in-plane polarization ($P = 1$, [45]). Any deviation from $P = 1$ is thus based on relativistic effects [46] or, in the case of RI, on the binding of the initial electron.

Fig.12 compares the polarization relating to the projectile-frame doubly differential cross section for RI from Au^{79+}+ H and one-electron Ar with the bremsstrahlung result [47] for e+ Au at $\tilde{E}_0 = 500$ keV and $\omega' = 450$ keV as a function of photon angle θ'_k. The RI result for Au^{79+}+ H is indistinguishable from the bremsstrahlung theory for e+ Au^{79+}. The deviations between this theory and experiment may partly be ascribed to the screening by the passive electrons in the neutral Au target used in the experiment, and partly to the inaccuracy of the Sommerfeld-Maue functions for Au^{79+}. When a heavier target is used in the RI calculations, P decreases. This is true in most cases [12].

The determination of the degree of photon polarization when the outgoing electron is not observed can be interpreted as a kind of averaging procedure. Thereby some information on the elementary process of bremsstrahlung is lost [36]. In fact, the polarization of the photons which are detected in coincidence with the outgoing electrons is much different from the one obtained by integrating over the electron emission angles [48]. Fig.13a depicts P resulting from the projectile-frame triply differential RI cross section, defined in (3.6), for the same collision parameters (and one-electron targets) as in Fig.12. The target dependence of P is largest for photons emitted close to the beam direction (θ'_k near 0° or 360°) and it gets stronger when the SWL is approached. In comparison with $d^3\sigma'/d\Omega'_f d\omega' d\Omega'_k$ (summed over λ, Fig.13b) one notes that a large depolarization coincides with the cross section minima. The explanation is simple. When the momenta of the outgoing particles are chosen such that a large momentum transfer to the (projectile) nucleus is required, this can only be achieved by a close collision for which the cross section is small. In close collisions, on the other hand, relativistic effects become particularly important, such that P is lowered. This behaviour is different for the noncoplanar geometry. There is a strong variation of P with φ such that the correspondence between the cross section minima and the minima in P is lost [12]. In fact, for $\varphi \neq 0$ one can have strong deviations from $P = 1$ even in the nonrelativistic case. A striking feature in Fig.13a is the asymmetry of P (with respect to reflection at $\theta'_k = 180°$) because the electron momentum $\boldsymbol{k}'_f$ is slightly tilted away from the beam axis. This asymmetry is much larger than that of the underlying cross section and emphasizes the supplementary information contained in a polarization measurement.

Let us now turn to the polarization related to the fourfold differential RI cross section, and to its dependence on the projectile and the target. In Fig.14 the polarization for RI from Ag^{47+} colliding with H, C, Ar and from U^{92+}, C^{6+} colliding with H is given. The electron energy (at the SWL) is fixed, as is ϑ_f (in the forward direction). At each photon emission angle ϑ_k, the frequency ω is chosen to coincide with the peak frequency (2.11) determined from the tip of the target Compton profile. Under this condition the polarizations corresponding to the triply and fourfold differential cross sections, respectively, show a similar dependence on the photon angle, and for a hydrogen target there is even complete agreement if ω is not too close to the SWL. The dependence of P on the target-frame emission angle ϑ_k changes, however, again drastically if an additional integration over the electron angles is made (see e.g. Fig.2 and Fig.20).

From Fig.14 it follows that the depolarization and the ϑ_k-variation of P is reduced when the projectile charge increases, a fact confirmed by bremsstrahlung experiments [49]. When the target gets heavier the depolarization increases, as mentioned earlier. If $\vartheta_f = 0°$, P drops to zero at $\vartheta_k = 0°$ and 180°. In that case of collinear particle emission one has cylindrical symmetry with respect to the beam axix, causing $d\sigma_{\lambda_1} = d\sigma_{\lambda_2}$ in (3.8).

We note that the shape of the angular distribution of the photons and of their degree of polarization changes strongly when one switches between the target and the projectile frame of reference [12]. This frame dependence arises from the fact that the threshold T_0 and the peak frequency ω_{peak} depend on ϑ_k (in contrast to the nonrelativistic case) whereas these quantities are angular independent in the projectile frame of reference.

3.4 Sommerfeld-Maue results in comparison with accurate calculations

In section 3.2 we have established the close relation between RI from a hydrogen target and the elementary process of bremsstrahlung. The bremsstrahlung results using accurate relativistic wavefunctions can therefore serve as a test for the applicability of the semirelativistic Sommerfeld-Maue functions for the unbound projectile electron. There is a considerable number of publications where the two theoretical approaches are compared with experiment (see e.g. [39, 50, 51]). Below we compile some representative bremsstrahlung literature results for very heavy nuclei which we have supplied with RI calculations (that extend the published Elwert-Haug bremsstrahlung results [38]).

In Fig.15 the singly differential bremsstrahlung cross section $d\sigma'/d\omega'$ from 50 keV and 500 keV $e+$ Au^{79+} is shown as a function of photon energy. The calculations from Lee et al [52] employ a partial-wave expansion of the relativistic electronic states in the field of Au^{79+}. Comparison is made with the experiments by Motz [53]. We have included the frame-transformed RI results from Au^{79+}+ H at the same collision energies (which are indistinguishable from the Elwert-Haug results given in [54] for this system). It is seen that at the lower collision energy ($\gamma = 1.1$) the Sommerfeld-Maue functions do quite well, but they get worse when the SWL is approached. At the higher energy ($\gamma = 2$) the deviations reach a factor of 2. For a systematic investigation of the different models at the SWL, see [55].

Now we proceed from the singly differential cross section to the doubly differential cross section for the same system (Fig.16). The accurate bremsstrahlung calculations [39] which compare well with the experimental data from Aiginger are, nearly independently of the photon emission angle, underestimated in our model by roughly a factor of 2 for $\omega' = 480$ keV (which is slightly below the SWL). In Fig.16 are included the results for an Al^{13+} nucleus. For this smaller charge, the Sommerfeld-Maue functions provide a good approximation even at this high collision energy.

Finally, in Fig.17 the triply differential bremsstrahlung cross section for 300 keV $e+$ Au at a photon energy well below the SWL ($\omega' = 150$ keV) is displayed [51]. For the forward electrons considered, there is quite good agreement between the Elwert-Haug theory and the accurate calculations for an Au^{79+} nucleus if θ_k' is not too large. From this figure it is also seen that accounting for the presence of the passive electrons in Au^0 lowers the cross section in the peak region. To our knowledge, a comparison of the two theoretical models for the triply differential cross section near the SWL covering the whole angular range has not yet been made.

4 Radiative electron capture and its relation to RI

The radiative electron capture, in which a bound target electron is captured into a bound state of a fast, highly stripped projectile with the simultaneous emission of a photon, plays an important role in spectroscopic studies of highly stripped heavy ions and has been investigated in great detail. The early work on REC started with a close collaboration between experimentalists and theoreticians [1, 14, 56]. Relativistic kinematics was first considered by Spindler et al [57]. Their predictions concerning the approximate cancellation between the relativistic retardation and the frame transformation (3.1) in the target-frame photon angular distribution was further verified experimentally by Anholt et al [58] in the case of highly relativistic projectiles.

A consistent relativistic prescription of REC was put forth by Eichler and coworkers [7, 27] who extended the nonrelativistic Kleber model [1]. The differential cross section for photoionization serves as their starting point from which, employing inverse kinematics, the differential cross section for radiative recombination, $d\sigma'_{RR}/d\Omega'_k$, is obtained. This cross section has eventually to be convoluted with the momentum distribution of the target state, as dictated by the impulse approximation. Relativistic kinematics leads to [7]

$$\frac{d^2\sigma_{REC}}{d\omega d\Omega_k} \;=\; \frac{\omega}{\omega'\gamma}\int d\boldsymbol{q}\; \frac{d\sigma'_{RR}(\boldsymbol{q}'_0)}{d\Omega'_k}\; |\tilde{\varphi}_{i,T}(\boldsymbol{q}_0)|^2\; \delta(E'_f + \omega' - E_i^T/\gamma + q_z v) \tag{4.1}$$

where the prefactor arises from $\frac{d^2\sigma}{d\omega d\Omega_k} = \frac{\omega}{\omega'}\frac{d^2\sigma'}{d\omega' d\Omega'_k}$ and $dq_{0z} = dq_z/\gamma$ (recall the relation $q_{0z} = E_i^T v/c^2 + q_z/\gamma$). $\tilde{\varphi}_{i,T}(\boldsymbol{q}_0)$ is the nonrelativistic target function and $\boldsymbol{q}'_0$ the Lorentz transform of $\boldsymbol{q}_0$. By means of this step-by-step method, accurate relativistic calculations of the photoionization process [59] are sufficient for the determination of REC.

4.1 Relativistic theory for REC

In the following a relativistic REC theory is outlined which is strictly derived from the formal scattering theory ([18, 22], without employing the photoelectric effect) and which extends the nonrelativistic formalism developed earlier [1, 60]. The Kleber-Eichler formula (4.1) is then recovered by means of additional approximations.

Starting point of the theory is the transition amplitude (2.1), where now $\psi_{f,P}^{(\sigma_f)'}(x')$ represents a bound projectile eigenstate. For heavy projectiles the off-shell approximation (2.3) is made which retains some influence of the target potential that is not accounted for in the Kleber-Eichler method. The impulse approximation is recovered by going on-shell, and the deviations between the (nonrelativistic) on-shell and off-shell results are somewhat larger than for RI [20]. For relativistic velocities they are not expected to play any significant role, however.

The IA transition amplitude $a_{fi,\lambda}^{REC}$ derived in this way has the form (2.5) where now E'_f is the energy of the bound final state and $\boldsymbol{W}_{rad}^{REC}(\sigma_f, s, \boldsymbol{q})$ describes the free-bound transition mediated by the photon field. For light targets we can describe the initial-state momentum-space function by a Darwin function [23]. For spherically symmetric states it has a simple product form [22], $\varphi_{i,T}^{(\sigma_i)}(\boldsymbol{q}_0) = N_i^T\, a_i^{(\sigma_i)}(\boldsymbol{q}_0)\, \tilde{\varphi}_{i,T}(\boldsymbol{q}_0)$, where

$$N_i^T \;=\; \left[1 + \left(\frac{Z_T\mu}{n_i}\right)^2\right]^{-\frac{1}{2}}, \quad a_i^{(1)}(\boldsymbol{q}_0) \;=\; \begin{pmatrix} 1 \\ 0 \\ \mu q_{0z} \\ \mu q_+ \end{pmatrix}, \quad a_i^{(2)}(\boldsymbol{q}_0) \;=\; \begin{pmatrix} 0 \\ 1 \\ \mu q_- \\ -\mu q_{0z} \end{pmatrix} \tag{4.2}$$

with $q_\pm = q_x \pm iq_y$, $\mu = c/(E_i^T + c^2)$.

Then the doubly differential cross section for REC follows from

$$\frac{d^2\sigma^{REC}}{d\omega d\Omega_k} \;=\; \frac{\omega\omega'}{2c^3}\sum_{\lambda,\sigma_i,\sigma_f}\int d^2\boldsymbol{b}\; |a_{fi,\lambda}^{REC}|^2$$

$$=\; \frac{\omega}{\omega'\gamma}\int d\boldsymbol{q}\; F(\boldsymbol{q})\; |\tilde{\varphi}_{i,T}(\boldsymbol{q}_0)|^2\; \delta(E'_f + \omega' - E_i^T/\gamma + q_z v) \tag{4.3}$$

where

$$F(\boldsymbol{q}) \;=\; \frac{(2\pi)^4\,\omega'^2}{2c^3\, v}\sum_{\lambda,\sigma_i,\sigma_f}\left| N_i^T \sum_{s=1}^{4} \boldsymbol{A}'_\lambda\, \boldsymbol{W}_{rad}^{REC}(\sigma_f, s, \boldsymbol{q}) \right. \tag{4.4}$$

$$\cdot \left[u_q^{(s)+} \sqrt{\frac{1+\gamma}{2\gamma}} \left(1 - \frac{\gamma v/c}{1+\gamma}\, \alpha_z\right)\, a_i^{(\sigma_i)}(\boldsymbol{q}_0)\right]\Bigg|^2 .$$

For the sake of comparison we furnish the differential cross section for the radiative recombination of a free electron with momentum $\boldsymbol{q}_0'$,

$$\frac{d\sigma'_{RR}}{d\Omega'_k}(\boldsymbol{q}_0') \;=\; \frac{(2\pi)^4\,\omega_0'^2}{2c^3\,v} \sum_{\lambda,\sigma_i,\sigma_f} \left| \boldsymbol{A}'_\lambda\, \boldsymbol{W}_{rad}^{REC}(\sigma_f,\sigma_i,\boldsymbol{q}_0')\right|^2 . \tag{4.5}$$

Here, $\omega_0' = \gamma_0 c^2 - E_f'$ follows from energy conservation and $\gamma_0 c^2 = \sqrt{(q_0' c)^2 + c^4}$ is the collision energy of the electron.

In the limit of vanishing target field, where $\tilde{\varphi}_{i,T}(\boldsymbol{q}_0)$ turns into a δ-function and $q_{0z}' = \gamma\,(-vE_{q_0}/c^2 + q_{0z}) \to -\gamma v$, the integral of (4.1) with respect to ω' provides, as expected, the differential cross section for radiative recombination,

$$\lim_{Z_T\to 0} \int_0^\infty d\omega'\, \frac{d^2\sigma'_{REC}}{d\omega' d\Omega'_k} \;=\; \frac{d\sigma'_{RR}}{d\Omega'_k}(-\gamma\boldsymbol{v}). \tag{4.6}$$

A straightforward calculation, following the lines of the derivation of the bremsstrahlung limit of RI [12], shows that the r.h.s. of (4.6) is also the $Z_T \to 0$ limit of $F(\boldsymbol{q})$ as required. In this limit one has $\omega' = \omega_0' = \gamma c^2 - E_f' > 0$. When Z_T is finite, the deviations between (4.1) and (4.3) depend on γ and increase with Z_T.

In the evaluation of the radiation matrix element $\boldsymbol{W}_{rad}^{REC}(\sigma_f, s, \boldsymbol{q})$ in (4.4) we use the semirelativistic wavefunctions to obtain a closed expression. The Darwin function for the final $1s_{1/2}$ ground state is given by

$$\psi_{f,P}^{(\sigma_f)'}(x') \;=\; N_f^P\, a_f^{(\sigma_f)}\, \tilde{\psi}'_{f,P}(\boldsymbol{x}')\, e^{-iE_f' t}, \tag{4.7}$$

$$a_f^{(1)} \;=\; \begin{pmatrix} 1 \\ 0 \\ -i\lambda\partial_{z'} \\ -i\lambda\partial_+ \end{pmatrix}, \qquad a_f^{(2)} \;=\; \begin{pmatrix} 0 \\ 1 \\ -i\lambda\partial_- \\ i\lambda\partial_{z'} \end{pmatrix}, \qquad N_f^P \;=\; \frac{1}{\sqrt{1\,+\,(Z_P\lambda)^2}}$$

where $\lambda = \frac{c}{E_f'+c^2}$, $E_f' = c^2\sqrt{1-(Z_P/c)^2}$ and $\partial_\pm = \partial_{x'} \pm i\partial_{y'}$. $\tilde{\psi}'_{f,P}(\boldsymbol{x}') = \pi^{-1/2}\, Z_P^{3/2}\, e^{-Z_P r'}$ with $r' = |\boldsymbol{x}'|$, is the nonrelativistic bound-state function.

The Sommerfeld-Maue function for the intermediate unbound state is defined in terms of the derivative of a confluent hypergeometric function [38],

$$\psi_{q,P}^{(s)'}(\boldsymbol{x}') \;=\; N_q e^{i\boldsymbol{q}\boldsymbol{x}'}(1 - \frac{ic}{2E_q}\boldsymbol{\alpha\nabla})\, {}_1F_1(i\eta_q, 1, i(qr' - \boldsymbol{q}\boldsymbol{x}'))\; u_{\boldsymbol{q}}^{(s)}, \tag{4.8}$$

$$u_{\boldsymbol{q}}^{(1)} \;=\; \sqrt{\frac{E_q + c^2}{2E_q}} \begin{pmatrix} 1 \\ 0 \\ \nu q_z \\ \nu q_+ \end{pmatrix}, \qquad u_{\boldsymbol{q}}^{(2)} \;=\; \sqrt{\frac{E_q + c^2}{2E_q}} \begin{pmatrix} 0 \\ 1 \\ \nu q_- \\ -\nu q_z \end{pmatrix},$$

with $N_q = (2\pi)^{-\frac{3}{2}}\, e^{\pi\eta_q/2}\,\Gamma(1 - i\eta_q)$, $E_q = \sqrt{q^2c^2 + c^4}$, $\nu = c/(E_q + c^2)$ and the Sommerfeld parameter $\eta_q = Z_P E_q/(qc^2)$.

In consistency with the accuracy of the semirelativistic functions, only terms up to $O(\alpha^2)$ are kept in the radiation matrix element (like in the bremsstrahlung theory [38]). This means that for the small components of $\psi_{f,P}^{(\sigma_f)'}$, the $\boldsymbol{\alpha\nabla}$ term in (4.8) is disregarded.

We make use of the relations $r' e^{-Z_P r'} = -\frac{\partial}{\partial Z_P}\, e^{-Z_P r'}$ and $\boldsymbol{\nabla}\,{}_1F_1(i\eta_q, 1, i(qr' - \boldsymbol{q}\boldsymbol{x}')) = -\frac{q}{r'}\, [\boldsymbol{\nabla}_{s_0}\, {}_1F_1(i\eta_q, 1, i(s_0 r' - \boldsymbol{s}_0\boldsymbol{x}'))]_{\boldsymbol{s}_0 = \boldsymbol{q}}$. Then the radiation matrix element can be based on a single integral [61]

$$I_0(Z_P, \boldsymbol{p}, \boldsymbol{s}_0) \;\equiv\; \int d\boldsymbol{x}'\, \frac{1}{r'}\, e^{-Z_P r'}\, e^{i\boldsymbol{p}\boldsymbol{x}'}\; {}_1F_1(i\eta_q, 1, i(s_0 r' - \boldsymbol{s}_0\boldsymbol{x}'))$$

$$= 4\pi \frac{1}{[Z_P^2 + p^2]^{1-i\eta_q}} \left[(\boldsymbol{p} - \boldsymbol{s}_0)^2 - (s_0 + iZ_P)^2\right]^{-i\eta_q}. \tag{4.9}$$

We write the 4-spinor $a_f^{(\sigma_f)}$ from $\psi_{f,P}^{(\sigma_f)'}$ in the following way,

$$a_f^{(\sigma_f)} = \begin{pmatrix} \chi^{(\sigma_f)} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{r'}\, g^{(\sigma_f)} \end{pmatrix} \quad \text{with } \chi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{4.10}$$

where $g^{(\sigma_f)}$ contains the derivatives (which act only on $e^{-Z_P r'}$) according to (4.7). One obtains from (2.4) with (4.7) and (4.8),

$$\boldsymbol{W}_{rad}^{REC}(\sigma_f, s, \boldsymbol{q}) = \frac{Z_P^{3/2}}{\sqrt{\pi}} N_q N_f^P (\boldsymbol{M}_1 + \boldsymbol{M}_2 + \boldsymbol{M}_3), \tag{4.11}$$

$$\boldsymbol{M}_1 = -\left[\begin{pmatrix} \chi^{(\sigma_f)} \\ 0 \end{pmatrix}^+ \boldsymbol{\alpha}\, u_{\boldsymbol{q}}^{(s)}\right] \frac{\partial}{\partial Z_P} I_0(Z_P, \boldsymbol{q} - \boldsymbol{k}', \boldsymbol{q})$$

$$\boldsymbol{M}_2 = \frac{icq}{2E_q} \left\{\left[\begin{pmatrix} \chi^{(\sigma_f)} \\ 0 \end{pmatrix}^+ \boldsymbol{\alpha}\, (\boldsymbol{\alpha} \boldsymbol{\nabla}_{\boldsymbol{s}_0})\, u_{\boldsymbol{q}}^{(s)}\right] I_0(Z_P, \boldsymbol{q} - \boldsymbol{k}', \boldsymbol{s}_0)\right\}_{\boldsymbol{s}_0 = \boldsymbol{q}}$$

$$\boldsymbol{M}_3 = \left\{\left[\begin{pmatrix} 0 \\ g^{(\sigma_f)} \end{pmatrix}^+ \boldsymbol{\alpha}\, u_{\boldsymbol{q}}^{(s)}\right] I_0(Z_P, \boldsymbol{\varrho} - \boldsymbol{k}', \boldsymbol{q})\right\}_{\boldsymbol{\varrho} = \boldsymbol{q}}.$$

The evaluation of $\boldsymbol{M}_3$ is done by means of $\partial_{z'} e^{-Z_P r'} = -\frac{Z_P}{r'}\, z' e^{-Z_P r'}$ followed by $z' e^{i\boldsymbol{\varrho}\boldsymbol{x}'} = -i \frac{\partial}{\partial \varrho_z}\, e^{i\boldsymbol{\varrho}\boldsymbol{x}'}$ and

$$\left[-\frac{\partial}{\partial \boldsymbol{\varrho}} I_0(Z_P, \boldsymbol{\varrho} - \boldsymbol{k}', \boldsymbol{q})\right]_{\boldsymbol{\varrho} = \boldsymbol{q}} = \frac{8\pi}{[Z_P^2 + (\boldsymbol{q} - \boldsymbol{k}')^2]^{2-i\eta_q}} \left[k'^2 - (q + iZ_P)^2\right]^{-i\eta_q}$$

$$\cdot \left\{\boldsymbol{q}(1 - i\eta_q) - \boldsymbol{k}' \left(1 - i\eta_q + i\eta_q \frac{Z_P^2 + (\boldsymbol{q} - \boldsymbol{k}')^2}{k'^2 - (q + iZ_P)^2}\right)\right\}. \tag{4.12}$$

This leads to the doubly differential REC cross section for the emission of a photon with polarization direction $\boldsymbol{e}_\lambda$,

$$\frac{d^2\sigma_\lambda^{REC}}{d\omega d\Omega_k} = \frac{(2\pi)^3(1+\gamma)\omega\omega' Z_P^3}{2c^3\gamma^2 v} |N_i^T N_f^P|^2 \int d\boldsymbol{q}\ |\tilde{\varphi}_{i,T}(\boldsymbol{q}_0)|^2\ \delta(E_f' + \omega' - E_i^T/\gamma + q_z v)$$

$$\cdot |N_q|^2 \sum_{\sigma_i, \sigma_f} \left| \boldsymbol{A}_\lambda' \,(\tilde{\boldsymbol{M}}_1 + \tilde{\boldsymbol{M}}_2 + \tilde{\boldsymbol{M}}_3)\, (1 - \frac{\gamma v/c}{1+\gamma}\, \alpha_z)\, a_i^{(\sigma_i)}(\boldsymbol{q}_0) \right|^2, \tag{4.13}$$

where $\boldsymbol{M}_l \equiv \tilde{\boldsymbol{M}}_l u_{\boldsymbol{q}}^{(s)},\ \ l = 1, 2, 3$ and the completeness relation for $u_{\boldsymbol{q}}^{(s)}$ was used.

The present theory has to be contrasted against the Sauter formula [62, 63] for the photoionization in the Kleber-Eichler model. Sauter also employs the Sommerfeld-Maue wavefunctions, but in addition makes an expansion in η_q to lowest order which invalidates his theory for the heavy projectiles.

4.2 Sommerfeld-Maue results in comparison with accurate calculations

There exists a systematic investigation of the angular dependence of the REC photons using accurate relativistic wavefunctions [7]. At the relativistic collision energies considered the binding of the initial-state electron plays only a minor role (if an integration over the photon energies is performed [7, Fig.32]). Therefore these calculations are actually done for the radiative recombination instead of REC.

In Fig.18 REC induced by 100 MeV/amu bare projectiles of charge $Z_P = 50$ and 70 is shown. For $Z_P = 50$, the difference between the Sommerfeld-Maue result and the accurate partial-wave calculations is very small except for the forward photons. At the velocity $v = 58.68$ a.u. ($\gamma = 1.1$) the deviations from the nonrelativistic impulse approximation [20] are already quite strong. For $Z_P = 70$ (Fig.18b) the Sommerfeld-Maue functions become poor for $\vartheta_k < 20°$. In particular they fail to correctly reproduce the strong spin-flip transitions which are responsible for the entire REC photon intensity at $\vartheta_k = 0°$ [7]. The high forward cross sections predicted by the partial-wave theory were recently verified experimentally for K-shell REC by U^{91+} and U^{92+} projectiles [64, 65]. The failure of the Sommerfeld-Maue functions in correctly predicting the spin-flip contributions for REC into strongly bound projectile eigenstates is due to their missing relativistic spatial contraction. Therefore the electron is not adequately described when being close to the nucleus. In this context we remark that spin-flip transitions during RI are of lesser importance than for REC because in RI the final state is unbound and therefore not localized so close to the nucleus.

4.3 REC in comparison with radiative ionization

In Fig.19 the photon angular distribution for RI in 100 MeV/amu Xe^{54+}+ H collisions, resulting from an integration over the electronic degrees of freedom, is compared to the REC results. This system was chosen because, according to Fig.18a, the Sommerfeld-Maue functions provide reliable REC results for all ϑ_k except in a small forward cone. Since the relativistic effects are more important for REC than for RI, it follows that the Sommerfeld-Maue wavefunctions are then also appropriate for RI.

The plotted RI results are for photon energies ω_{peak} corresponding, respectively, to the SWL and to electrons with a kinetic energy of approximately half the Xe^{53+} ground-state binding energy ($E'_{f,kin} = 21.8$ keV). For the doubly differential REC cross section we have also chosen $\omega = \omega_{peak}^{REC}$. This REC peak position is obtained from (4.13) – in analogy to (2.11) – as

$$\omega_{peak}^{REC}(\vartheta_k) \;=\; \frac{\gamma E_i^T - E'_f}{\gamma(1 - \frac{v}{c}\cos\vartheta_k)} \tag{4.14}$$

with $E'_f < mc^2$ the ground-state energy. It is seen that the angular distribution of the REC photons in the peak maximum differs very much from the RI results at the ionization threshold. In contrast, the RI into higher-lying continuum states has a photon distribution similar to the one at the SWL. This shows that it is not the final-state energy E'_f but the final-state momentum distribution which is the decisive quantity in determining the shape of the cross section. For the sake of comparison we have included the REC angular distribution obtained from the photon-energy integrated cross section. The similarity between $d\sigma^{REC}/d\Omega_k$ and the angular distribution from the doubly differential REC cross section in the peak maximum is obvious. This behaviour corresponds to the previously discussed similarity between the triply and fourfold differential RI cross sections below the SWL.

4.4 Photon linear polarization

For the photoionization process the polarization correlations were studied in great detail [66], using the relativistic partial-wave formalism [67]. These investigations were motivated by realizing that the photoeffect can serve as a polarizer of electrons, a transmitter of polarization from photons to electrons, or an analyzer of polarized radiation. It is the latter effect which translates to the observation of linearly polarized REC photons caused by unpolarized quasifree electrons. Within the Kleber-Eichler model, the correlation parameter C_{10} of photoionization [66] corresponds to the degree P of linear polarization in REC.

Compared to photoionization the REC process has the advantage that the photon escapes from the target more or less unperturbed whereas in the inverse process the emitted electron may undergo successive collisions inside the target. As an application of REC it was suggested [68] to use the polarization of the emitted photon to gain information on the spin polarization of ion beams. In a recent pilot experiment, P was measured for K-shell REC in 400 MeV/amu U^{92+}+ N_2 collisions [69] and was found to be in accord with the predictions from accurate REC calculations [70, 71].

Aiming at a comparison of the angular dependence of P which results from REC and threshold-RI, respectively, we recall a common property of P which renders the deviations between the two processes smaller than in the case of the photon distributions. For symmetry reasons the RI polarization vanishes when the photon is emitted parallel or antiparallel to the beam axis ($\vartheta_k = 0°, 180°$) provided the emitted electron – if observed – is ejected into the beam direction too. For the photoeffect it was shown that a consistent relativistic theory also leads to a vanishing P for $\vartheta_k = 0°$ and 180° [66], and the same is true for REC [68, 71]. When Sommerfeld-Maue functions are used, P decreases strongly near 0° and 180° but does not vanish, whereas for RI, one does get $P = 0$. This is another indication that the semirelativistic functions are more appropriate for RI than they are for REC.

From Fig.20 where P resulting from K-shell REC and threshold-RI in 300 MeV/amu Ar^{18+}+ H collisions is shown, it is seen that at photon angles between 40° and 130° the photon degree of polarization is indeed quite similar for the two processes. However, its decrease to zero towards 0° and 180° is much steeper in the case of REC. For a heavier projectile the angular dependence of P from REC and threshold-RI is similar to the one in Fig.20 (see [7, Fig.54]), but the two processes differ somewhat more from each other [12]. A comparison with REC results using accurate wavefunctions [7] shows that the semirelativistic wavefunctions are appropriate for an argon projectile (except for photons ejected into the forward direction), whereas they start to deteriorate for $Z_P > 50$.

5 Conclusion

We have discussed the significant features of the momentum distributions of the outgoing photon and electron in the process of radiative electron capture to the projectile continuum. The underlying model was the relativistic impulse approximation, derived from scattering theory, with the use of semirelativistic Sommerfeld-Maue functions for the electronic states as the only additional approximation. Relying on the fact that bremsstrahlung is the inverse process of RI for a vanishing target field, the accuracy of our model, agreeing with the Elwert-Haug theory in this limit, could be tested against available theoretical results which use an accurate relativistic partial-wave representation of the wavefunctions. We conjecture that for collision velocities in the weak-relativistic regime ($\gamma \lesssim 2$) our model is accurate for not too heavy projectiles ($Z_P \lesssim 50$) in the region where the cross sections are large, but it may underestimate the cross sections when they are very small. For uranium projectiles the structures in the momentum distributions are qualitatively correctly predicted, but the absolute values come out a factor of 2 too low. This underprediction for uranium is confirmed by the comparison with experimental singles photon spectra which have been put on an absolute scale by normalizing to the well-established REC peak maxima.

Concerning the process occurring simultaneously with RI, the Coulomb capture into the projectile continuum, we have given predictions for which collision parameters both processes provide comparable electron intensities in the forward peak (cusp) region. The importance of this particular region of the electron spectrum is, on the one hand, its sensitivity to relativistic effects resulting from the close electron-projectile collisions. On the other hand, it seems to be the only region where radiative ionization can be the dominating process. We have established the strong dependence of the ratio between RI and ECC on the target species and on the collision velocity by using the same theoretical model for both processes. Recent coincidence experiments on 90 MeV/amu U^{88+}+ N_2, allowing for the detection of the RI and ECC cusp electron spectra in one measurement, have confirmed the strong dominance of ECC predicted by theory for this collision system. In addition, they have established the shape of the RI cusp with its skewness to the opposite side of the ECC cusp. They have thus proven the feasibility of measuring the short-wavelength limit of bremsstrahlung with the tool of inverse kinematics.

If comparison is made between the frame-transformed RI and the bremsstrahlung results, there are considerable differences in the photon and electron angular distributions as well as in the photon linear polarization for the heavier targets. These differences increase when the photon frequency approaches the short-wavelength limit. When the target field is decreased to zero and the photon frequency well below the SWL, there is a smooth transition to the bremsstrahlung limit. At the SWL, on the other hand, the RI limit for vanishing target charge falls a factor of 2 below the bremsstrahlung theory. A related phenomenon concerns the deviations in the (photon or electron) angular distributions when calculated, respectively, from the triply and fourfold differential RI cross sections (the latter taken at the electron energy that provides the peak value of the cross section) which are present at the SWL but absent at lower frequencies. All these SWL-pecularities arise from the fact that the cusp-like forward peak structure is superimposed on a background which is shaped by the Compton profile [22].

For the radiative electron capture to bound states we have derived a relativistic formulation of the impulse approximation and have again employed the Sommerfeld-Maue wavefunctions for the electron. We have compared it to results from the nonrelativistic impulse approximation and have found similar photon angular distributions, but a lower global intensity if the proper relativistic prescription is used. We have also contrasted the REC photon angular distributions to those from the doubly differential RI cross section at the short-wavelength limit. Whereas for the collision system investigated the RI angular distribution is comparatively flat with shallow minima near 0° and 180°, the REC distribution has a pronounced maximum near 100° with very deep minima at 0° and 180°. We ascribe this dissimilarity to the different momentum distributions of the outgoing electron in the ground state and near the ionization threshold, respectively. The supplementary comparison of our model with available partial-wave results for REC is not so conclusive for RI as in the case of bremsstrahlung. We attribute the considerable deviations between our model and the literature results for the forward-emitted REC photons in collisions with very heavy projectiles to the necessity of very close collisions when the electron is captured into the K-shell. For electrons in unbound final states, on the contrary, the small-distance part of the wavefunction, where the semirelativistic functions fail, is expected to be not so important.

We conclude that the radiative ionization presents itself indeed as a link between bremsstrahlung and radiative capture to bound states. However, RI exhibits a good deal of peculiarities which make it worth while an object of study for its own sake. Further experiments, in particular on an absolute scale which is independent of theory, are highly desirable.

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Figure Captions

Fig.1
Doubly differential RI cross section from 223.2 MeV/amu U^{90+}+ N_2 ($v = 80.77$ a.u.) for an emission angle $\vartheta_k = 132°$ as a function of photon energy ω. Shown are RI from the K-shell (———) and L-shell ($- - -$) of N as well as the total RI from N_2 ($- \cdot - \cdot -$) as described in the text. Experimental data (■) from Ludziejewski et al [10].

Fig.2
Doubly differential RI cross section per electron from 223.2 MeV/amu U^{90+}+N_2 at $\omega = 65$ keV as a function of photon emission angle ϑ_k. Theory: ——— . Experiment: ■, Ludziejewski et al [10]. Also shown is the photon linear polarization P relating to $d^2\sigma/d\omega d\Omega_k$ for this process (left-hand scale without dimension, $- \cdot - \cdot -$).

Fig.3
Doubly differential cross section for electron emission at $\vartheta_f = 1.5°$ in 30 MeV/amu Ar^{18+}+ H collisions (a) and at $\vartheta_f = 15°$ in 30 MeV/amu Kr^{36+}+ He collisions (b) as a function of kinetic electron energy $E_{f,kin} = E_f - mc^2$. ——— , RI; $- \cdot - \cdot -$, ECC.

Fig.4
Doubly differential cross section for electron emission at $\vartheta_f = 3°$ in 90 MeV/amu U^{88+}+ N collisions as a function of $E_{f,kin}$. ——— , RI; $- \cdot - \cdot -$, ECC.

Fig.5
Doubly differential cross section for RI (a) and ECC (b) from 90 MeV/amu U^{88+}+ N_2 at $\vartheta_f = 0 \pm 1.9°$. Experiment (■, from Nofal et al [11, 35]) is normalized to theory in the maximum.

Fig.6
Doubly differential RI cross section for one target K-shell electron in 90 MeV/amu U^{88+}+ N collisions at $\vartheta_f = 0$ averaged over the detector resolution θ_0 as a function of kinetic electron energy. $\theta_0 = 0.5°$ ($- \cdot - \cdot -$), $1°$ ($- - - -$), $3°$ (———), $5°$ ($\cdots\cdots$) and $10°$ (- - - -). The peak maxima are marked by vertical lines, and the vertical line on the abscissa marks the cusp energy $(\gamma - 1)c^2 = 49.37$ keV.

Fig.7
Doubly differential cross section for electron emission at $\vartheta_f = 3°$ from U^{88+}+ N collisions in the respective peak maximum for RI (———) and ECC ($- \cdot - \cdot -$) as a function of collision momentum γv. The experimental data for $\gamma v = 61.67$ a.u.(■) are from Nofal [35]. The RI datum point is normalized to theory; ECC is calculated from the measured ratio.

Fig.8
Crossing momentum γv_{cr} as a function of target nuclear charge Z_T. Shown is γv_{cr} as obtained from the doubly differential RI and ECC cross sections evaluated in the RI-peak maximum at $\vartheta_f = 5°$ for one-electron capture from the target K-shell ($- \cdot - \cdot -$) and for the summed capture from all shells of a neutral target (———) by a Xe^{54+} projectile. The black dots are the calculated values and the lines are eye-guides.

Fig.9
Projectile-frame triply differential cross section for K-shell RI from Xe^{54+} + T in coplanar

geometry for $\tilde{E}_0 = (\gamma - 1)mc^2 = 100$ keV, $\theta'_f = 10°$, $\varphi = 0$ as a function of photon angle θ'_k. $\theta'_k > 180°$ corresponds to $2\pi - \theta'_k$ for $\varphi = 180°$. φ is the relative azimuthal angle between photon and electron. T = Ar ($- \cdot - \cdot -$), C ($- - - -$), H ($\cdots\cdots$), $Z_T = 0.3$ (- - - -) and the bremsstrahlung result ($Z_T = 0$, ———). The upper bunch of curves is for $\omega' = 60$ keV, the lower bunch is at the SWL ($\omega' = 94.71,\ 99.4,\ 99.97,\ 99.984,\ 99.985$ keV for $Z_T = 18,\ 6,\ 1,\ 0.3,\ 0$, respectively).

Fig.10
Projectile-frame triply differential cross section for K-shell RI from Xe^{54+}+ T in coplanar geometry ($\varphi = 0$) for $\tilde{E}_0 = 100$ keV, $\theta'_k = 30°$ as a function of electron angle θ'_f. The steep upper curves correspond to $\omega' = 60$ keV, the lower curves to the SWL. T = Ar ($- \cdot - \cdot -$), C (- - - - -) and H (———).

Fig.11
Projectile-frame triply differential cross section for K-shell RI from Xe^{54+}+ T in noncoplanar geometry for $\tilde{E}_0 = 100$ keV, $\theta'_f = 10°$, $\theta'_k = 30°$ as a function of φ. The upper curves correspond to $\omega' = 60$ keV, the lower curves to the SWL. T = Ar ($- \cdot - \cdot -$) and H (———).

Fig.12
Polarization corresponding to $d^2\sigma'_\lambda/d\omega' d\Omega'_k$ for K-shell RI from collisions of Au^{79+} with H (———) and Ar ($- \cdot - \cdot -$) at $\tilde{E}_0 = 500$ keV, $\omega' = 450$ keV as a function of θ'_k. Comparison is made with the polarization of bremsstrahlung from equivelocity ($v = 118.25$ a.u.) $e+$ Au^0 collisions (•, [47]).

Fig.13
Polarization P (a) and projectile-frame triply differential cross section $d^3\sigma'/d\Omega'_f d\omega' d\Omega'_k$ (b) for K-shell RI from collisions of Au^{79+} with H (——), C($- - -$) and Ar ($- \cdot - \cdot -$) at $\tilde{E}_0 = 500$ keV, $\omega' = 450$ keV, $\theta'_f = 10°$ and $\varphi = 0$ as a function of θ'_k.

Fig.14
Polarization corresponding to (target-frame) $d^4\sigma_\lambda/dE_f d\Omega_f d\omega d\Omega_k$ for RI at the SWL from collisions of Ag^{47+} with one electron targets H (——), C ($- - -$) and Ar ($- \cdot - \cdot -$) (a) and of C^{6+} ($- - - -$), Ag^{47+} (———) and U^{92+} ($- \cdot - \cdot -$) with H (b) as a function of ϑ_k. The parameters are $\tilde{E}_0 = 300$ keV $= E_{f,kin}$, $\vartheta_f = 1°$, $\varphi = 0$ and $\omega = \omega_{peak}(\vartheta_k)$ from (2.11).

Fig.15
Singly differential projectile-frame cross section $d\sigma'/d\omega'$ for bremsstrahlung from Au as a function of ω'. Partial-wave results for 50 keV (——) and 500 keV ($- \cdot - \cdot -$) electrons colliding with Au^{79+} [52, 54]. Experimental data for 500 keV $e+$ Au^0 (■, [53]). Frame-transformed RI results for Au^{79+}+ H at $\tilde{E}_0 = 50$ keV ($- - - -$) and 500 keV (- - - - -).

Fig.16
Doubly diffferential projectile-frame cross section $d^2\sigma'/d\omega' d\Omega'_k$ for bremsstrahlung from Au (upper curves) and Al (lower curves) at a collision energy of $\tilde{E}_0 = 500$ keV and $\omega' = 480$ keV as a function of photon emission angle. Partial-wave results (——) for Au^{79+} and Al^{13+}, respectively [39]. The experimental data for Au^0 are from Aiginger (■, taken from [39]) and from Motz (•, [53]). The data for Al^0 are from Motz (∘, [53]). Frame-transformed RI results for equivelocity Au^{79+}+ H ($- \cdot - \cdot -$) and Al^{13+}+ H ($- - - -$).

Fig.17
Triply differential bremsstrahlung cross section $d^3\sigma'/d\Omega'_f d\omega' d\Omega'_k$ from 300 keV $e+$ Au collisions at $\omega' = 150$ keV and $\theta'_f = 0$. Shown are the partial-wave results ($- - -$) and Elwert-Haug results (- - - -) for Au^{79+} as well as for a screened target (———, partial-wave result; $\cdots\cdots$, modified Elwert-Haug theory). Taken from [51].

Fig.18
Singly differential cross section for REC into the K-shell of bare projectiles with $Z_P = 50$ (a) and 70 (b) at 100 MeV/amu as a function of photon angle ϑ_k. The partial-wave results from

[7] are for a free initial electron (———), the present REC results use a hydrogen target (– – – –). For comparison, the nonrelativistic result for $Z_P = 50$ colliding with H ($- \cdot - \cdot -$) is included in (a). In (b), the spin-flip contributions to the singly differential cross section are shown separately ($- \cdot - \cdot -$, partial-wave expansion; - - - -, Sommerfeld-Maue functions).

Fig.19
Doubly differential cross section for photons from the K-shell REC in 100 MeV/amu Xe^{54+}+ H collisions at the REC peak frequency (4.14) (———) and for RI photons at threshold (ω from (2.11) for $E'_{f,kin} = 10^{-3}$ keV, $- \cdot - \cdot -$) and for $E'_{f,kin} = 21.8$ keV (– – – –). Included is the singly differential REC cross section $d\sigma/d\Omega_k$ (left-hand scale in b/sr, $\cdots\cdots$). All calculations use semirelativistic wavefunctions.

Fig.20
Polarization of photons from collisions with a 300 MeV/amu Ar^{18+} projectile as a function of photon emission angle. Shown are accurate REC results for capture of a free electron into the argon K-shell (———) and REC results using Sommerfeld-Maue functions and a hydrogen target (– – – –). The difference between P relating to $d^2\sigma/d\omega d\Omega_k$ at ω_{peak}^{REC} and to $d\sigma/d\Omega_k$ is indistinguishable. Included are RI results relating to $d^2\sigma/d\omega d\Omega_k$ for ω from (2.11) corresponding to the SWL ($E'_{f,kin} = 10^{-3}$ keV, $- \cdot - \cdot -$).