Polarization transfer in relativistic electron-nucleus bremsstrahlung

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Abstract

The emission of circularly polarized photons during the scattering of fast spin-polarized electrons from heavy nuclei is studied within the Dirac–Sommerfeld-Maue approach. Predictions are made for the dependence of the polarization correlations C_{32} and C_{12} on collision energy, photon energy and nuclear charge. A comparison with pilot measurements of the transmission asymmetry, sensitive to C_{32}, for 3.5 MeV e + Pb verifies that the polarization transfer increases with photon energy for small emission angles.

The precise knowledge of the polarization transfer from the beam particles to the photons emitted in a bremsstrahlung process, combined with an accurate measurement of the photon polarization via recently developed Compton polarimetry [1]-[4], provides a promising method to determine the degree of beam polarization and its change with time during an experiment [5]. Such knowledge is important for nuclear structure investigations. Bremsstrahlung may also be used as a source of radiation for a variety of applications, among those for the study of parity conservation in photofission experiments which requires a high degree of circular polarization of the photons [6]. Early theoretical predictions [7, 8], based on the Sommerfeld-Maue (SM) approximation introduced by Bethe and Maximon [9], indicated that the polarization transfer is particularly large at the short-wavelength limit (SWL). There, the electron transfers all its kinetic energy to the photon such that relativistic (spin) effects become highly important. These SM results for the polarization correlations were topped by accurate relativistic partial-wave calculation for a selection of collision systems [10]-[12], and only recently a systematic partial-wave study up to collision energies of 3 MeV has become available [13]. At even higher energies the partial-wave theory (which then suffers from convergence problems) may be supplemented at the SWL by the Dirac–Sommerfeld-Maue (DSM) theory [14, 15] where the slow scattered electron is described by an exact Dirac state and the fast incoming electron by an SM function.

We consider the case where the target is a bare nucleus with charge number Z and where the scattered electron is not observed. The doubly differential cross section $d\sigma$ for the emission of a photon with momentum $\boldsymbol{k}$, energy $\omega = kc$ and polarization $\boldsymbol{e}_\lambda$ into the solid angle $d\Omega_k$ is given (in atomic units, $\hbar = m = e = 1$) by [16, 17]

$$d\sigma \equiv \frac{d^2\sigma}{d\omega\, d\Omega_k} = \frac{4\pi^2 \omega k_f E_f}{c^3\, v} \sum_{\sigma_f} \int d\Omega_f \, |\boldsymbol{e}_\lambda^* \, \boldsymbol{W}_{rad}(\sigma_f, \sigma_i)|^2 \,,$$

$$\boldsymbol{e}_\lambda^* \, \boldsymbol{W}_{rad}(\sigma_f, \sigma_i) = \int d\boldsymbol{r} \, \psi_f^{(\sigma_f)+}(\boldsymbol{r}) \, (\boldsymbol{\alpha} \boldsymbol{e}_\lambda^*) \, e^{-i\boldsymbol{k}\boldsymbol{r}} \, \psi_i^{(\sigma_i)}(\boldsymbol{r}). \tag{1}$$

In (1), $\psi_i^{(\sigma_i)}$ and $\psi_f^{(\sigma_f)}$ describe, respectively, the initial and final electronic states with total energy E_i and $E_f = E_i - \omega$, momentum $\boldsymbol{k}_i$ and $\boldsymbol{k}_f$ and spin projection σ_i and σ_f. The vector $\boldsymbol{\alpha}$ comprises the Dirac

matrices, v is the collision velocity and $d\Omega_f$ the solid angle of the scattered electron. In the DSM theory, the partial-wave representation of the Dirac state $\psi_f^{(\sigma_f)}$ is employed. In order to avoid the inclusion of more partial waves, we resort to the SM theory when ω is sufficiently below the short-wavelength limit. In the SM theory $\psi_i^{(\sigma_i)}$ and $\psi_f^{(\sigma_f)}$ are both described by Sommerfeld-Maue functions, providing $\boldsymbol{W}_{rad}$ analytically.

Denoting the spin polarization vector of the incoming electron by $\boldsymbol{n}_s = (\zeta_1, \zeta_2, \zeta_3)$ and characterizing the photon polarization by $\xi_k,\ k = 1, 2, 3$, Tseng and Pratt [11] parametrized the (doubly differential) cross section in the following way,

$$d\sigma \;=\; \frac{1}{2}\, d\sigma_0 \,(1 + C_{03}\xi_3 + C_{11}\zeta_1\xi_1 + C_{12}\zeta_1\xi_2 + C_{20}\zeta_2 + C_{23}\zeta_2\xi_3$$

$$+ C_{31}\zeta_3\xi_1 + C_{32}\zeta_3\xi_2), \tag{2}$$

where $d\sigma_0$ is the cross section for unpolarized electrons (including a sum over the photon index λ). We choose a coordinate system with the z-axis ($\boldsymbol{e}_z$) along the beam direction $\boldsymbol{k}_i$ and the x-axis ($\boldsymbol{e}_x$) along $\boldsymbol{k}$ for the photon emission angle $\theta_k = 90°$ ($\boldsymbol{k}_i$ and $\boldsymbol{k}$ define the reaction plane). The polarization $\boldsymbol{e}_\lambda$ can be written in terms of the basis vectors $\boldsymbol{e}_{\lambda_1} = (0, 1, 0)$ and $\boldsymbol{e}_{\lambda_2} = (-\cos\theta_k, 0, \sin\theta_k)$. Right (+) and left (−) circularly polarized photons are, respectively, represented by

$$\boldsymbol{e}_\pm \;=\; \frac{1}{\sqrt{2}}\,(\boldsymbol{e}_{\lambda_2} \pm i\,\boldsymbol{e}_{\lambda_1}). \tag{3}$$

This definition of $\boldsymbol{e}_\pm$ corresponds to $\xi_1 = \xi_3 = 0$ and $\xi_2 = \pm 1$, such that for circularly polarized photons the parametrization (2) reduces to

$$d\sigma \;=\; \frac{1}{2}\, d\sigma_0 \,(1 + C_{12}\zeta_1\xi_2 + C_{20}\zeta_2 + C_{32}\zeta_3\xi_2). \tag{4}$$

Defining the spin polarization direction $\boldsymbol{n}_s$ with respect to the basis $\hat{\boldsymbol{\zeta}}_1 = \boldsymbol{e}_x,\ \hat{\boldsymbol{\zeta}}_2 = -\boldsymbol{e}_y,\ \hat{\boldsymbol{\zeta}}_3 = -\boldsymbol{e}_z$ as done in our previous work [14], we can eliminate the polarization correlation C_{20} from (4) by requiring that $\boldsymbol{n}_s$ lies in the reaction plane ($\zeta_2 = 0$). Introducing the angle α_s between $\boldsymbol{n}_s$ and the z-axis, $\boldsymbol{n}_s$ can be written as $\boldsymbol{n}_s = \hat{\boldsymbol{\zeta}}_1 \sin\alpha_s - \hat{\boldsymbol{\zeta}}_3 \cos\alpha_s$ (such that in (4), $\zeta_1 = \sin\alpha_s,\ \zeta_3 = -\cos\alpha_s$).

The Stokes parameter P_3 for circularly polarized photons is defined by the asymmetry with respect to the emission of right- and left-circularly polarized photons,

$$P_3(\alpha_s) \;=\; \frac{d\sigma(\boldsymbol{e}_+) - d\sigma(\boldsymbol{e}_-)}{d\sigma(\boldsymbol{e}_+) + d\sigma(\boldsymbol{e}_-)} \;=\; C_{12}\sin\alpha_s \;-\; C_{32}\cos\alpha_s. \tag{5}$$

Thus the polarization correlations C_{32} and C_{12} can be calculated from

$$C_{32} \;=\; -P_3(0) \;\equiv\; -P_{3\|}, \qquad C_{12} \;=\; P_3(90°) \;\equiv\; P_{3\perp}, \tag{6}$$

where the subscripts $\|$ and $\perp$ indicate that the electron spin is aligned with the beam axis, respectively, is perpendicular to it.

In Fig.1 we show predictions for C_{32} for bare Au and Cu targets in the collision energy range 5 – 15 MeV where the collision energy is defined by the kinetic energy $E_{i,kin} = E_i - c^2$. The increase of C_{32} with $E_{i,kin}$ at the forward angles, already predicted by Haug [8] for low-Z targets and supported by partial-wave results [11, 13] for $Z \leq 79$ in the energy range between 0.05 – 3 MeV, is seen to continue at the higher energies, and for $E_{i,kin} \gtrsim 10$ MeV, the polarization transfer at the SWL is complete at least up to $\theta_k \approx 120°$. For a lower nuclear charge C_{32} is slightly higher at the SWL, but it becomes independent of Z for $\theta_k \to 0$. This Z-independence at $\theta_k = 0$ holds also in the ultrarelativistic limit [7]. However, it is neither true at smaller collision energies (below, say, 1 MeV [5, 11]) nor at the larger photon angles. When the photon energy gets smaller, C_{32} strongly decreases in modulus and also its dependence on $E_{i,kin}$ and Z weakens considerably (Fig.1).

In the experiments, performed with 3.5 MeV electrons colliding with Pb, the intensity difference for the emission of right- and left-circularly polarized photons is measured with the help of a magnetized absorber. The photons colliding with the polarized absorber electrons suffer Compton scattering, the cross section for which is polarization-dependent according to the Klein-Nishina formula [18]. The measured transmission asymmetry $A = (I_+ - I_-)/(I_+ + I_-)$ where $\pm$ corresponds, respectively, to right- and left-circularly polarized photons, is thus not simply given by the product of C_{32} with the polarization degree of the electron beam, but is in addition proportional to the polarization-dependent part of the Compton scattering cross section [3, 5]. The upper theory curve in Fig.2 is a calculation of A [3] using a beam polarization degree of 8 percent, the C_{32} predictions from the ultrarelativistic Olsen formula [7] (neglecting the electron rest energy), and the scattering cross section based on the Klein-Nishina formula. In the lower theory curve, which is in better accord with the preliminary data points, the Olsen predictions for C_{32} are replaced by calculations of C_{32} within the DSM model (at $\omega = 3.5$ MeV), respectively with the SM theory (for the lower ω).

When the electrons are spin-polarized perpendicular to the beam axis, such that the polarization transfer is given in terms of the polarization correlation C_{12}, the angular dependence changes dramatically. As seen from Fig.3, C_{12} does not only vanish at $\theta_k = 0$ and 180° (like all other polarization correlations except C_{32} [11]), but at the SWL it exhibits a double-well structure with a strong minimum at the backmost angles, which is the deeper, the higher the collision energy. We expect, however, that at energies exceeding 15 MeV this minimum will shrink again similar to the case of the spin asymmetry correlation C_{20} [14] when going beyond 10 MeV. As concerns the maximum at small angles, ultrarelativistic estimates provide the bound $C_{12} \lesssim 0.1$ [20] in agreement with our results. Also the Z-dependence of C_{12} is much more pronounced than for C_{32}.

The accuracy of the present calculations for the polarization correlation C_{32} related to longitudinally spin-polarized electrons is quite satisfactory. This may be inferred from Fig.4 where for a gold target the DSM results at $E_{i,kin} = \omega = 3$ MeV are compared with the screened-atom partial-wave results of Yerokhin and Surzhykov [13]. We recall that for the polarization correlations screening effects are unimportant [11], being at most 10 percent for impact energies between 0.1 – 1 MeV, and decreasing with $E_{i,kin}$ beyond 1 MeV [13]. Even in the case of $E_{i,kin} = 1$ MeV and $\omega = 0.9$ MeV (and more so for decreasing ω) the differences between the SM approximation and the partial-wave results are small. This result is somewhat unexpected, taken into consideration that the Sommerfeld-Maue functions, particularly for electrons close to the nucleus, are inappropriate at low energies [19]. In fact, for the Stokes parameter P_2, related to linearly polarized photons, a systematic comparison with partial-wave results has shown that the SM functions become only reliable at sufficiently high electron energies E such that $Zc/E \lesssim 0.02$ [15]. For C_{12} the present calculations indeed are in severe disagreement with partial-wave results in a situation comparable to Fig.3 (3 MeV e + Au, $\omega = 1.5$ MeV [21]). We therefore expect that, for a gold target, $E_{i,kin} \gtrsim 15$ MeV is needed for an adequate representation of C_{12} within the DSM model.

In conclusion, we have applied the Dirac–Sommerfeld-Maue theory (and for less energetic photons the SM theory) for predicting the angular dependence of the polarization correlations related to circularly polarized photons and electrons spin-polarized in the reaction plane. It was shown that for C_{32}, describing the polarization transfer from longitudinally spin-polarized electrons, the DSM respective SM theory performs well even for heavy targets at energies down to 1 MeV. Also the measured dependence of the photon transmission asymmetry on the photon energy for 3.5 MeV longitudinally spin-polarized electrons colliding with lead is well reproduced. Experiments with fast, transversely spin-polarized electrons, sensitive to C_{12}, are highly welcome to test the theoretical predictions.

Acknowledgments

References

[1] S.Tashenov et al, Phys. Rev. Lett. 97 (2006) 223202.

[2] S.Tashenov, T.Bäck, R.Barday, B.Cederwall, J.Enders, A.Khaplanov, Yu. Poltoratska, K.-U.Schässburger, A.Surzhykov, submitted for publication.

[3] F.Nillius, K.Aulenbacher, PESP Proceedings (2010).

[4] R.Märtin et al, Contributed paper to CAARI, Denton, Texas (2010).

[5] R.Barday et al, PESP Proceedings (2010).

[6] S.Tashenov, private communication.

[7] H.Olsen, L.C.Maximon, Phys. Rev. 114 (1959) 887.

[8] E.Haug, Phys. Rev. 188 (1969) 63.

[9] H.A.Bethe, L.C.Maximon, Phys. Rev. 93 (1954) 768.

[10] H.Brysk, C.D.Zerby, S.K.Penny, Phys. Rev. 180 (1969) 104.

[11] H.K.Tseng, R.H.Pratt, Phys. Rev. A 7 (1973) 1502.

[12] I.J.Feng, I.B.Goldberg, Y.S.Kim, R.H.Pratt, Phys. Rev. A 28 (1983) 609.

[13] V.A.Yerokhin, A.Surzhykov, Phys. Rev. A 82 (2010) 062702.

[14] D.H.Jakubassa-Amundsen, Phys. Rev. A 82 (2010) 042714.

[15] D.H.Jakubassa-Amundsen, A.Surzhykov, Eur. Phys. J. D (2011) in print.

[16] W.Heitler, *Quantum Theory of Radiation*, Clarendon Press, Oxford, 1954, 3rd edition, §25.

[17] H.K.Tseng, R.H.Pratt, Phys. Rev. A 3 (1971) 100.

[18] O.Klein, Y.Nishina, Z. Phys. 52 (1929) 853; Y.Nishina, Z. Phys. 52 (1929) 869.

[19] J.K.Fink, R.H.Pratt, Phys. Rev. A 7 (1973) 392.

[20] H.Olsen, L.C.Maximon, Phys. Rev. 110 (1958) 589.

[21] V.A.Yerokhin, private communication.

Figure captions

Fig.1
Polarization correlation C_{32} for longitudinally spin-polarized electrons as a function of photon angle θ_k.
a) Au^{79+}: upper curves, SWL (DSM results) for beam energy $E_{i,kin} = 5$ MeV (————-), 10 MeV $(-\cdot-\cdot-)$ and 15 MeV $(----)$. Lower curves, $\omega/E_{i,kin} = \frac{1}{5}$ (SM results) for $E_{i,kin} = 5$ MeV (———) and 10 MeV $(-\cdot-\cdot-)$.
b) Au^{79+} (————-) and Cu^{29+} $(-\cdot-\cdot-)$ for 5 MeV: uppermost curves, SWL (DSM results); middle curves, $\omega = 3$ MeV ($\omega/E_{i,kin} = \frac{3}{5}$), lowermost curves, $\omega = 1$ MeV ($\omega/E_{i,kin} = \frac{1}{5}$) (all SM results).

Fig.2
Transmission asymmetry A (in percent) for 3.5 MeV longitudinally spin-polarized electrons colliding with a lead target as a function of photon energy ω for $\theta_k = 0$. The experimental data ($\bullet$) are from Nillius and

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Aulenbacher [3], calculations from Nillius and Aulenbacher using Olsen's formula for C_{32} $(----)$, and scaled down by the ratio C_{32}(present)$/C_{32}$(Olsen) (———).

Fig.3
Polarization correlation C_{12} for transversely spin-polarized electrons at the SWL as a function of photon angle θ_k. Au^{79+}: 5 MeV (———), 10 MeV $(-\cdot-\cdot-)$, 15 MeV $(----)$. Cu^{29+}: 5 MeV $(\cdots\cdots\cdots)$ (DSM results).

Fig.4
Polarization correlation C_{32} for a gold target as a function of photon angle θ_k. Shown are (screened-target) partial-wave results (———) from [13] for 3 MeV electrons at the SWL ($\omega = 3$ MeV, upper curve), for 1 MeV electrons and photons of energy $\omega = 0.9$ MeV (middle curve) and $\omega = 0.5$ MeV (lower curve), as well as results (for Au^{79+}) within the DSM model $(-\cdot-\cdot-$, SWL at $E_{i,kin} = 3$ MeV) and the SM approach ($E_{i,kin} = 1$ MeV and $----$, $\omega = 0.9$ MeV; $\cdots\cdots$, $\omega = 0.5$ MeV).