MATHEMATICAL GAUGE THEORY 1

FLORIN BELGUN

Gauge theory plays a central role in quantum field theories (the Standard Model) and has very important applications to the differential geometry and topology of low-dimensional manifolds.

The goal of this introductive lecture series is to describe the mathematical background of the Maxwell equations, of the Yang-Mills functional, of characteristic classes and describe some applications of the classical index Theorem of Atiyah and Singer.

More precisely, the main topics include:

- Vector fields, Flows, the Frobenius Theorem
- Lie Groups and Lie Algebras, homogeneous spaces
- Principal bundles, connections and holonomy
- The curvature tensor, gauge transformations and gauge invariance
- Chern classes and Chern-Weil theory
- The Atiyah-Singer index theorem
- Mathematical expression of Maxwell's equations and of the Yang-Mills functional
- Applications

The lectures will be given in German or English depending on the audience.

References:

- K. Nomizu, *Lie Groups and Differential Geometry*, Mathematical Society of Japan, 1956.
- S. Kobayashi, K. Nomizu, *Foundations of Differential Geometry I, II*, Interscience Publishers, 1963–1969.
- D. Bleeker, *Gauge Theory and Variational Principles*, Addison Wesley, 1981.
- F.W. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, Springer, 1983.
- J. Milnor, J.D. Stasheff, *Characteristic Classes*, Princeton University Press, 1974.

Prerequisites: Basic knowledge of differentiable manifolds (e.g. as in the lectures *Differenzierbare Mannigfaltigkeiten*. Additional knowledge of vector bundles, differential forms and Riemannian metrics is recommended, but these notions will be revised in the first lectures.

Schedule: 2 lecture + 1 exercise class per week (2 x 90 + 90 minutes). The lecture takes place on Monday at 4 p.m. and Wednesdays at 10 a.m. in **B 040**. The excercise class is scheduled for Wednesdays 8 a.m. also in B040. Because of numerous holidays in the summer term, some of the excercise classes will sometimes be used as lecture classes.

Validation: The lecture series will finish around June 15 (after about 20 lectures), hence it corresponds to **6 ECTS** points.

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