Evolutionary PDE's in perfectly plastic fluid theory

Dominic Breit and Joachim Naumann

The equation of motion for an incompressible perfectly plastic fluid on a bounded domain $\Omega \subset \mathbb{R}^d$, $d = 2, 3$, during the time intervall $(0, T)$ reads as

$$-\partial_t u + \operatorname{div} \sigma = \nabla \pi - f \quad \text{on} \quad Q := \Omega \times (0, T).$$

Here $u : Q \to \mathbb{R}^d$ is the velocity field, $\pi : Q \to \mathbb{R}$ the pressure, $\sigma : Q \to \mathbb{R}^{d\times d}$ denotes the stress deviator and $f : Q \to \mathbb{R}^d$ an external system of volume forces. Between σ and the symmetric gradient $\varepsilon(u)$ of the velocity field we have the following relation (constitutive law) which was introduced by von Mises in 1913 (g is the yield value)

$$\varepsilon(u) = 0 \quad \Rightarrow \quad |\sigma| \leq g, \quad \varepsilon(u) \neq 0 \quad \Rightarrow \quad \sigma = \frac{g}{|\varepsilon(u)|}\varepsilon(u).$$

We show the existence of a weak solution

$$(u, \sigma) \in L^1(0, T; \mathrm{BD}_{\mathrm{div}}(\Omega)) \times L^\infty(Q, \mathbb{R}^{d\times d})$$

to the equation above where the constitutive law has to be understand in a measure theoretical fashion. The space $\mathrm{BD}(\Omega)$ denotes the class of L^1-functions whose distributional symmetric gradient generates a bounded Radon measure.