On the global regularity of solutions of the singular p-Laplacian system

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In this talk I am going to present some regularity results for a nonlinear system of p-Laplacian type under Dirichlet boundary conditions:

$$\begin{cases} -\nabla\cdot\left[\,(\,\mu+|\,\nabla\, u|\,)^{p-2}\,\nabla\, u\,\right]=\,f \;\; \text{in}\;\Omega\,, \\ u=\,0 \;\; \text{on}\;\partial\Omega\,. \end{cases}$$

The vector fields $u=(u_1(x),\cdots,u_N(x))$ and $f=(f_1(x),\cdots,f_N(x))$ are defined on a bounded and smooth domain $\Omega\subset\mathbb{R}^n$, $n\geq 3$. Here $\mu\geq 0$ and $p\in(1,2]$. When $\mu=0$, the system is the well-known singular p-Laplacian system. Our interests concern up to the boundary, full regularity of the first derivatives and integrability of the second derivatives of the solutions. We prove $W^{2,q}(\Omega)$ regularity, for any arbitrarily large q. Therefore, by a standard embedding, we get the α- Hölder continuity, up to the boundary, of the gradient of the solution, for any $\alpha<1$. The results are obtained for p belonging to some interval $[p_0,2)$, for a suitable lower exponent p_0. In particular, if Ω is a convex domain, solutions belong to $W^{2,2}(\Omega)$ for any $p\in(1,2]$.