Bohmian Mechanics

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Quantum Mechanics and Reality. While quantum mechanics, as presented in physics textbooks, provides us with a formalism, it does not attempt to provide a description of reality. The formalism is a set of rules for computing the probability distribution of the outcome of essentially any experiment (within the realm of quantum mechanics). A description of reality, in contrast, would tell us what processes take place on the microscopic level that lead to the random outcomes that we observe, and would thus explain the formalism. While the correctness of the formalism is almost universally agreed upon, the description of the reality behind the formalism is controversial. It has also been doubted whether a description of reality needs to conform with ordinary standards of logical consistency, and whether to have such a description is desirable at all. Indeed, it has often been claimed that quantum theory forces us to reject the reality of an external world that exists objectively, independently of the human mind.

Bohmian Mechanics and Quantum Mechanics. Bohmian mechanics, which is also called the de Broglie-Bohm theory, the pilot-wave model, and the causal interpretation of quantum mechanics, is a version of quantum theory discovered by Louis de Broglie in 1927 (de Broglie, 1928) and rediscovered by →David Bohm in 1951 (Bohm, 1952). It is a theory providing a description of reality, compatible with all of the quantum formalism and all of ordinary logic. In Bohmian mechanics a system of particles is described in part by its wave function, evolving according to Schrödinger's equation, the central equation of quantum theory. However, the wave function provides only a partial description of the system. This description is completed by the specification of the actual positions of the particles. The latter evolve according to the "guiding equation," which expresses the velocities of the particles in terms of the wave function. Thus in Bohmian mechanics the configuration of a system of particles evolves via a deterministic motion choreographed by the wave function. In particular, when a particle is sent into a two-slit apparatus, the slit through which it passes and where it later arrives on a screen are completely determined by its initial position and wave function.

As such, Bohmian mechanics is a counterexample to the claim that quantum theory is incompatible with the reality of an objective external world. It is a "realistic quantum theory," and, since its formulation makes no reference to observers, it is also a "quantum theory without observers." For historical reasons, it has been called a "hidden-variables theory." The existence of Bohmian mechanics shows that many of the radical epistemological consequences usually drawn from quantum mechanics by physicists and philosophers alike are unfounded. It shows that there is no need for contradictory notions such as "complementarity"; that there is no need to imagine a particle as somehow being in two places at the same time or physical quantities as having unsharp values; and that there is no need to assume that human consciousness intervenes in physical process (by, e.g., collapsing wave functions). Bohmian mechanics resolves all of the paradoxes of quantum mechanics, eliminating its weirdness and mystery.

The Measurement Problem. The most commonly cited of the conceptual difficulties that plague quantum mechanics is the measurement problem, or, what amounts to more or less the same thing, the paradox of Schrödinger's cat. The problem is as follows: Suppose that the wave function of any individual system provides a complete description of that system. When we analyze the process of measurement in quantum mechanical terms, we find that the after-measurement wave function for system and apparatus arising from Schrödinger's equation for the composite system typically involves a superposition over terms corresponding to what we would like to regard as the various possible results of the measurement—e.g., different pointer orientations. It is difficult to discern in this description of the after-measurement situation the actual result of the measurement—e.g., some specific pointer orientation. By contrast, if, like Einstein, one regards the description provided by the wave function as incomplete, the measurement problem vanishes: With a theory or interpretation like Bohmian mechanics, in which the description of the after-measurement situation includes, in addition to the wave function, at least the values of the variables that register the result, there is no measurement problem. In Bohmian mechanics pointers always point.

The Equations of Bohmian Mechanics. Bohmian mechanics is the minimal completion of Schrödinger's equation, for a nonrelativistic system of particles, to a theory describing a genuine motion of particles. For Bohmian mechanics the state of a system of N particles is described by its wave function $\psi = \psi(\mathbf{q}_1, \ldots, \mathbf{q}_N) = \psi(q)$, a complex- (or spinor-) valued function on the space of possible configurations q of the system, together with its actual configuration Q defined by the actual positions $\mathbf{Q}_1, \ldots, \mathbf{Q}_N$ of its particles. The theory is then defined by two evolution equations: *Schrödinger's equation*

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi \,,$$

for $\psi = \psi_t$, the wave function at time t, where H is the nonrelativistic (Schrödinger) Hamiltonian, containing the masses of the particles and a potential energy term, and a first-order evolution equation, *the guiding equation*:

$$\frac{d\mathbf{Q}_j}{dt} = \frac{\hbar}{m_j} \mathrm{Im} \frac{\psi^* \nabla_j \psi}{\psi^* \psi}(\mathbf{Q}_1, \ldots, \mathbf{Q}_N) \,,$$

for $Q = Q(t)$, the configuration at time t, the simplest first-order evolution equation for the positions of the particles that is compatible with the Galilean (and time-reversal) covariance of the Schrödinger evolution. Here $\hbar$ is Planck's constant divided by 2π, m_j is the mass of the j-th particle, and ∇_j is the gradient with respect to the coordinates of the j-th particle. If ψ is spinor-valued, the products in numerator and denominator should be understood as scalar products. If external magnetic fields are present, the gradient should be understood as the covariant derivative, involving the vector potential. For an N-particle system these two equations (together with the detailed specification of the Hamiltonian, including all interactions contributing to the potential energy) completely define the Bohmian mechanics.

It is perhaps worth noting that the guiding equation is intimately connected with the de Broglie relation $\mathbf{p} = \hbar\mathbf{k}$, proposed by de Broglie in late 1923, the consideration of which quickly led Schrödinger to the discovery of his wave equation in late 1925 and early 1926. The de Broglie relation connects a particle property, momentum $\mathbf{p} = m\mathbf{v}$, to a wave property, the wave vector $\mathbf{k}$ of a plane wave $\psi(\mathbf{q}) = e^{i\mathbf{k}\cdot\mathbf{q}}$. From this one can easily guess the guiding equation as the simplest possibility for an equation of motion for Q for the case of a general wave function ψ.

Bohmian mechanics inherits and makes explicit the nonlocality implicit in the notion, common to just about all formulations and interpretations of quantum theory, of a wave function on the configuration space of a many-particle system (→John Bell and Bell's Theorem). It accounts for all of the phenomena governed by nonrelativistic quantum mechanics, from spectral lines and scattering theory to superconductivity and quantum computing. In particular, the usual measurement postulates of quantum theory, including collapse of the wave function, probabilities given by the absolute square of probability amplitudes constructed from the wave function, and the role of self-adjoint operators as observables emerge from an analysis of the two equations of motion—Schrödinger's equation and the guiding equation.

Quantum Randomness. The statistical significance of the wave function was first recognized in 1926 by Max Born, just after Schrödinger discovered his famous wave equation. Born postulated that the configuration Q of a quantum system is random, with probability distribution given by the density $|\psi(q)|^2$. Under the influence of the developing consensus in favor of the Copenhagen interpretation, $|\psi(q)|^2$ came to be regarded as giving the probability of *finding* the configuration Q were this to be measured, rather than of the configuration actually *being* Q, a notion that was supposed to be meaningless. In accord with these quantum probabilities, quantum measurements performed on a system with definite wave function ψ typically yield random results.

For Bohmian mechanics the $|\psi(q)|^2$-distribution has a particularly distinguished status. As an elementary consequence of Schrödinger's equation and the guiding equation, it is *equivariant*, in the sense that these equations are compatible with respect to the $|\psi(q)|^2$-distribution. More precisely, this means that if, at some time t, the configuration $Q(t)$ of a Bohmian system were random, with distribution given by $|\psi_t(q)|^2$, then this would also be true for any other time. This distribution is thus called the *quantum equilibrium distribution*.

A Bohmian universe, though deterministic, evolves in such a manner that an *appearance* of randomness emerges, precisely as described by the quantum formalism. To understand how this comes about one must first appreciate that in a world governed by Bohmian mechanics, measurement apparatuses, too, are made of Bohmian particles. In a Bohmian universe tables, chairs and other objects of our everyday experience are simply agglomerates of particles, described by their positions in physical space, and whose evolution is governed by Bohmian mechanics.

Then, for the analysis of quantum measurements, the following observation is crucial: to the extent that the result of any quantum measurement is registered configurationally, at least potentially, the predictions of Bohmian mechanics for the result must agree with those of orthodox quantum theory (assuming the same Schrödinger equation for both) provided that the configuration Q (of the largest system required for the analysis of the measurement, with wave function ψ) is random, with probability density in fact given by the quantum equilibrium distribution, the quantum mechanical prediction for the distribution of Q.

To justify this quantum equilibrium hypothesis is a rather delicate matter, one that has been explored in considerable detail (Dürr, Goldstein, and Zanghì, 1992): it can be shown that the probabilities for positions given by the quantum equilibrium distribution $|\psi(q)|^2$ emerge naturally from an analysis of "equilibrium" for the deterministic dynamical system defined by Bohmian mechanics, in much the same way that the Maxwellian velocity distribution emerges from an analysis of classical thermodynamic equilibrium.

Typicality. Thus, with Bohmian mechanics, the statistical description in quantum theory indeed takes, as Einstein (1949, p. 672) anticipated, "an approximately analogous position to the statistical mechanics within the framework of classical mechanics." A key ingredient for appreciating the status and origin of such a statistical description is the notion of *typicality*, a notion that, historically, goes back to Ludwig Boltzmann's mechanical analysis of the second law of thermodynamics. In Bohmian mechanics, a property P is typical if it holds true for the overwhelming majority of histories $Q(t)$ of a Bohmian universe. More precisely, suppose that Ψ_t is the wave function of a universe governed by Bohmian mechanics; a property P, which a solution $Q(t)$ of the guiding equation for the entire universe can have or not have, is called *typical* if the set $S_0(P)$ of all initial configurations $Q(0)$ leading to a history $Q(t)$ with the property P has size very close to one,

$$\int_{S_0(P)} |\Psi_0(q)|^2 dq = 1 - \varepsilon\,, \quad 0 \le \varepsilon \ll 1\,,$$

with "size" understood relative to the $|\Psi_0|^2$ distribution on the configuration space of the universe. For instance, think of P as the property that a particular sequence of experiments yields results that look random (accepted by a suitable statistical test), governed by the appropriate quantum distribution. One can show, using the *law of large numbers*, that P is a typical property; see (Dürr, Goldstein, and Zanghì, 1992) for a thorough discussion.

Operators as Observables. It would appear that because orthodox quantum theory supplies us with probabilities for a huge class of quantum observables and not merely for positions, it is a much richer theory than Bohmian mechanics, which seems exclusively concerned with positions. In this regard, as with so much else in the foundations of quantum mechanics, the crucial remark was made by Bell (1987, p. 166): "[I]n physics the only observations we must consider are position observations, if only the positions of instrument pointers. It is a great merit of the de Broglie-Bohm picture to force us to consider this fact. If you make axioms, rather than definitions and theorems, about the 'measurement' of anything else, then you commit redundancy and risk inconsistency."

In Bohmian mechanics, the standard quantum observables, represented by self-adjoint operators, indeed arise from an analysis of quantum experiments, as "definitions and theorems": For any quantum experiment, take as the relevant Bohmian system the combined system that includes the system upon which the experiment is performed as well as all the measuring instruments and other devices used in performing the experiment (together with all other systems with which these have significant interaction over the course of the experiment). The initial configuration is then transformed, via the guiding equation for the big system, into the final configuration at the conclusion of the experiment. With the quantum equilibrium hypothesis, i.e., regarding the initial configuration of this big system as random in the usual quantum mechanical way, with distribution given by $|\psi|^2$, the final configuration of the big system, including in particular the orientation of instrument pointers, will be distributed according to $|\psi|^2$ at the final time. If the experiment happens to be "measurement-like," and the outcomes of the experiment are calibrated by an assignment of numerical values to the different pointer orientations, then the induced probability distributions of these results will be given by the familiar quantum measurement postulates—i.e., by the spectral measure, relative to the wave function of the system upon which the experiment is performed, of a self-adjoint operator A associated with the experiment (Dürr, Goldstein, and Zanghì, 2004), in which case we speak, in orthodox quantum theory, of a "measurement of A."

The Stern–Gerlach experiment provides an illuminating example: By means of a suitable interaction (with a magnetic field), the parts of the wave function that lie in different eigenspaces of the relevant spin operator become spatially separated, and the result ("up" or "down") is thus a function of the final, detected position of the particle, concerning which we can only predict that it is random and distributed according to $|\psi|^2$ at the final time. By calibrating the outcomes of the experiment with numerical values, e.g., $+1$ for upper detection, and -1 for lower detection, it is not difficult to see that the probability distribution for these values can be conveniently expressed in terms of the quantum mechanical spin operators—for a spin-1/2 particle given by the Pauli spin matrices.

Contextuality and Naive Realism About Operators. Since the result of a Stern–Gerlach experiment depends upon, not just the initial position and the initial wave function of the particle, but also on a choice among several magnetic fields that could be used to perform a Stern–Gerlach measurement of the same spin operator, this experiment is not a genuine measurement in the literal sense, i.e., it does not reveal a preexisting value associated with the spin operator itself. In fact, there is nothing the least bit mysterious, or even nonclassical, about the nonexistence of such values associated with operators. Thus the widespread idea that in a realistic quantum theory all quantum observables should possess actual values, which is in fact impossible by the Kochen–Specker theorem, was from the outset not as reasonable at it may have appeared, but rather was based on taking operators as observables too seriously—an attitude, almost implicit in the word "observable," that can be called "naive realism about operators."

Another consequence concerns *contextuality*, the notion that the result of an experiment depends not just on "what observable the experiment measures" but on more detailed information that conveys the "context" of the experiment. Contextuality is often regarded as deep, mysterious and even close to Bohr's complementarity. However, in Bohmian mechanics it boils down to the trivial insight that the result of an experiment depends on the experiment.

Collapse of the Wave Function. According to the quantum formalism, performing an ideal quantum measurement on a quantum system causes a random jump or "collapse" of its wave function into an eigenstate of the observable measured. But while in orthodox quantum theory the collapse is merely superimposed upon the unitary evolution of the wave function, without a precise specification of the circumstances under which it may legitimately be invoked—and this ambiguity is nothing but another facet of the measurement problem—Bohmian mechanics consistently embodies both the unitarity evolution and the collapse of the wave function as appropriate. Concerning the evolution of the wave function, Bohmian mechanics is indeed formulated in terms of Schrödinger's equation alone. However, since observation implies interaction, a system under observation cannot be a closed system but rather must be a subsystem of a larger system that is closed, e.g., the entire universe. And there is no reason a priori why a subsystem of a Bohmian universe should itself be a Bohmian system, even if the subsystem happens to be "closed." Indeed, it is not even clear a priori what should be meant by the wave function of a subsystem of a Bohmian universe.

The configuration Q of this larger system, this universe, naturally splits into X, the configuration of the subsystem, and Y, the configuration of its environment. Suppose the universe has wave function $\Psi = \Psi(q) = \Psi(x, y)$. According to Bohmian mechanics, this universe is then completely described by Ψ, evolving according to Schrödinger's equation, together with X and Y. Thus there is a rather obvious choice for what should be regarded as the wave function of the subsystem, namely the *conditional wave function* $\psi(x) = \Psi(x, Y)$, obtained by plugging the actual configuration of the environment into the wave function of the universe. Moreover, taking into account the way that the conditional wave function $\psi_t(x) = \Psi_t(x, Y(t))$ depends upon time, it is not difficult to see that it obeys Schrödinger's equation for the subsystem when that system is suitably decoupled from its environment and, using the quantum equilibrium hypothesis, that it randomly collapses according to the usual quantum mechanical rules under precisely those conditions on the interaction between the subsystem and its environment that define an ideal quantum measurement.

Uncertainty. It follows from the quantum equilibrium hypothesis and the definition of the conditional wave function that when the (conditional) wave function of a subsystem is ψ, its configuration must be random, with distribution $|\psi(x)|^2$, even if its full microscopic environment Y—itself grossly more than what we could conceivably have access to—were taken into account. In other words, the (conditional) wave function ψ of a subsystem represents maximal information about its configuration X. Thus, in a universe governed by Bohmian mechanics there are sharp, precise, and irreducible limitations on the possibility of obtaining knowledge, limitations which can in no way be diminished through technological progress leading to better means of measurement. This *absolute uncertainty* is in precise agreement with Heisenberg's uncertainty principle. [The fact that knowledge of the configuration of a system must be mediated by its (conditional) wave function may partially account, from a Bohmian perspective, for how orthodox physicists could identify the state of a quantum system—its complete description—with its (collapsed) wave function without encountering any practical difficulties.]

Objections. A great many objections have been and continue to be raised against Bohmian mechanics. Most of these objections have little or no merit. The most serious one is that Bohmian mechanics does not account for phenomena such as pair creation and annihilation characteristic of quantum field theory. However, this is not an objection to Bohmian mechanics per se, but merely a recognition that quantum field theory explains a great deal more than does nonrelativistic quantum mechanics, whether in orthodox or Bohmian form. It does, however, underline the need to find an adequate, if not compelling, Bohmian version of quantum field theory, and of gauge theories in particular, a problem that is pretty much wide open.

A related objection is that Bohmian mechanics cannot be made Lorentz invariant, by which it is presumably meant that no Bohmian theory—no theory that could be regarded somehow as a natural extension of Bohmian mechanics—can be found that is Lorentz invariant. The main reason for this belief is the manifest nonlocality of Bohmian mechanics.

But nonlocality, as John Bell has argued and the experiments have shown, is a fact of nature (→John Bell and Bell's Theorem). Moreover, concerning the widespread belief that standard quantum theories have no difficulty incorporating relativity while Bohmian mechanics does, there is much less here than meets the eye. On the one hand, one should keep in mind that the empirical import of orthodox quantum mechanics relies on both the unitary evolution of the state vector (or the equivalent unitary evolution of the operators in the Heisenberg representation) and the collapse or reduction of the state vector (or any other equivalent devise that incorporates the effect of observation or measurement). But the Lorentz invariance of this part of the theory has rarely been considered in a serious way—most of the empirical import of standard relativistic quantum mechanics is in the so-called "scattering regime." But if this were done, arguably, the tension between Lorentz invariance and quantum nonlocality would soon become manifest. On the other hand, a variety of approaches to the construction of a Lorentz invariant Bohmian theory have in fact been proposed, and some toy models formulated.

What is a Bohmian Theory? Finding a satisfactory relativistic version of Bohmian mechanics and extending Bohmian mechanics to quantum field theory are topics of current research and we shall not attempt to give an overview here. (Some remarks, however, are given in the next section.) Rather, we shall briefly sketch what we consider to be the general traits of any theory that could be regarded as a natural extension of Bohmian mechanics. Three requirements seem essential to us: 1. The theory should be based upon a clear ontology, the *primitive ontology* representing what the theory is fundamentally about—the basic kinds of entities (such as the particles in Bohmian mechanics) that are to be the building blocks of everything else, including tables, chairs, and measurement apparatuses. 2. There should be a quantum state vector, a wave function, that evolves according to the unitary quantum evolution and whose role is to somehow generate the motion for the variables describing the primitive ontology. 3. The predictions should agree (at least approximately) with those of orthodox quantum theory—at least to the extent that the latter are unambiguous. Note that we do not regard as essential either the deterministic character of the dynamics of the primitive ontology, or that the latter should be given by particles described by their positions in physical three-dimensional space—a field ontology, or a string ontology would do just as well.

In short, a "Bohmian theory" is merely a quantum theory with a coherent ontology. But when the theory is regarded in these very general terms, an interesting philosophical lesson emerges: in the structure of a Bohmian theory one can recognize some general features that are indeed common to all "quantum theories without observers," that is, to all precise formulations of quantum theory not based on such vague and imprecise notions as "measurement" or "observer"—such as Ghirardi-Rimini-Weber-Pearle's "dynamical reduction" models or Gell-Mann and Hartle's "decoherent histories" approach. One essential feature is the primitive ontology of the theory—what the theory is fundamentally about. The other very general and crucial feature is the sort of explanation of physical phenomena the theory should provide: an *explanation based on typicality.* Not just for a Bohmian theory, but for any physical theory with probabilistic content, the physical import of the theory must arise from its provision of a notion of typical space-time histories, specified for example via a probability distribution on the set of all possible histories of the primitive ontology of the theory.

History and Present Status. In 1951 Bohm rediscovered de Broglie's 1927 pilot-wave model and showed that the quantum measurement formalism, based on non-commuting operators as observables, emerged from the basic principles of de Broglie's theory. Since then Bohmian mechanics has been developed and refined: noteworthy are Bell's clarification of the axioms of the theory and the analysis of the status of probability and the role of typicality (Bell, 1987; Dürr, Goldstein, and Zanghì, 1992), as well as the investigations of quantum non-equilibrium (Valentini, 2002). Several ways of extending Bohmian mechanics to quantum field theory have been proposed. One (Bohm, 1952), for bosons (i.e., force fields), is based on an actual field configuration on physical three-dimensional space that is guided by a wave functional according to an infinite-dimensional analogue of the guiding equation (see also Bohm and Hiley, 1993; Holland, 1993). Another proposal (Dürr, Goldstein, Tumulka, and Zanghì, 2004) relies on seminal work by Bell (1987, p. 173) and ascribes trajectories to the electrons or whatever sort of particles the quantum field theory is about; however, in contrast to the original Bohmian mechanics, this proposal involves a stochastic dynamics, according to which particles can be created and annihilated.

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