Two Arrows of Time in Nonlocal Particle Dynamics

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Abstract

Considering what the world would be like if backwards causation were possible is usually mind-bending. Here we discuss something that is easier to study, a model that incorporates a very restricted sort of backwards causation. Whereas it probably prohibits signalling to the past, it allows nonlocality while being fully covariant. And that is what constitutes its value: it may be a step towards a fully covariant version of Bohmian mechanics.

In this paper I will introduce to you a dynamical system—a law of motion for point particles—that has been invented [4] as a toy model in Bohmian mechanics; for more about Bohmian mechanics, see Detlef Dürr's contribution to this volume. What makes it remarkable is that it has two arrows of time, and that precisely its having two arrows of time is what allows it to perform what it was designed for: to have effects travel faster than light from their causes (in short, *nonlocality*) without breaking Lorentz invariance. Why should anyone desire such a behavior of a dynamical system? Because Bell's nonlocality theorem [1] teaches us that any dynamical system violating Bell's inequality must be nonlocal in this sense. And Bell's inequality, after all, is violated in Nature.

Well, it is easy to come up with a nonlocal theory if one assumes that one of the Lorentz frames is preferred to the others: simply assume a mechanism of cause and effect (a sort of interaction in the widest sense) that operates *instantaneously* in the preferred frame. That is what nonrelativistic theories usually do. In other frames, these nonlocal effects will either travel at a

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superluminal ($> c$) but finite velocity or precede their causes by a short time span. This doesn't mean that causal loops could arise since in the preferred frame effects never precede causes; but the entire notion of a preferred frame is, of course, against the spirit of relativity. Without a preferred frame, to find a nonlocal law of motion is tricky, and much agonizing has been spent on this. About one way to achieve this you will learn below.

Let's come back first to the two arrows of time. They are opposite arrows, in fact. But unlike the arrows considered in Lawrence Schulman's contribution to this volume, they are not both thermodynamical arrows. One of the two is the thermodynamical arrow. Let's call it Θ. It arises, as emphasized first by Ludwig Boltzmann and in this conference by Schulman, not from whatever asymmetry in the microscopic laws of motion, but from boundary conditions. That is, from the condition that the initial state of the universe be taken from a particular subset of phase space (corresponding to, say, a certain low entropy macrostate), while the final state is not subjected to any such conditions—except in the scenarios studied by Schulman. The dynamical laws considered in discussions of the thermodynamic arrow of time are usually time reversal invariant. But not so ours! It explicitly breaks time symmetry, and that is how another arrow of time comes in: an arrow of microscopic time asymmetry, let's call it C. Such an arrow must be assumed before writing down the equation of motion (6). In addition, the equation of motion is easier to solve in the direction C than in the other direction. Doesn't it seem ugly and unnatural to introduce a time asymmetry? Sure, but we will see it buys us something: Lorentz invariant nonlocality.

Remember that such an arrow is simply absent in Newtonian mechanics or time symmetric theories. So it is not surprising that the microscopic arrow C is not the source of the macroscopic time arrow Θ, even more, the direction of Θ is completely independent of the direction of C. Θ depends on boundary conditions, and not on the details of the microscopic law of motion. And in our case, Θ will even be opposite to C. Since inhabitants of a hypothetical universe will regard the thermodynamical arrow as their natural time arrow, related to macroscopic causation, to memory, and to apparent free will, you should always think of Θ as pointing towards the future, whereas C is pointing to what we call the past.

It's time to say what the equation of motion is. The equation is trying to be as close to Bohmian mechanics as possible, to be an immediate generalization, and to have Bohmian mechanics as its nonrelativistic limit. To remind you of how Bohmian mechanics works, you take the wave function (which is supposed to solve Schrödinger's equation—without ever having to collapse), plug in the positions of all the particles (here is where a notion of simultaneity comes in), and from that you compute the velocity of any particle by applying a certain formula, Bohm's law of motion, which amounts to dividing the probability current by the probability density. Now, for a Lorentz-invariant version, we first have to worry about the wave function.

There are three respects in which the wave function of nonrelativistic quantum mechanics (or Bohmian mechanics, for that matter) conflicts with relativity: (a) the dispersion relation $E = p^2/2m$ at the basis of the Schrödinger equation is nonrelativistic, (b) the wave function is a function of $3N$ position but only one time coordinate, (c) the collapse of the wave function is supposed instantaneous. Physicists were very successful at solving (a) by means of the Klein–Gordon or Dirac equation, but it is a little too early for enthusiasm since we still face (b) and (c). We will worry about (c) later, and focus on (b) now. The obvious answer is to introduce a wave function ψ of $4N$ coordinates, that is one time coordinate for each particle, in other words ψ is a function on (space-time)^N. You get back the nonrelativistic function of $3N + 1$ coordinates after picking a frame and setting all time coordinates equal. Multi-time wave functions were first considered by Dirac *et al.* in 1932 [2], but what they didn't mention was that the N time evolution equations

$$\mathrm{i}\hbar \frac{\partial \psi}{\partial t_i} = H_i \psi \tag{1}$$

needed for determining ψ from initial data at $t = 0$ do not always possess solutions. They are usually inconsistent. They are only consistent if the following condition is satisfied:

$$[H_i, H_j] = 0 \text{ for } i \neq j\,. \tag{2}$$

This is easy to achieve for non-interacting particles and tricky in the presence of interaction. Indeed, to my knowledge it has never been attempted to write down consistent multi-time equations for many interacting particles, although this would seem an immediate and highly relevant problem if one desires a a manifestly covariant formulation of relativistic quantum mechanics. We will here, however, stay on the easy side and simply consider a system of noninteracting particles. We take the multi-time equations to be Dirac equations in an external field A_μ,

$$\mathbf{1} \otimes \cdots \otimes \underbrace{\gamma^\mu}_{i\text{th place}} \otimes \cdots \otimes \mathbf{1} \left(\mathrm{i}\frac{\partial}{\partial x_i^\mu} - eA_\mu(x_i)\right) \psi = m\psi \tag{3}$$

where ψ : (space-time)$^N \to (\mathbb{C}^4)^{\otimes N}$, and e and m are charge and mass, respectively. The corresponding Hamiltonians commute trivially since the derivatives act on different coordinates and the matrices on different indices.

A Dirac wave function naturally defines a tensor field

$$J^{\mu_1\cdots\mu_N} := \overline{\psi}\,\gamma^{\mu_1}\otimes\cdots\otimes\gamma^{\mu_N}\,\psi\,, \tag{4}$$

and according to the variant of Bohmian mechanics for Dirac wave functions, the velocity of particle i is, in the preferred frame,

$$\dot{Q}_i^\mu \propto J^{0\ldots\overset{i}{\mu}\ldots 0}(Q_1\ldots Q_N) \tag{5}$$

where the proportionality factor depends on the choice of parametrization of the world line $Q_i^\mu(s)$ (and thus is physically irrelevant). The coordinates taken for the other particles are their positions *at the same time*, $Q_j^0 = Q_i^0$. Instead of a Lorentz frame, one can take any foliation of space-time into spacelike hypersurfaces for the purpose of defining simultaneity-at-a-distance [3]. The theory I'm about to describe, in contrast, uses the hypersurfaces naturally given by the Lorentzian structure on space-time: the light cones. More precisely: the future light cones—and that is how the time asymmetry comes in.

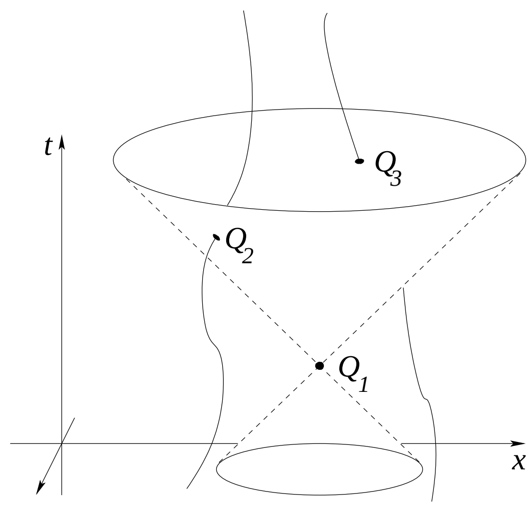

Figure 1: How to choose the N space-time points where to evaluate the wave function, as described in the text.

So here are the steps: first solve (3), so you know ψ on (space-time)^N. Then, compute the tensor field J on (space-time)^N according to (4). For determining the velocity of particle i at space-time point Q_i, find the points Q_j where the other particles cross the future light cone of Q_i, as depicted in figure 1. Plug these N space-time points into the field J and get a single tensor. Find out what the 4-velocities $u_j^{\mu_j}$ of the other particles at Q_j are. Use these to contract all but one index of J. By definition, the resulting vector is, up to an irrelevant proportionality factor, the 4-velocity we've been looking for:

$$\dot{Q}_i^{\mu_i} \propto J^{\mu_1 \cdots \mu_N}(Q_1 \ldots Q_N) \prod_{j \neq i} u_{j\mu_j}(Q_j)\,. \tag{6}$$

One can show [4] that this 4-velocity is always timelike or null.

This law of motion is what can be called an ordinary differential equation with advanced arguments, because the velocity depends on the positions (and velocities) of other particles at future times, indeed with a *variable* delay span $Q_j^0 - Q_i^0$. It may seem to complicate things considerably that what happens here depends on the *future* rather than past behavior of the other particles, but that is an artifact of perspective: look at the equation of motion (6) in the other time direction, that is in the direction C, and notice it now has only *retarded* arguments. That is a more familiar sort of differential delay equation that gives rise to no logical or causal problems. So this theory, although involving a mechanism of backwards causation, is provably paradox free, since no causal loops can arise: first solve the wave equation for ψ in the usual direction Θ, then solve the equation of motion in the opposite direction C.

Unfortunately, there is no obvious probability measure on the set of solutions to (6). This is different from the situation in Bohmian mechanics, where the $|\psi|^2$ distribution is conserved, a fact crucial for the probability predictions of that theory. The lack of such a measure for the model considered here makes it impossible to say whether or not this theory violates Bell's inequality, which is a relation between probabilities. But this law of motion takes what is perhaps the biggest hurdle on the way towards a fully covariant law of motion conserving the $|\psi|^2$ distribution, what Bell's theorem says is a necessary condition: nonlocality. I should add that in the nonrelativistic limit, the future light cone approaches the hyperplane $t =$ const. and the law of motion approaches the "hypersurface Bohm–Dirac law" (5) conserving $|\psi|^2$.

How does nonlocality come about in this model? That has to do with the two arrows of time, pointing in opposite directions. Had we chosen them to point in the same direction, the theory would have been local, because what happens at Q_i would only depend on (what we call) the past light cone. But in this model, we evaluate ψ on the future light cone of Q_i, which means ψ has, in its multi-time evolution, gone through all the external fields at spacelike separation from Q_i. And that is how the velocity at Q_i may be influenced by the field imposed by an experimenter at spacelike separation from Q_i.

And what is the story then about problem (c) above, the instantaneous collapse? The first thing to say is that collapse is not among the basic rules of this model, or any Bohmian theory. That simply disposes of problem (c). But something more should be said, since the collapse rule can be derived in Bohmian mechanics: even if the wave function of Schrödinger's cat remains forever a superposition, *the cat* (formed by the particles, of course) is either dead or alive, with probabilities determined by $|\psi|^2$, and the wave packet of the dead cat (i.e., the corresponding term in the superposition) is too far away in configuration space to influence the motion of the live cat. In the model we are concerned with here, everything just said still applies, except the probabilities of course.

To this day, thinking about time, time's arrows, and relativity remains a source of the unexpected.

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