On the diagonalization of large matrices

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Abstract

We will present a procedure of how to diagonalize large matrices by viewing them as operators on a infinite dimensional Hilbert space and solve a corresponding eigenvalue problem on it. For several types of matrices this eigenvalue problem becomes extremely simple and its solutions give very good approximations to the original problem. As a special case we will be able to diagonalize *all sparse large Toeplitz matrices* almost exactly - because of its importance in applications this case will be explained on an instructive example in some detail. The qualitative (and to a very good accuracy also the quantitative) behaviour of the spectra and the eigenstates is captured entirely by means of a very simple calculation. Additionally the procedure provides for interesting links between recursion relations, differential equations and matrices - problems which may seem very different to start with turn out to be equivalent.

The procedure has advantages: it works and it is very elementary. However, it has the disadvantage that it is only useful in certain cases and at the moment it seems that rather few general statements can be made.