LMU Munich — Prof. Dr. Sabine Jansen, Dr. Robert Helling

Summer term 2018 — S. Das, S. Aristarkhov

Exercises on Mathematical Statistical Physics
Math Sheet 5

Problem 1 Let $(\Omega, \mathcal{F}, P)$ be a probability space and $C_1, \ldots, C_n$ be a partition of Ω, define $\mathcal{C}$ as for Ex. 1 sheet 4. Assume that

$$\forall i : P(C_i) > 0 \tag{1}$$

holds. Consider a function

$$f : \Omega \to \mathbb{R}, \tag{2}$$

that is measureable and satisfies $\int_\Omega |f|^2 dP < \infty$, define $\varphi(\omega) := \sum_{i=1}^n 1_{C_i}\varphi_i$ with for each $i \in \{1, \ldots, n\}$

$$\varphi_i := \frac{\int_{C_i} |f|^2(\omega) dP(\omega)}{P(C_i)}. \tag{3}$$

As always you are allowed to use results from previous exercises on the same or a different sheet.

a) Check that $\int_\Omega |f| dP < \infty$ and

$$\int_\Omega f dP = \sum_{j=1}^n \varphi_j P(C_i) = \int_\Omega \varphi dP \tag{4}$$

hold.

b) Show that for each $\mathcal{C}$-measureable function $g : \Omega \to \mathbb{R}$

$$\int_\Omega fg dP = \int_\Omega \varphi g dP \tag{5}$$

holds.

c) Show that φ is the orthogonal projection of f onto $L^2(\Omega, \mathcal{C}, P)$ with respect to $\langle\cdot,\cdot\rangle$.

Problem 2 Let $\Omega := \{-1,1\}^{\mathbb{Z}^d}$ and $\mathcal{F}$ be given by the product σ-algebra introduced in the lecture. Consider the event $A := \{\omega = (\omega_x)_{x\in\mathbb{Z}^d} \in \Omega \mid \exists x \in \mathbb{Z}^d : \omega_x = 1\}$ that there is at least one site with spin up.

a) Show that A is measureable.

b) Show that there is a sequence $(f_n)_{n\in\mathbb{N}}$ of local maps such that $f_n \to 1_A$ (indicator function of the set A) pointwise.

c) Show that 1_A is not quasi-local.

Problem 3 Let $\Omega = \{-1,1\}^{\mathbb{Z}}$ and f be given by

$$f(\omega) = \begin{cases} \lim_{n\to\infty} \frac{1}{2n+1} \sum_{k=-n}^{n} \omega_k & \text{if the limit exists} \\ 0 & \text{else} \end{cases}. \tag{6}$$

Prove or disprove whether f is quasilocal.

Please, hand in into the letter box not later than Wednesday 23.05 14:00.