Bell-Type Quantum Field Theory

Work with Detlef Dürr, Shelly Goldstein, & Roderich Tumulka

Nino Zanghì

Università di Genova

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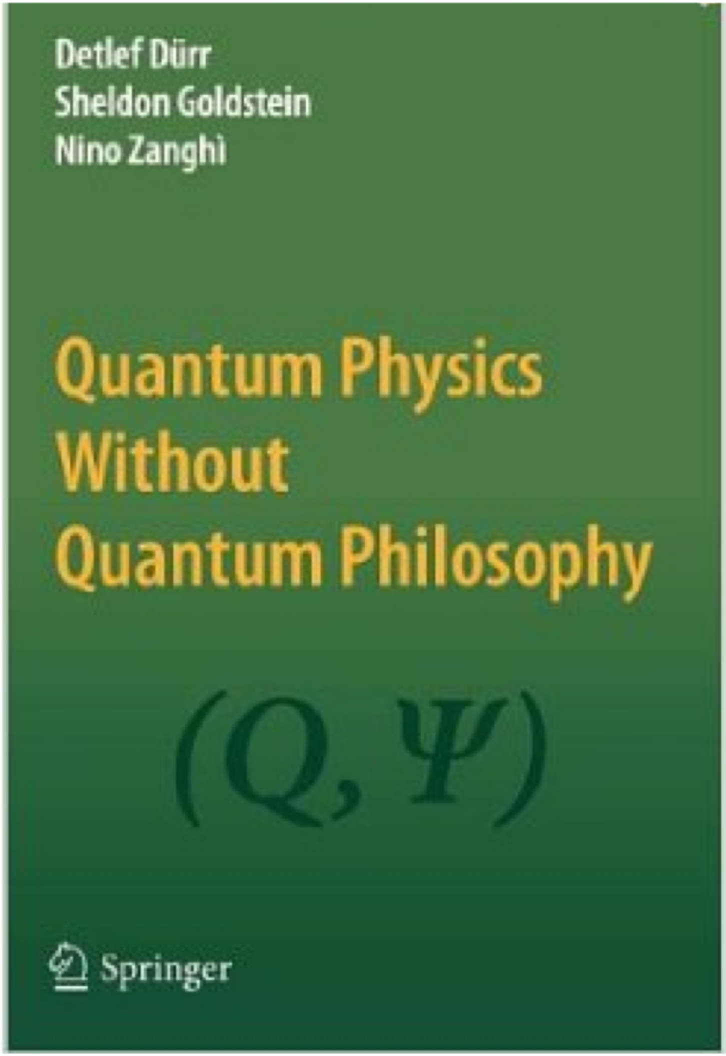
Detlef Dürr
Sheldon Goldstein
Nino Zanghì
Quantum Physics
Without
Quantum Philosophy
(Q, Ψ)
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Two Parts:

I From Orthodox Quantum Theory to Bohmian Mechanics

II From Bohmian Mechanics to Orthodox Quantum Fields

Quotes with small (but non zero) relation with the talk

It may perhaps now be expected that I define extension [space] to be either substance or accident or else simply nothing. But not at all so: for it has a certain mode of existence proper to itself, which suits neither substances nor accidents. (Newton, *De Gravitatione*)

. . . what can be said at all should be said clearly. . . (Wittgenstein, *Tractatus*)

What the true definition of Pragmatism may be, I find it very hard to say; but in my nature it is a sort of instinctive attraction for living facts. (C. S. Peirce, *Harvard Lectures on Pragmatism*)

. . . pragmatism is, in itself, no doctrine of metaphysics, no attempt to determine any truth of things. It is merely a method of ascertaining the meanings of hard words and of abstract concepts. (C. S. Peirce, *Pragmatism*)

Part I: From Orthodox Quantum Theory to Bohmian Mechanics

Orthodox Quantum Theory

- $\text{Prob}(Z \in \Delta|\psi) = \langle\psi, P_A(\Delta)\psi\rangle$

  A s.a. operator $\quad [\,|\langle\psi|\alpha\rangle|^2\,]$

- $\psi_0 \;\to\; \psi_t$ unitary evolution when no measurements are performed

- $\psi \;\to\; \psi_\alpha$ collapse after measurement with result $Z = Z_\alpha$

Orthodox Quantum Theory

"what is ψ" $\quad\rightarrow\quad$ "***what is the role of*** ψ"

In a recent admired work on Analytic Mechanics it is stated that we understand precisely the effect of force, but what force itself is we do not understand! This is simply a self-contradiction. . . . ***if we know what the effects of force are, we are acquainted with every fact which is implied in saying that a force exists, and there is nothing more to know*** (C. S. Peirce, **"How to make our ideas clear"**)

Orthodox Quantum Theory

ψ has a role in the behavior of macroscopic objects ("measurement instruments") during "quantum measurements".

State

$$(Z, \psi)$$

$\psi = \psi(\mathbf{q}_1, \ldots, \mathbf{q}_N)$, Z: macroscopic variable

Local beables (PO)

The ***local beables*** (primitive ontology) are

the mathematical counterparts in the theory to real events at definite places and times in the real world (as distinct from the many purely mathematical constructions that occur in the working out of physical theories, as distinct from things which may be real but not localized, and as distinct from the 'observables' of other formulations of quantum mechanics, for which we have no use here.) [J.S. Bell]

Local beables (PO)

This is a pretentious name for a theory which hardly exists otherwise, but which ought to exist. The name is deliberately modelled on 'the algebra of local observables'. The terminology, be-able as against observ-able, is not designed to frighten with metaphysic those dedicated to realphysic. It is chosen rather to help in making explicit *some notions already implicit in, and basic to, ordinary quantum theory*. For, in the words of Bohr, 'it is decisive to recognize that, however far the phenomena transcend the scope

of classical physical explanation, the account of all evidence must be expressed in classical terms'. It is the ambition of the theory of local beables to bring these 'classical terms' into the equations, and not relegate them entirely to the surrounding talk.

Bohmian Mechanics

Complete state description (Q, ψ)

ψ as above, Q: microscopic variable, e.g.,

$$Q = (\mathbf{Q}_1, \ldots, \mathbf{Q}_N)$$

$\mathbf{Q}_i$ positions of particles (LB)

BM is fundamentally about ***microscopic LB*** (particles, or fields or strings ...), what we call PO (primitive ontology). The role of ψ is to govern the motion of the PO.

Bohmian Mechanics

The equations of motion

$$\frac{d\mathbf{Q}_k}{dt} = \frac{\hbar}{m_k}\mathrm{Im}\frac{\psi^*\nabla_k\psi}{\psi^*\psi}(\mathbf{Q}_1,\ldots,\mathbf{Q}_N)$$

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi \qquad \left[H = -\sum_{k=1}^{N}\frac{\hbar^2}{2m_k}\nabla_k^2 + V\right]$$

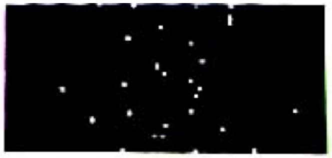
Bohmian Mechanics

Implications of BM

1. the wf of a (sub-)system
2. quantum randomness
3. operators as observables
4. absolute uncertainty
5. collapse of the wave packet
6. formal scattering
7. familiar (macroscopic) reality

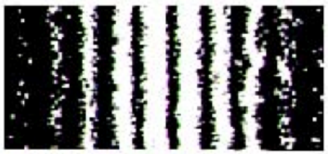
Bohmian Mechanics

Quantum Randomness



(a) After 28 electrons

(b) After 1000 electrons



(c) After 10,000 electrons

(d) Two slit electron pattern

Ψ WF universe

ψ wf subsystem

$$\frac{1}{N}\sum_{k}^{N}\delta(\mathbf{q}-\mathbf{Q}_k) \approx |\psi(\mathbf{q})|^2$$

$P_\Psi\,(\text{exceptions}|\ \text{preparation})$

small for N large

Bohmian Mechanics

Operators as Observables

macroscopic variable

$$Z = F(Q)$$

$\mathrm{Prob}(Z \in \Delta | \psi) = \langle \psi, P_A(\Delta)\psi \rangle$

A s.a. operator

Bohmian Mechanics

When the cogency of Bohm's reasoning is admitted, a ***final protest*** is often this: it is all nonrelativistic. This is to ignore that Bohm himself, in an appendix to one of the 1952 papers, already applied his scheme to the electromagnetic field. And application to scalar fields is straightforward. However until recently, to my knowledge, no extension covering Fermi fields had been made. Such an extension will be sketched here. (J.S. Bell, "Beables for quantum field theory")

Part II: From Bohmian Mechanics to Orthodox Quantum Fields

Standard Model Interaction Lagrangian

$$\mathbf{L}_{int} = \sum_{f=e,u,d} e\,Q_f\, \underbrace{\Psi_f \gamma^\mu \Psi_f \overset{\text{photon}}{A_\mu}}_{\text{QED}} +$$

$$g_2/\cos\theta_w \sum_{f=\nu,e,u,d} [\underbrace{\Psi_{Lf}\gamma^\mu\Psi_{Lf}(T^3_{fL} - Q_f \sin^2\theta_w) Z_\mu}_{\text{Left handed neutral weak int.}} + \underbrace{\Psi_{Rf}\gamma^\mu\Psi_{Rf}(- Q_f\sin^2\theta_w)Z_\mu}_{\text{Right handed neutral weak int.}}]$$

$$+ g_2/\sqrt{2}\left[\underbrace{(u_L\gamma^\mu d_L + \nu_e\gamma^\mu e_L) W^+_\mu}_{\text{flavor changing weak interactions}} + \underbrace{(d_L\gamma^\mu u_L + e_L\gamma^\mu\nu_e) W^-_\mu}_{\text{flavor changing weak interactions}}\right]$$

$$+ g_3/2 \sum_{q=u,d} \underbrace{\sum_a}_{\text{Sum over gluons}} (q_{red}, q_{green}, q_{blue})\,\gamma^\mu \underset{\text{SU(3) generators}}{\lambda^a} \begin{pmatrix} q_{red}\\ q_{green}\\ q_{blue}\end{pmatrix} \overset{\text{Gluons - carry color/anti-color}}{G^a_\mu}$$

Shows only first generation interactions.

B. Hale 11-17-10

(Towards) a General (Bohmian) Theory of Motion

Configuration of the local beables of the theory

$$Q$$

WF

$$\Psi = \Psi(q)$$

evolving according to

$$i\frac{\partial \Psi}{\partial t} = H\Psi\,, \tag{1}$$

Equivariance (statistical transparency)

From (1)

$$\frac{\partial|\Psi|^2}{\partial t}(q,t) = 2\,\mathrm{Im}\left[\Psi^*(q,t)\,(H\Psi)(q,t)\right] \tag{2}$$

If

$$H = -\frac{1}{2}\Delta + V \quad \text{(Schrödinger Hamiltonian)} \tag{3}$$

Then

$$2\,\mathrm{Im}\left[\Psi^*(q,t)\,(H\Psi)(q,t)\right] = -\mathbf{div}\,\left[\mathrm{Im}\,\Psi^*(q,t)\nabla\Psi(q,t)\right] \tag{4}$$

so that (2) becomes

$$\frac{\partial|\Psi|^2}{\partial t} = -\mathbf{div}\,\left(v\;|\Psi|^2\right)\;\;\textbf{with}\;\; v = \mathrm{Im}\,\frac{\Psi^*\nabla\Psi}{\Psi^*\,\Psi}\,. \tag{5}$$

$\Rightarrow$ deterministic motion with velocity $v = v^{\Psi,H}(q)$

$$
\begin{tikzcd}
\Psi \arrow[r] \arrow[d, "\text{Schrödinger evolution}"'] & |\Psi|^2 \arrow[d, "\text{Transport along the Bohmian flow}"] \\
\Psi_t \arrow[r] & |\Psi_t|^2
\end{tikzcd}
$$

Equivariance is an expression of the compatibility between the Schrödinger evolution for the wave function and the law $\dot{Q} = v(Q)$ governing the motion of the actual configuration.

Abstract characterization

$$
L|\Psi_t|^2 = \frac{\partial|\Psi_t|^2}{\partial t} \tag{6}
$$

where $L(\bullet) = -\text{div}\ (v\,\bullet)$ is the generator of a Markov deterministic process.

There are other motions, ***stocastic motions***, with different generators that satisfy (1), but they are not ***minimal*** (in a suitable precise mathematical sense)

Examples:

(a) $Q = (\mathbf{Q}_1, \ldots, \mathbf{Q}_N)$ (N particles)

(b) $Q = B(\mathbf{r})$ (Bosonic Field)

(c) Nelson Stochastic Mechanics (actually a 2-parameter family with different diffusions constants and drifts) for particles or bosonic fields (not minimal)

What if H is not of Schrödinger's type?

Eq. (2)

$$\frac{\partial |\Psi|^2}{\partial t}(q,t) = 2\,\mathrm{Im}\left[\Psi^*(q,t)\,(H\Psi)(q,t)\right]$$

is always valid. Find the minimal Markov generator L such that

$$L|\Psi_t|^2 = 2\,\mathrm{Im}\left[\Psi^*(q,t)\,(H\Psi)(q,t)\right]$$

Integral Operators Correspond to Jump Processes

If

$$(H\Psi)(q) = \int dq'\,\langle q|H|q'\rangle\,\Psi(q')\,. \tag{7}$$

Then the minimal Markov generator is

$$L\rho(q) = \int\limits_{q' \in \mathcal{Q}} \Big(\sigma(q|q')\rho(q') - \sigma(q'|q)\rho(q) \Big) dq' \,, \tag{8}$$

with rates

$$\sigma(q|q') = \frac{\big[2\,\mathrm{Im}\, \Psi^*(q)\, \langle q | H | q' \rangle\, \Psi(q') \big]^+}{\Psi^*(q')\, \Psi(q')}. \tag{9}$$

Process Additivity

$$L_{H_0 + H_I} = L_{H_0} + L_{H_I}$$

QFT

Fermions and Bosons

$F(\mathbf{r})$ fermionic fields; $B(\mathbf{r})$ bosonic fields are the building blocks of the Hamiltonian.

$$[B(\mathbf{r}), B(\mathbf{r}')] = 0\,, \quad [F(\mathbf{r}), F(\mathbf{r}')] \neq 0$$

Beable: fermion number ($[N(\mathbf{r}), N(\mathbf{r}')] = 0$)

The distribution of fermion number in the world certainly includes the positions of instruments, instrument pointers, ink on paper, ... and much much more. (J.S. Bell, *Beables for quantum field theory*)

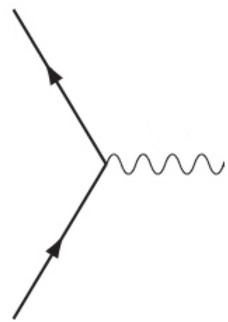
Configuration q:

$n(\mathbf{r})$ = eigenvalues of Fermion number $F^*(\mathbf{r})F(\mathbf{r})$

H_0 = FREE DIRAC $\Rightarrow L_0$ (deterministic motion of $n(\mathbf{r})$)

$$n(\mathbf{r},t) = \sum_k^N \delta(\mathbf{r} - \mathbf{Q}_k(t))$$

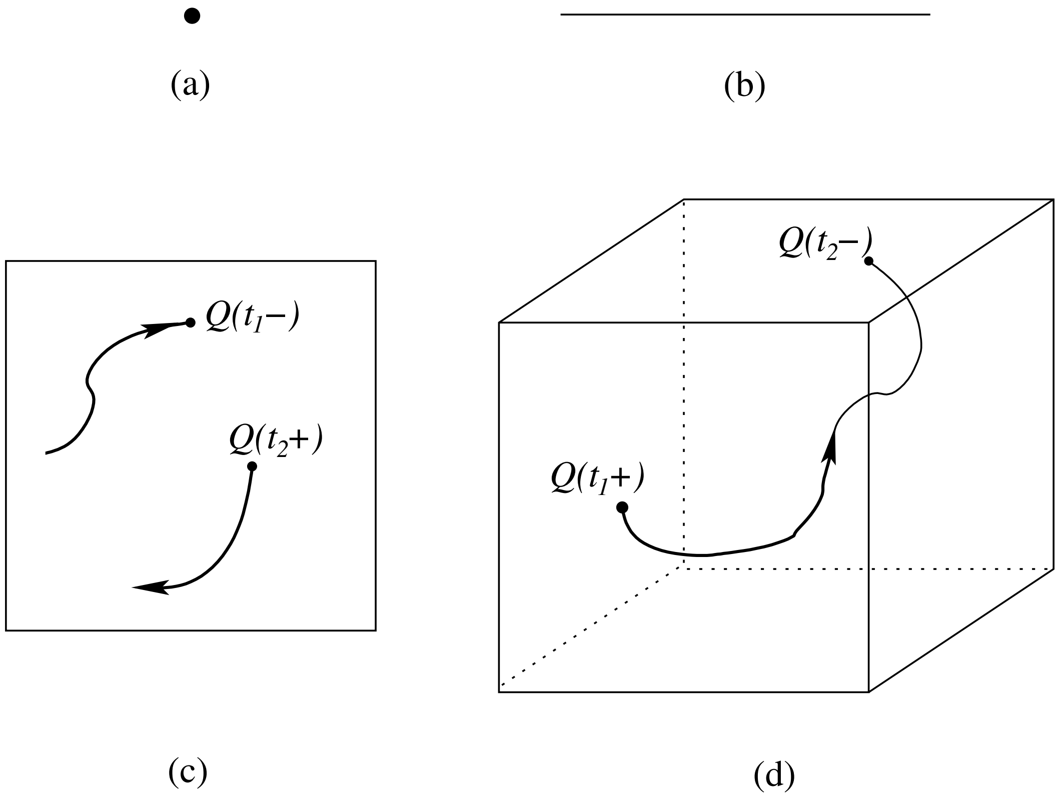
Structure of H_I (simplest model)



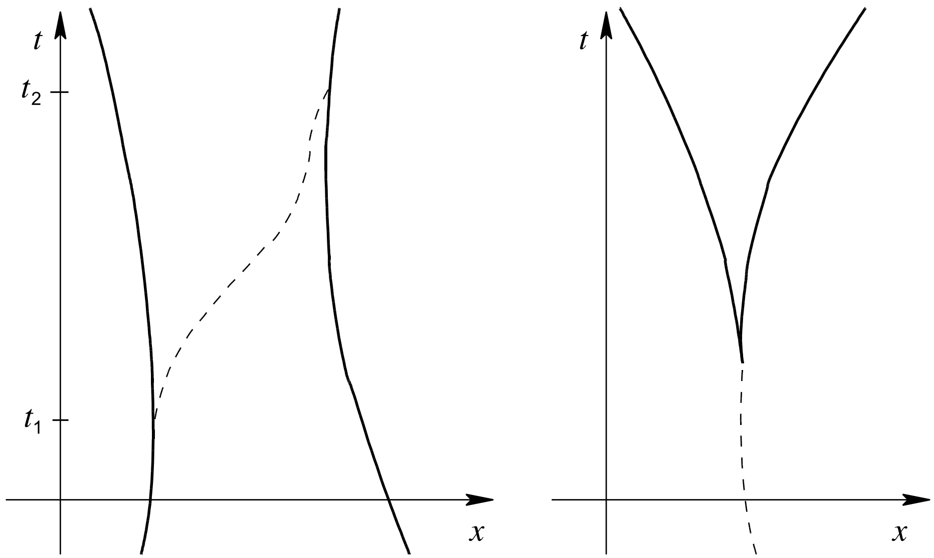
$$H_I = \int d^3\mathbf{r} B(\mathbf{r}) F^*(\mathbf{r}) F(\mathbf{r})$$

It is an integral operator in the fermion number representation $\Rightarrow L_I =$ jump process

The "$L_0 + L_I$"-motion looks like this



or



(a) (b)

(here also boson number is a beable)

Possibilities

(1) Only fermion number is a beable (Bell)

(2) Fermion number & bosonic fields (Bohm)

(3) Fermion number & boson number

(4) Only boson number

(5) Only boson fields (Struyve)

All except 3 have asymmetry between bosons (photons) and fermions (electrons)

[not all scintillations on a screen are a sign that there is a particle]

Open problems Many!!!

Somebody should work on them!

(1) Make clear the various possibilities listed above

(2) Use axiomatic frameworks (such as Wightman's) to improve what we did.

(3) Use modern renormalization methods to study the limits starting from a well defined theory with cut-offs.

(4) Try to understand the geometry of (non-Abelian) gauge theories from the Bohmian point of view.

(5) $q = g^3$ (quantum gravity).

(6) . . .

Morals

For those who do not like field operators:

The field operators are just what the doctor ordered for the efficient construction of a theory describing the creation, motion, and annihilation of particles.(DGTZ, *Bohmian Mechanics and Quantum Field Theory*)

For those who like only field operators:

But I insist that my concern is strictly professional. I think that conventional formulations of quantum theory, ***and of quantum field theory in particular***, are unprofessionally vague and ambiguous. Professional theoretical physicists ought to be able to do better. Bohm has shown us a way. (J.S. Bell, *Beables for quantum field theory*)