Quantum Physics without Quantum Philosophy

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Quantum Physics Without Quantum Philosophy

(Q, Ψ)

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Quantum philosophy, a peculiar twentieth century malady, is responsible for most of the conceptual muddle plaguing the foundations of quantum physics.

When this philosophy is eschewed, one naturally arrives at Bohmian mechanics, which is what emerges from Schrödinger’s equation for a nonrelativistic system of particles when we merely insist that “particles” means particles.
The Title

Quantum Physics
In this chapter, we shall tackle immediately the mysterious behavior in its most strange form. We choose to examine a phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the only mystery. We cannot explain the mystery in the sense of “explaining” how it works. We will tell you how it works. In telling you how it works we will have told you about the basic peculiarities of all quantum mechanics. (Richard Feynman)
Quantum theory shows us where classical logic goes awry.... It requires radically new ways of thinking. (W. Thirring)

It is clear that this result can in no way be reconciled with the idea that electrons move in paths.... In quantum mechanics there is no such concept as the path of a particle. (Landau and Lifshitz)
...the idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them... is impossible... (W. Heisenberg)
How it works

• $\text{Prob}(Z \in \Delta | \psi) = \langle \psi, P_A(\Delta) \psi \rangle$
  
  A s.a. operator $\left[ |\langle \psi | \alpha \rangle|^2 \right]$

• $\psi_0 \rightarrow \psi_t$ unitary evolution when no measurements are performed

• $\psi \rightarrow \psi_\alpha$ collapse after measurement with result $Z = Z_\alpha$
What is $\psi$?

**The mathematics is easy**

$$\psi : \mathbb{R} \times \mathbb{R}^{3N} \rightarrow \mathbb{C} \quad (\text{or } \mathbb{C}^{kN})$$

better

$$\psi : \mathbb{R} \times \mathbb{R}^3 \times \cdots \times \mathbb{R}^3 \rightarrow \mathbb{C} \quad (\text{or } \mathbb{C}^{kN})$$

**The physics is not easy (nonlocality)**

“what is $\psi$” $\rightarrow$ “what is the role of $\psi$”

[C. S. Peirce]
$\psi$ has a role in the behavior of macroscopic objects ("measurement instruments") during "quantum measurements".

### Complete state description

$$(Z, \psi)$$

$\psi = \psi(q_1, \ldots, q_N), \quad Z: \text{macroscopic variable}$$
The *local beables* are the mathematical counterparts in the theory to real events at definite places and times in the real world (as distinct from the many purely mathematical constructions that occur in the working out of physical theories, as distinct from things which may be real but not localized, and as distinct from the 'observables' of other formulations of quantum mechanics, for which we have no use here.) J.S. Bell]

the macro variables $Z$ of OQM are LB
**Complete state description** \((Q, \psi)\)

\(\psi\) as above, \(Q\): microscopic variable, e.g.,
\[
Q = (Q_1, \ldots, Q_N)
\]

\(Q_i\) positions of particles (LB)

BM is fundamentally about *microscopic* LB (particles, or fields or strings ...), what we call PO (primitive ontology). The role of \(\psi\) is to govern the motion of the PO.
The equations of motion

\[
\frac{dQ_k}{dt} = \frac{\hbar}{m_k} \text{Im} \frac{\psi^* \nabla_k \psi}{\psi^* \psi} (Q_1 \ldots, Q_N)
\]

\[
i\hbar \frac{\partial \psi}{\partial t} = H \psi
\]

\[
H = - \sum_{k=1}^{N} \frac{\hbar^2}{2m_k} \nabla^2_k + V
\]

\[
\text{SE} \quad \Rightarrow \quad \frac{\partial |\psi(q)|^2}{\partial t} = - \text{div} \left( \text{Im} \frac{\psi^* \nabla_k \psi}{\psi^* \psi} (q) |\psi(q)|^2 \right)
\]
de Broglie relation \( p = \hbar k \) connects a particle property, the momentum \( p = mv \), with a wave property, the wave vector \( k \):

\[
v = \frac{\hbar k}{m}
\]

But the wave vector \( k \) is defined only for a plane wave.

For a general wave \( \psi \), the obvious generalization of \( k \) is the local wave vector \( \nabla S(q)/\hbar \), where \( S \) is the phase of the wf.
• Modified Hamilton-Jacobi, \( \psi = R e^{i S} \), with \( R \) and \( S \) real. \( v = \nabla S/m \)

• The quantum continuity equation, for quantum probability density \( \rho \) and a quantum probability current \( J \): \( v^\psi = J/\rho \).

• Symmetry. Invariance of the law under rotations, translations, time-reversal, and Galilean boosts.

• Wigner distribution:
  \[
  v^\psi(q) = \int \frac{p}{m} W^\psi(q, p) \, dp
  \]
\textbf{Heisenberg representation:}

\[ v(q, t) = -\frac{1}{\hbar} \text{Im} \frac{\langle \psi | [\hat{\mathcal{Q}}(dq, t), \hat{Q}_i(t)] | \psi \rangle}{\langle \psi | \hat{\mathcal{Q}}(dq, t) | \psi \rangle} (q = Q(t)) , \]

\[ \hat{\mathcal{Q}}(dq, t) \quad \text{PVM of} \quad \left( \hat{Q}_1(t), \ldots, \hat{Q}_N(t) \right) \]

\[ . . . \]
Implications of BM

1. the wf of a (sub-)system
2. quantum randomness
3. operators as observables
4. absolute uncertainty
5. collapse of the wave packet
6. formal scattering
7. familiar (macroscopic) reality
A final protest

When the cogency of Bohm’s reasoning is admitted, a final protest is often this: it is all nonrelativistic. This is to ignore that Bohm himself, in an appendix to one of the 1952 papers, already applied his scheme to the electromagnetic field. And application to scalar fields is straightforward. However until recently, to my knowledge, no extension covering Fermi fields had been made. Such an extension will be sketched here. (J.S. Bell, “Beables for quantum field theory”)
QFT Schrödinger’s equation

\[ i \frac{d\Psi}{dt} = -iH\Psi \quad (H \geq 0) \]

\[ n = n(r), \quad (r \in 3D \text{ lattice}) \text{ fermion number} \]

define

\[ dtT_{mn} \]

trans. prob. \( m \rightarrow n \text{ in time } dt \)

\[ T_{nm} = J_{nm}/D_{mm} \]
\[ J_{nm} = \sum_{qp} 2\text{Re}\langle \Psi | nq \rangle \langle nq | (-iH) | mp \rangle \langle mp | \Psi \rangle \]

\[ D_m = \sum_{q} |\langle mq | \Psi \rangle|^2 \]

If \( J_{nm} > 0 \). Otherwise \( T_{nm} = 0 \)

**stochastic process for fermionic number**

\[ \frac{dP_n}{dt} = \sum_{m} (T_{nm}P_n - T_{mn}P_n) \]
From Schrödinger’s equation

$$\frac{dD_n}{dt} = \sum_m (T_{nm}D_n - T_{mn}D_n)$$

so, if at some initial time

$$P_n(0) = D_n(0)$$

then for all times \( t \)

$$P_n(t) = D_n(t)$$

\[
\left[ \frac{\partial |\psi(q)|^2}{\partial t} = -\text{div} \left( \text{Im} \frac{\psi^* \nabla_k \psi}{\psi^* \psi} (q) |\psi(q)|^2 \right) \right]
\]
WF of a Subsystem

In a Bohmian universe with wf $\Psi = \Psi(x, y)$, what is meant by the wf $\psi = \psi(x)$ of a subsystem of that universe?

$$\psi(x) = \Psi(x, Y)$$
The evolution law for $\psi$

$$\psi_t(x) = \Psi_t(x, Y_t)$$

need not be Bohmian. Yet

$$\frac{dX}{dt} = \hbar \text{Im} \frac{\psi^* \nabla_x \psi}{\psi^* \psi}$$

(masses absorbed in the gradient)
CPF

\[ P_\Psi(X \in dx \mid Y) = |\psi(x)|^2 \, dx \]

Quantum Randomness and Absolute Uncertainty

(No external time (gravity): open problem, work in progress with Florian Hoffmann)
\[
\frac{1}{N} \sum_{k}^{N} \delta(q - Q_k) \approx |\psi(q)|^2
\]

\[P_\psi \text{ (exceptions| preparation)} \]

small for \( N \) large
macroscopic variable

\[ Z = F(Q) \]

\[ \text{Prob}(Z \in \Delta | \psi) = \langle \psi, P_A(\Delta) \psi \rangle \]

A s.a. operator
More on 2-slit

Experimental Bohmian trajectories:
Photons in a double slit set-up


Theoretical Bohmian trajectories

From: C. Philippidis, et al., Il nuovo cimento B (1979)
Weak Measurement of the Bohmian Trajectories

\[ v(x) = \frac{1}{m} \text{Re} \langle x | \hat{p} | \psi \rangle \langle x | \psi \rangle \]

\[ \text{Sw} \quad \text{where} \quad L_y, L_z \rightarrow I_w \propto p_x \]

WM of total current = WM of momentum

\[ \text{Ss} \quad \text{where} \quad L_y, L_z \rightarrow I_s \propto |r_s|^2 \]

Post-selection with position measurement

Damiano Marian 06-03-2015 25 / 29

Xavier Oriols, Damiano Marian, NZ
• There is a clear primitive ontology $\mathcal{X}$, and it describes matter in space and time.

• There is a state vector $\Psi$ in Hilbert space that evolves either unitarily or, at least, for microscopic systems very probably for a long time approximately unitarily.

• The state vector $\Psi$ governs the behavior of $\mathcal{X}$ by means of (possibly stochastic) laws.

• The laws are such that for typical histories of $\mathcal{X}$, the probability distribution of the variables representing $\mathcal{X}$ at time $t$ is, in a suitable sense, (approximately) $|\Psi_t|^2$. 