Time in Bohmian Mechanics

Roderich Tumulka

RUTGERS UNIVERSITY

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1. Bohmian Mechanics

Conventional Axioms For Quantum Mechanics

Schrödinger Equation

$\psi_t : (\mathbb{R}^3)^N \to \mathbb{C}^m$ wave function of a system. While the system is closed,

$$i\hbar \frac{\partial \psi}{\partial t} = -\sum_{k=1}^{N} \frac{\hbar^2}{2m_k} \nabla_k^2 \psi + V\psi$$

Measurement Postulate

If an observer measures the observable with operator $A = \sum_\alpha \alpha P_\alpha$ at time t, then

$$\mathrm{Prob}(\mathrm{outcome} = \alpha) = \langle \psi_t | P_\alpha \psi_t \rangle \,,$$

and if outcome $= \alpha$ then (collapse)

$$\psi_{t+0} = \frac{P_\alpha \psi_t}{\| P_\alpha \psi_t \|} \,.$$

BM as Axioms For Quantum Mechanics

Schrödinger Equation

$\psi_t : (\mathbb{R}^3)^N \to \mathbb{C}^m$ wave function of the universe. At all times,

$$i\hbar \frac{\partial \psi}{\partial t} = -\sum_{k=1}^{N} \frac{\hbar^2}{2m_k} \nabla_k^2 \psi + V\psi$$

Bohm's Equation of Motion

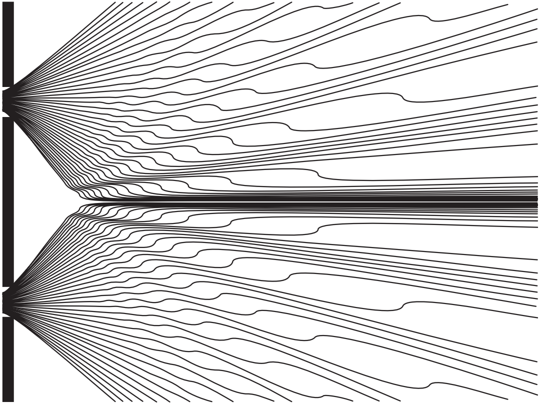
$Q_k(t) \in \mathbb{R}^3$ position of particle k, $Q(t) = (Q_1(t), \ldots, Q_N(t))$ config.,

$$\frac{dQ_k(t)}{dt} = \frac{j_k^\psi}{|\psi_t|^2}(Q(t)) = \frac{\frac{\hbar}{m}\mathrm{Im}\, \psi_t^* \nabla_k \psi_t}{\psi_t^* \psi_t}(Q(t)) \,.$$

Quantum Equilibrium Distribution

$\mathrm{Prob}(Q(t) \in dq) = |\psi_t(q)|^2$, i.e., $Q(t)$ is *typical* relative to $|\psi_t|^2$.

[Slater 1923, de Broglie 1926, Bohm 1952, Bell 1966]



Picture: Gernot Bauer (after Chris Dewdney)

BM allows analysis of “quantum measurements.”

BM $\Rightarrow$ quantum formalism

[Bohm 1952, Bell 1966, Dürr–Goldstein–Zanghì 1992, 2004]

Thus, BM is an explanation of QM, though “unromantic” (J. S. Bell).

Bohmian Mechanics is Time Symmetric

If

$$t \mapsto \big(Q(t), \psi(t)\big)$$

is a solution of the equations of Bohmian mechanics, then so is

$$t \mapsto \big(Q(-t), \psi^*(-t)\big),$$

where ψ^* is the (componentwise) complex conjugate.

Moreover, probabilities behave in the right way: for the reversed process, $\rho = |\psi(-t)|^2$.

Arrow of time: initial wave function looks different from the wave function at a later time.

BM in Relativistic Space-Time

Assume the existence of a time foliation:

- a physical object mathematically represented by a spacelike foliation = slicing of space-time M into spacelike hypersurfaces
- against the spirit of relativity (a price worth paying?)
- defines temporal order of spacelike separated events, or simultaneity at a distance.

Keep the Lorentzian metric $g_{\mu\nu}$.
Then BM possesses an analog on $(M, g_{\mu\nu})$ using the Dirac equation.

Versions compatible with the spirit of relativity?

For BM unknown, unlikely.
But $\exists$ for GRWf theory = Ghirardi–Rimini–Weber theory with flash ontology

2. Some Issues About Time—A Bohmian Perspective

Time Measurements

Surround a particle with detectors, measure when a detector clicks.

Why is there no time operator?

In BM: Why do operators play a role at all?

Operators in Bohmian Mechanics

Theorem in BM: statistics are given by a POVM

For every experiment on a system with Hilbert space $\mathscr{H}$ there is a POVM $E(\cdot)$ acting on $\mathscr{H}$ such that if the system has wave function ψ,

$$\mathrm{Prob}(\text{outcome} \in \Delta) = \langle \psi | E(\Delta) \psi \rangle$$

[Dürr–Goldstein–Zanghì 2004]

Def: POVM = positive-operator-valued measure on set Ω:
For $\Delta \subseteq \Omega$, $E(\Delta)$ is a positive operator on $\mathscr{H}$. $E(\Omega) = I$.
If $\Delta_1, \Delta_2, \ldots$ are pairwise disjoint then $E\Big(\bigcup_{i=1}^{\infty} \Delta_i\Big) = \sum_{i=1}^{\infty} E(\Delta_i)$.

Special case: $\Omega = \mathbb{R}$, $E(\Delta)$ projections $\leftrightarrow$ self-adjoint A.
($E(\Delta) = \sum_{\alpha \in \Delta} P_\alpha$ spectral projections of $A = \sum_\alpha \alpha P_\alpha$.)

Why is there no time operator?

- There is a POVM $E(dt)$ on $\Omega = [0, \infty)$ for the statistics of the time of detection.
- Operators-as-observables are attributed metaphysical meaning in orthodox QM, but not in BM.
- In BM, POVMs just encode probability distributions.
- In BM, self-adjoint observables are just special POVMs.
- Thus, in BM, it is not surprising that there is no time operator.

Tunnelling Time

How long did the particle stay inside the barrier?

To this question several answers have been proposed in QM, some implying faster-than-light motion [Enders–Nimtz 1992] or even negative tunnelling times. [Review: Leavens 1996]

In BM straightforward answer: read off from the trajectory!

A Problem of Time in Quantum Gravity

How can it be that the wave function of the Wheeler–de Witt equation is time-independent?

In BM straightforward answer: If the wave function is not everything, then a time-independent wave function does not mean that nothing moves! If $\psi(t)$ is constant, $Q(t)$ need not be.

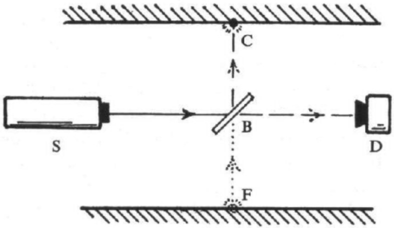
3. The Arrow of Time

Are Quantum Measurements Evidence of Time Asymmetry (Rather Than Macroscopic Irreversibility)?

Are Quantum Measurements Evidence of Time Asymmetry (Rather Than Macroscopic Irreversibility)?

Roger Penrose [The Road to Reality, 2004]: YES!

"Suppose there is a photon source S which emits single photons...aimed at a beam splitter B...angled at 45°. If a photon is transmitted through then it activates a detector D..., while if it is reflected, then it gets absorbed into the ceiling C."



"imagine...this particular experiment backwards in time..., given that there is a detection event at D. ... We would find a probability of 50% for emission at S and 50%...for the photon to come from the floor F. This...is an absurdity...completely the wrong answer!"

Are Quantum Measurements Evidence of Time Asymmetry (Rather Than Macroscopic Irreversibility)?

Bohmian mechanics: NO!

BM is a counter-example, as it is time symmetric and leads to the same probability predictions as the quantum formalism.

What went wrong?

We need to include the detector in the quantum mechanical description.

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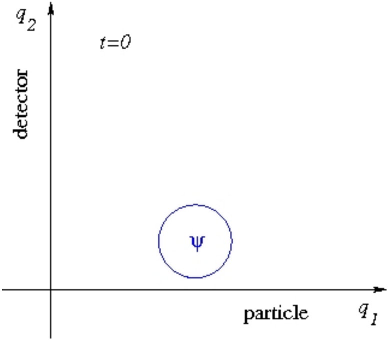

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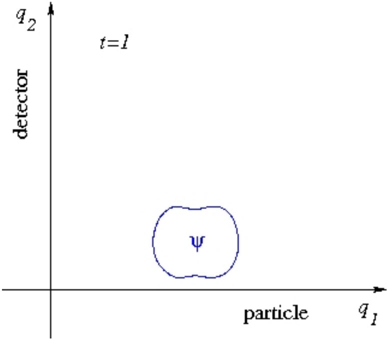

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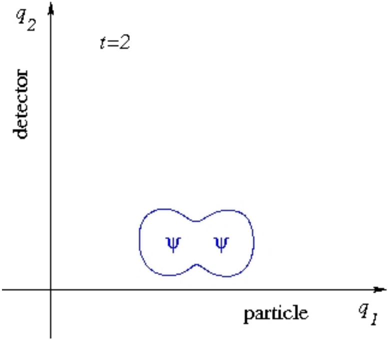

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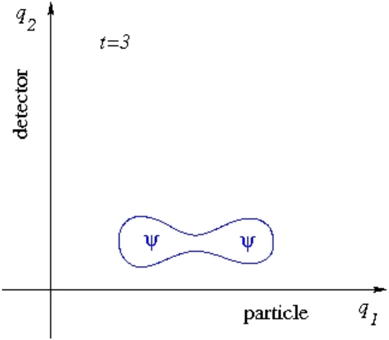

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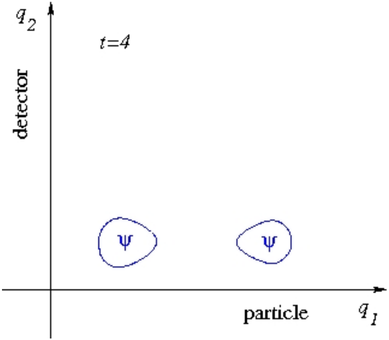

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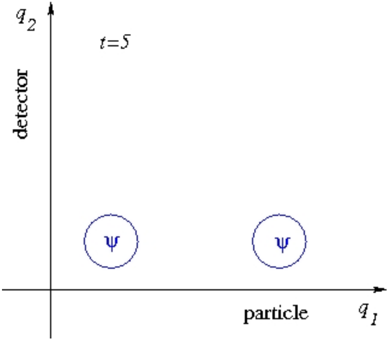

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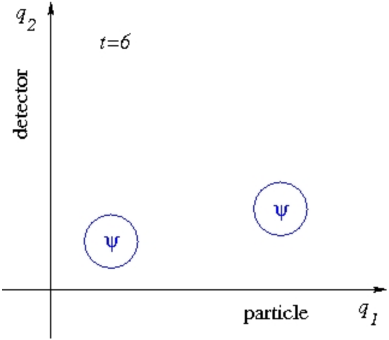

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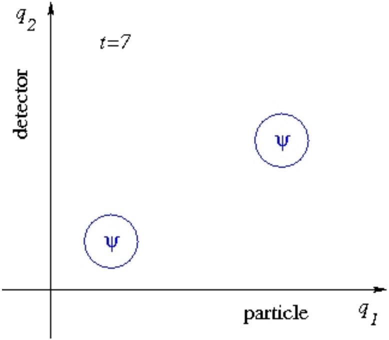

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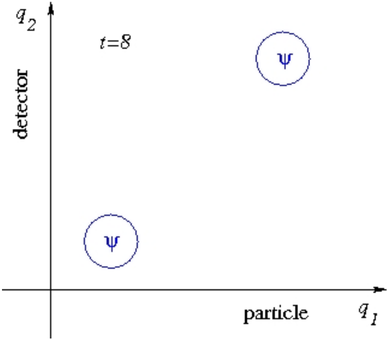

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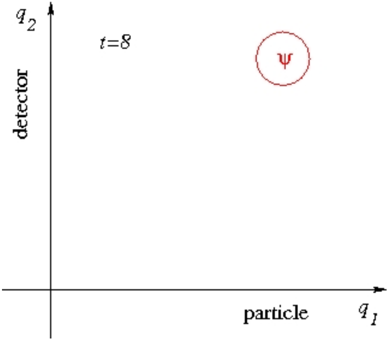

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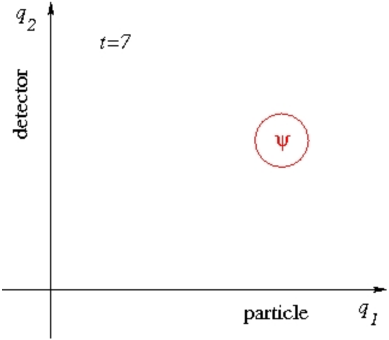

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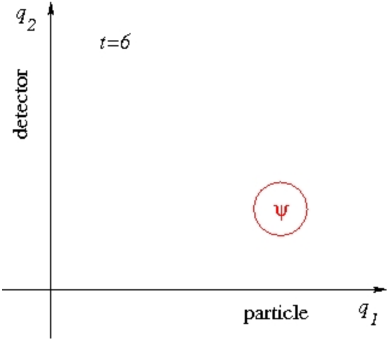

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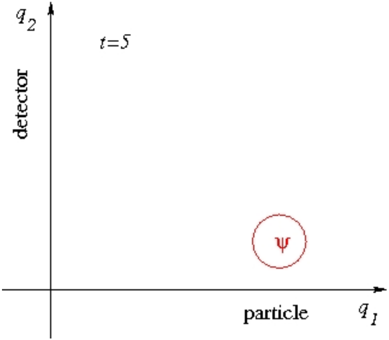

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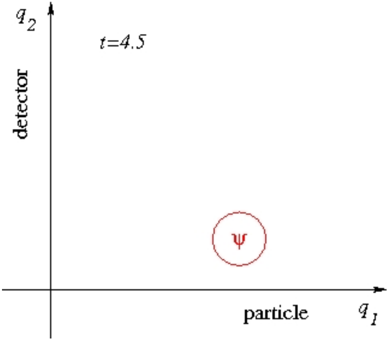

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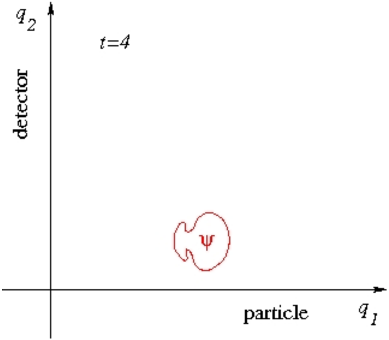

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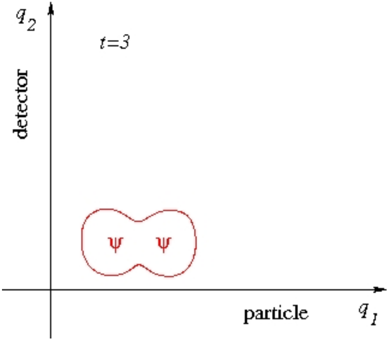

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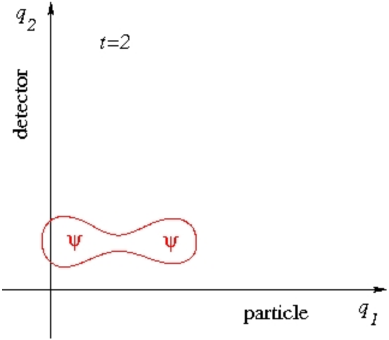

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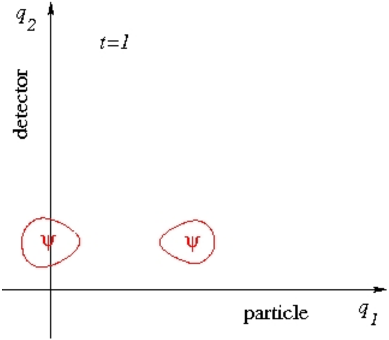

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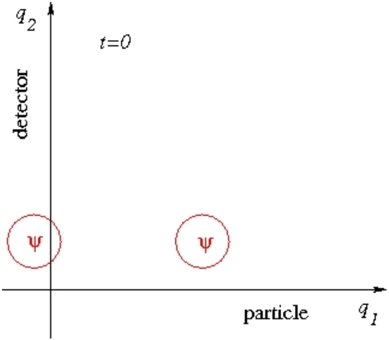
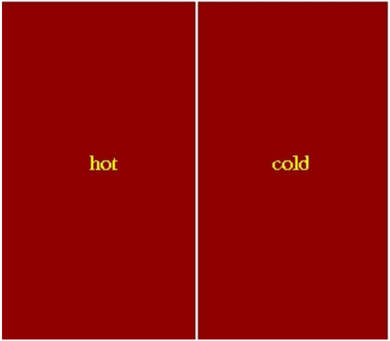


Boltzmann's Explanation of the (Thermodynamic) Arrow of Time, Classical and Quantum

Two bricks:



Entropy of ensemble

Classical: Gibbs entropy $S_{\mathrm{G}} = -k \int_{\mathbb{R}^{6N}} dq\, dp\, \rho(q,p) \log \rho(q,p)$
Quantum: von Neumann entropy $S_{\mathrm{vN}} = -k\, \mathrm{tr}(\hat\rho \log \hat\rho)$

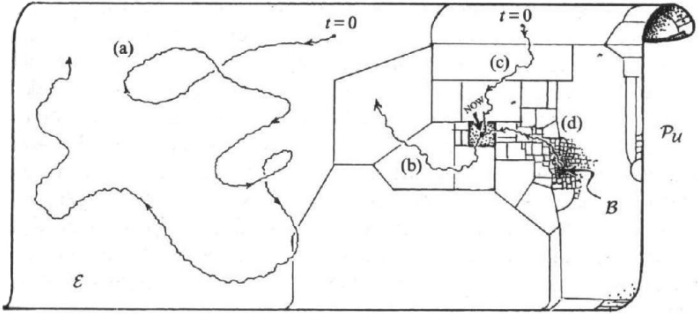
Does not increase, except by interaction, coarse-graining or $t \to \infty$.
Above all, inappropriate in BM where $\exists$ observer-independent truth.

Entropy of individual system

Classical: macrostates correspond to disjoint subsets Γ_ν of phase space such that $\mathbb{R}^{6N} = \cup_\nu \Gamma_\nu$ (partition)
Boltzmann entropy $S_{\mathrm{B}}(\nu) = k \log \mathrm{Vol}(\Gamma_\nu)$
Quantum: macrostates correspond to orthogonal subspaces $\mathscr{H}_\nu \subseteq \mathscr{H}$ such that $\mathscr{H} = \oplus_\nu \mathscr{H}_\nu$
quantum Boltzmann entropy $S_{\mathrm{qB}}(\nu) = k \log \dim \mathscr{H}_\nu$

Classical: Phase point typically moves to larger and larger cells Γ_ν. That's why $S_{\rm B}$ increases.

Drawing: R. Penrose



Quantum problem: Classically, every microstate (q, p) belongs to one and only one macrostate Γ_ν, and so has an unambiguous $S_{\rm B}(q, p)$. But in the quantum case, most microstates ψ do not belong to any $\mathscr{H}_\nu$, and so don't have any entropy value. Example: Schrödinger's cat.
foundations(QM) $\leftrightarrow$ foundations(statistical mechanics).

What could it even mean to say that $\log \dim \mathscr{H}_\nu$ increases with time?

Bohm helps:

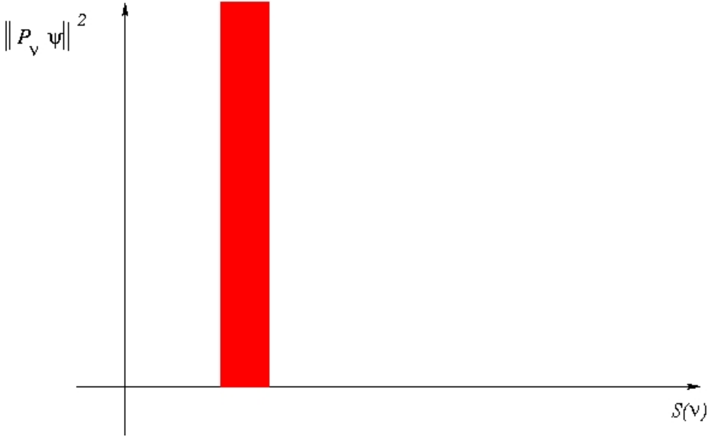
In BM, there is a fact about whether Schrödinger's cat is alive:
$\psi = 2^{-1/2}\big(|\mathrm{dead}\rangle + |\mathrm{alive}\rangle\big)$ but $Q = Q_{\mathrm{dead}}$ or $Q = Q_{\mathrm{alive}}$.

$$S(Q,\psi) = S_\nu \quad \Leftrightarrow \quad Q \in \mathrm{support}(P_\nu \psi)$$

where P_ν = projection to $\mathscr{H}_\nu$.

Q does not *always* select a unique macrostate ν (and thus a unique entropy S_ν), but in practically relevant cases.

Conjecture:



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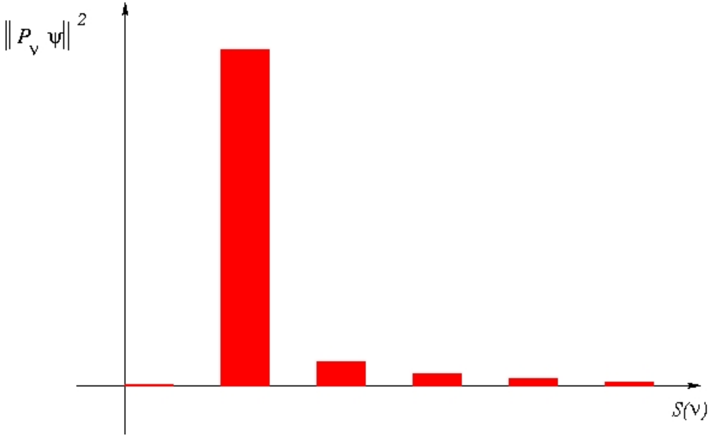

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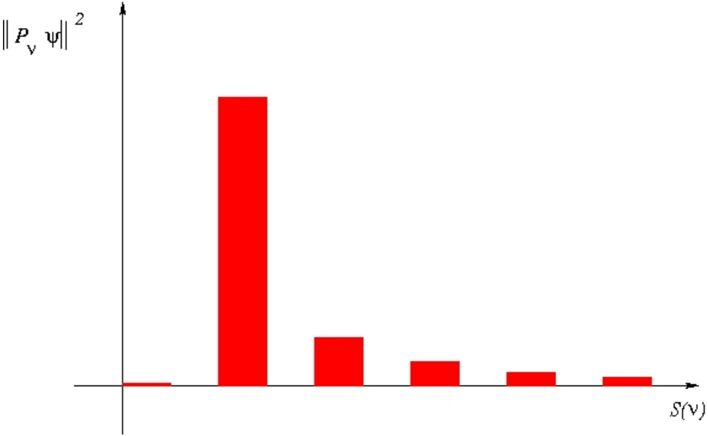

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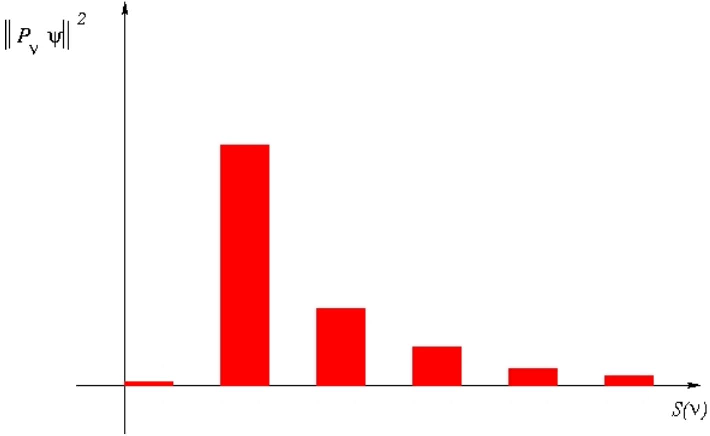

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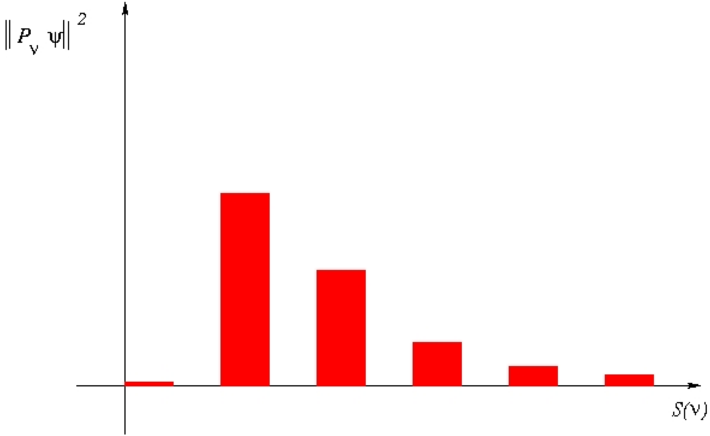

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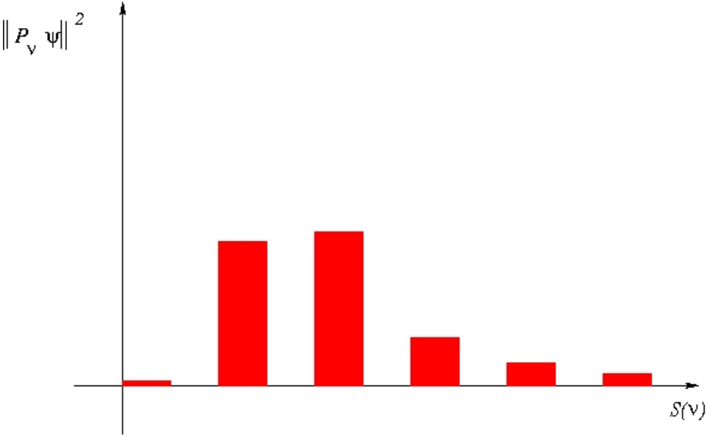

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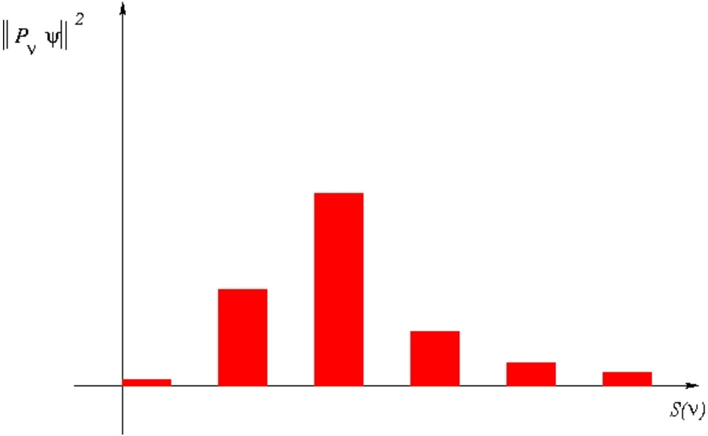

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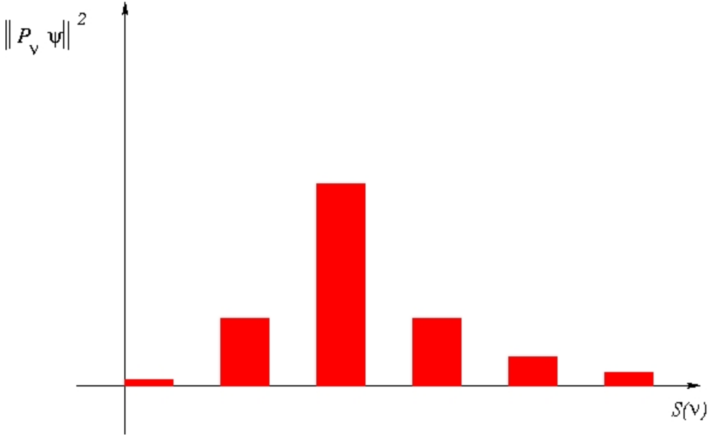

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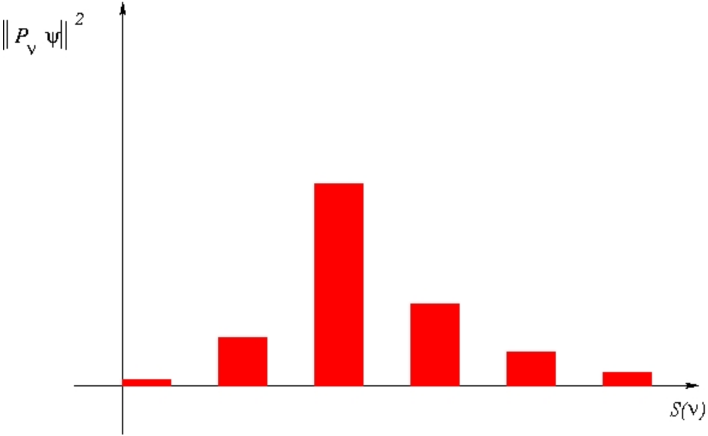

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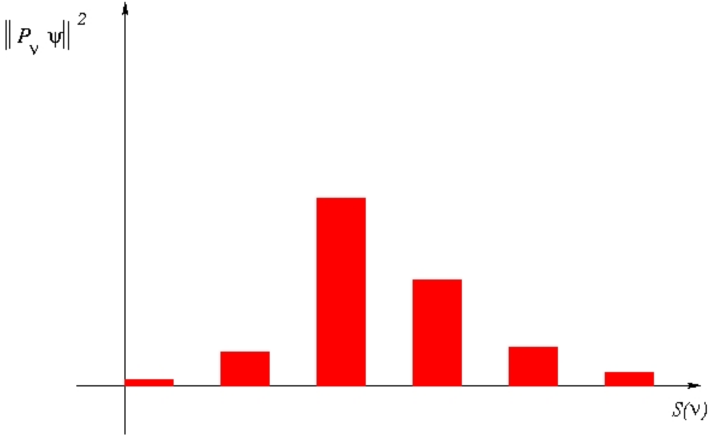

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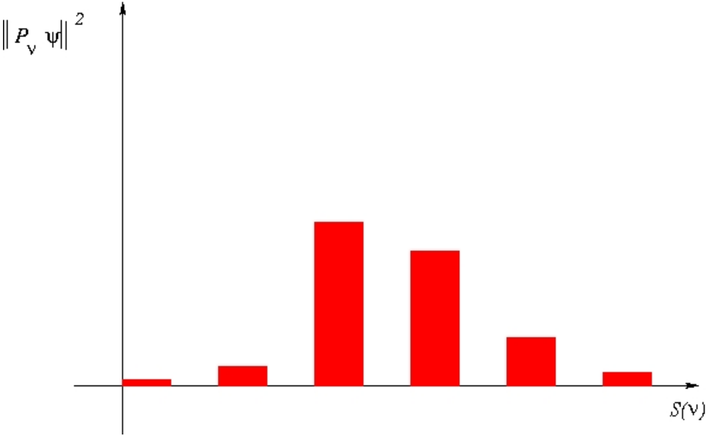

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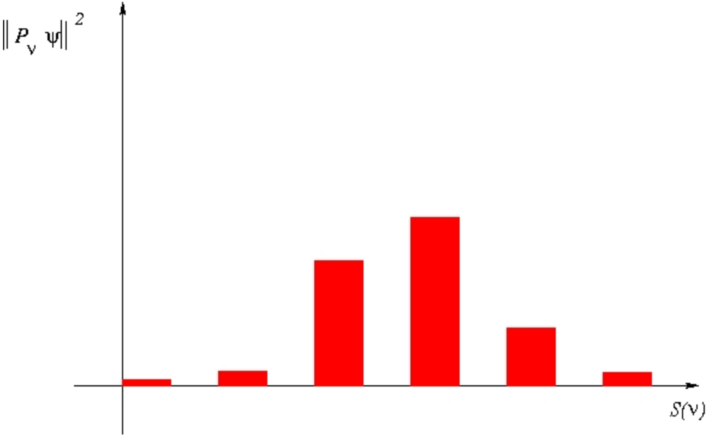

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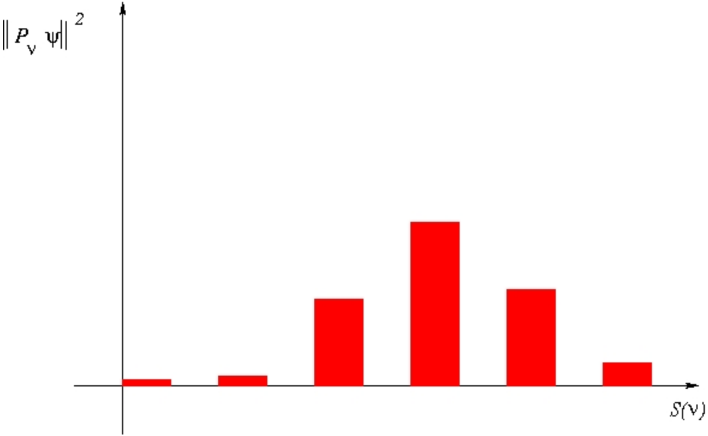

Conjecture:
In realistic situations, if $S(\nu') < S(\nu)$ then for typical $\psi_0 \in \mathscr{H}_\nu$ and for every $t \in \mathbb{R}$, $\|P_{\nu'}\psi_t\|^2$ is small.

It seems like
Conjecture + BM $\Rightarrow$ increase of $S_{\mathrm{qB}}(Q(t), \psi(t))$

while Conjecture alone does not prohibit entropy decrease:
For example, suppose that in an airport building there are 5,000 passengers at every time t, and furthermore that, at every t, 5 passengers undergo the security check: that is a small fraction 10^{-3}, but this fact does not contradict the statement that every passenger at some point undergoes the security check.

Thank you for your attention