Bohmian Mechanics as the Foundation of Quantum Mechanics

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Definition of Bohmian mechanics

Bohmian mechanics is a non-relativistic theory of point particles moving in 3-space along trajectories.

- N particles in 3-space, at positions $\mathbf{Q}_i(t) \in \mathbb{R}^3$ at time t. Equation of motion (a.k.a. "guidance equation"):

$$\frac{d\mathbf{Q}_i}{dt} = \frac{\hbar}{m_i} \operatorname{Im} \frac{\nabla_i \psi}{\psi} \big(\mathbf{Q}_1(t), \ldots, \mathbf{Q}_N(t), t\big)$$

  depending on some wave function $\psi(t) : \mathbb{R}^{3N} \to \mathbb{C}$.
- Time evolution of $\psi(\mathbf{q}_1, \ldots, \mathbf{q}_N, t)$ given by the Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\sum_{i=1}^{N} \frac{\hbar^2}{2m_i} \nabla_i^2 \psi + V(\mathbf{q}_1, \ldots, \mathbf{q}_N)\, \psi$$

- The initial configuration $Q(t=0) = \big(\mathbf{Q}(0), \ldots, \mathbf{Q}(0)\big)$ is typical relative to the $|\psi(0)|^2$ distribution, i.e., looks as if chosen randomly in $\mathbb{R}^{3N}$ with $|\psi(0)|^2$ distribution.

Conservation of $|\psi|^2$

Equivariance theorem

If $Q(t=0)$ is random with $|\psi(0)|^2$ distribution, then $Q(t)$ is random with $|\psi(t)|^2$ distribution for all $t \in \mathbb{R}$.

Proof: The equation of motion can be rewritten equivalently as

$$\frac{d\mathbf{Q}_i}{dt} = \frac{\mathbf{j}_i}{\rho}(Q(t), t)$$

with the quantities known in QM as the probability density $\rho = |\psi|^2$ and probability current

$$\mathbf{j}_i = \frac{\hbar}{m_i} \mathrm{Im}\left[\psi^* \nabla_i \psi\right].$$

As a well-known consequence of the Schrödinger equation, they satisfy a continuity equation

$$\frac{\partial \rho}{\partial t} = -\sum_{i=1}^{N} \nabla_i \cdot \mathbf{j}_i \, .$$

But this equation coincides with the equation for probability transport by the Bohmian motion.

Another basic property

When ψ factorizes,

$$\psi(\mathbf{q}_1, \ldots, \mathbf{q}_N) = \varphi(\mathbf{q}_1, \ldots, \mathbf{q}_M)\, \chi(\mathbf{q}_{M+1}, \ldots, \mathbf{q}_N)\,,$$

then it follows that the motion of one subsystem $(\mathbf{Q}_1, \ldots, \mathbf{Q}_M)$ is independent of the configuration or wave function χ of the other:

$$\frac{d\mathbf{Q}_i}{dt} = \frac{\hbar}{m_i} \operatorname{Im} \frac{\nabla_i \varphi}{\varphi}(\mathbf{Q}_1, \ldots, \mathbf{Q}_M) \qquad \text{for } i \leq M$$

(as long as ψ factorizes).

The key fact about Bohmian mechanics

As a consequence of the definition of the theory:

Observers inhabiting a Bohmian universe (made out of Bohmian particles) observe random-looking outcomes of their experiments whose statistics agree with the rules of quantum mechanics for making predictions.

A theory like this was believed to be impossible

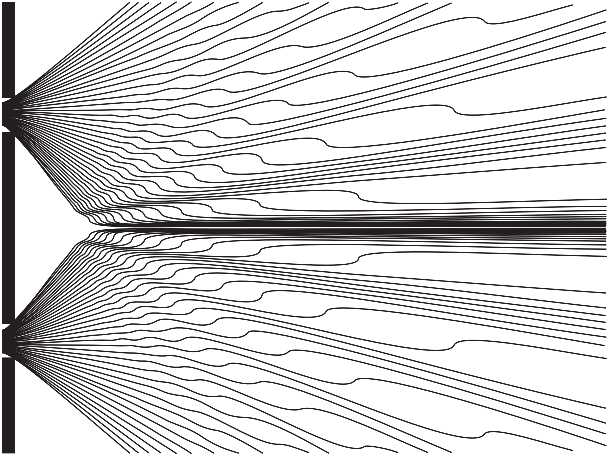
Werner Heisenberg in 1958:

"We can no longer speak of the behavior of the particle independently of the process of observation."
"The idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them [...], is impossible."

W. Heisenberg (1901–1976)

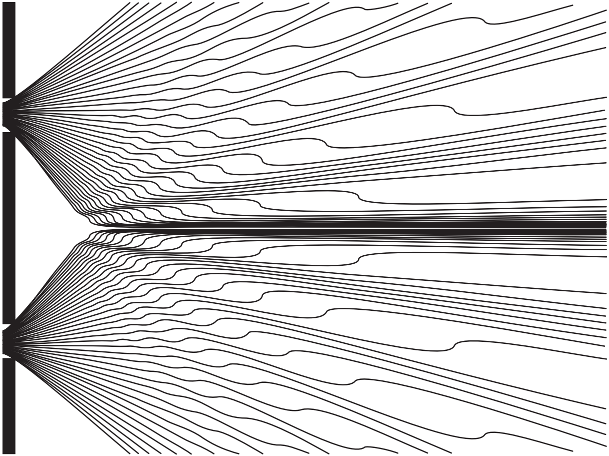
Heisenberg was wrong. Bohmian mechanics is a counter-example to the impossibility claim.

Example: the double-slit experiment



Picture: Gernot Bauer (after Chris Dewdney)

Shown: A double-slit and 80 possible paths of Bohm's particle. The wave passes through both slits, the particle through only one.



Most paths arrive where $|\psi|^2$ is large—that's how the interference pattern arises. If one slit gets closed, the wave passes through only one slit, which leads to different trajectories and no interference pattern. Bohmian mechanics takes wave–particle dualism literally: there is a wave, and there is a particle. The path of the particle depends on the wave.

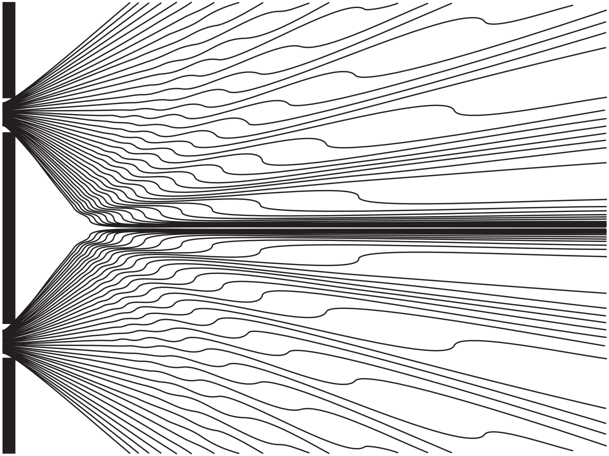
Limitations to control

As a consequence of the definition of the theory:

Observers or agents in a Bohmian universe (made out of Bohmian particles) can prepare a system to have a particular wave function φ_{sys}, but they cannot prepare the system's configuration Q_{sys} to be a particular configuration X, unless $\varphi_{\text{sys}}(q) = \delta(q - x)$. In fact, they cannot prepare Q_{sys} any more accurately, or any differently, than being random with distribution $|\varphi_{\text{sys}}|^2$.

$\mathbf{Q}_k(t)$ often called "hidden variable"—better: uncontrollable variable

Heisenberg's uncertainty relation in Bohmian mechanics



When the wave function is narrow then the spread in velocity is large.

Another mathematical way of thinking of Bohmian mechanics

The configuration $Q = (\mathbf{Q}_1, \ldots, \mathbf{Q}_N)$ moves in configuration space $\mathbb{R}^{3N}$ according to

$$\frac{dQ}{dt} = v^{\psi(t)}(Q(t)),$$

where v^ψ, the velocity vector field for ψ, is

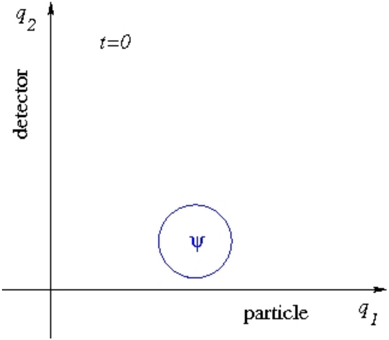
$$v = \left(\frac{\mathbf{j}_1}{\rho}, \ldots, \frac{\mathbf{j}_N}{\rho}\right)$$

Note that at any fixed time, $v^\psi(Q)$ depends only on $\psi(Q)$ and $\nabla\psi(Q)$ (while over time, other parts of ψ may propagate, reach Q, and influence Q).

Collapse of the wave function in Bohmian mechanics

The wave function of system and apparatus together does not collapse (but evolves according to the Schrödinger equation).
However, some parts of the wave function become irrelevant to the particles and can be deleted because of decoherence: Two packets of ψ do not overlap in configuration space and will not overlap any more in the future (for the next 10^{100} years).

Evolution of ψ in configuration space of particle + detector:



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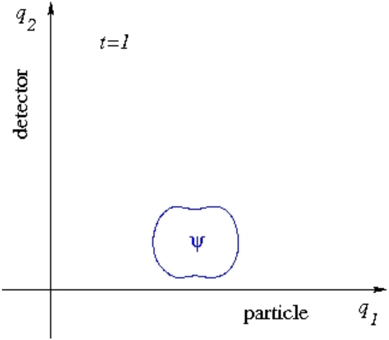

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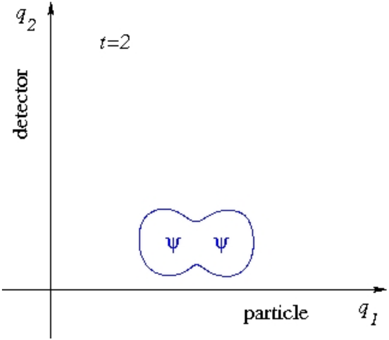

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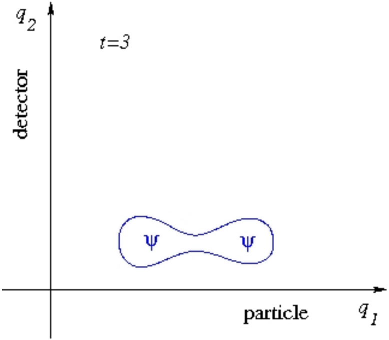

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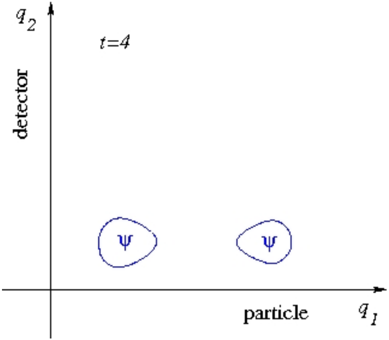

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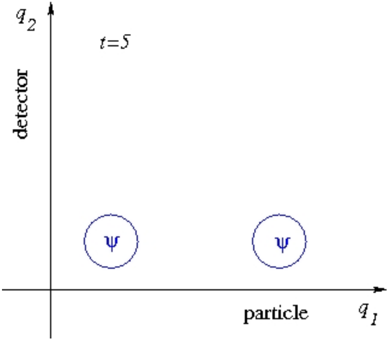

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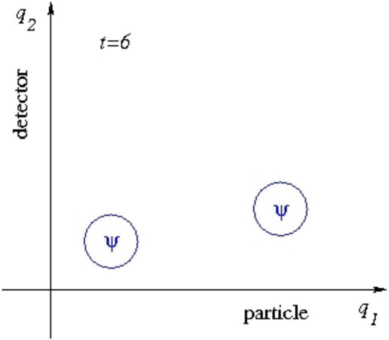

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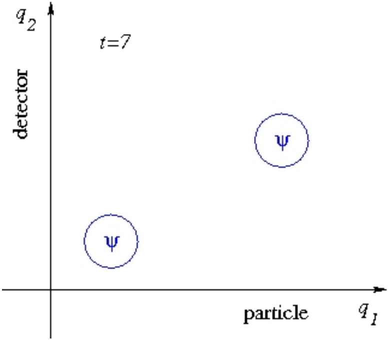

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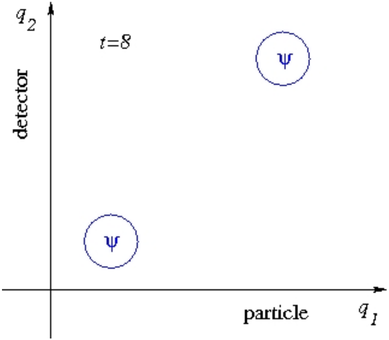
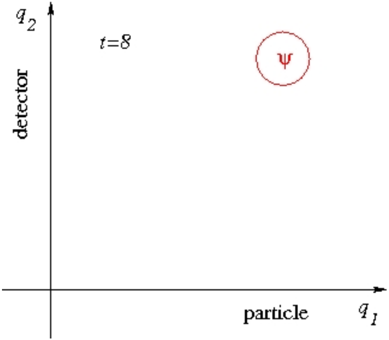


Evolution of ψ in configuration space of particle + detector:

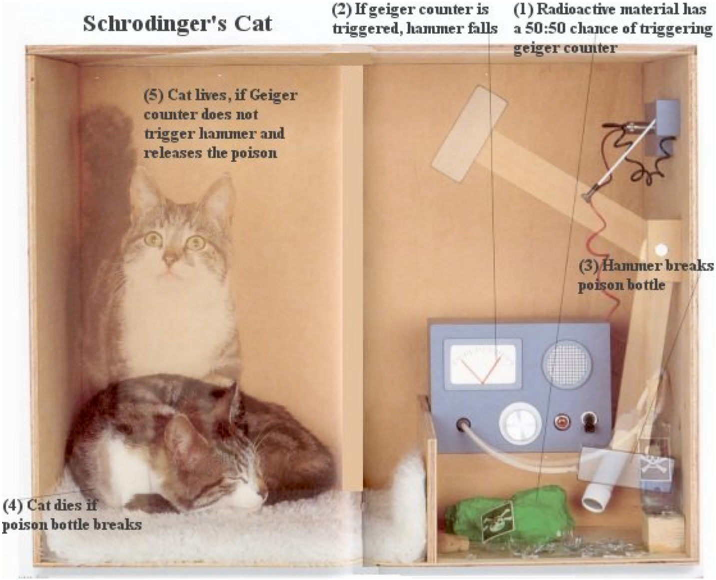


Collapse of the wave function in Bohmian mechanics

- If two packets of ψ do not overlap in configuration space and will not overlap any more in the future (for the next 10^{100} years), then only the packet containing Q will be relevant to the motion of Q (for the next 10^{100} years).
- So the other packets can safely be ignored from now on (although strictly speaking, they still exist) $\Rightarrow$ collapse of ψ
- Probability that ψ collapses to this packet $=$

  probability that Q lies in this packet $=$

  $\int_{\text{packet}} |\psi|^2 = ||\text{packet}||^2$
- Thus, the standard collapse rule comes out.

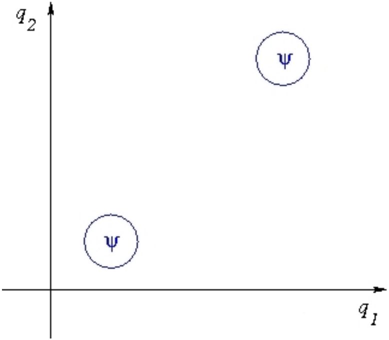

Schrodinger's Cat
(2) If geiger counter is triggered, hammer falls
(1) Radioactive material has a 50:50 chance of triggering geiger counter
(5) Cat lives, if Geiger counter does not trigger hammer and releases the poison
(3) Hammer breaks poison bottle
(4) Cat dies if poison bottle breaks

How Bohmian mechanics solves the problem of Schrödinger's cat

The wave function is indeed the superposition

$$\psi = \psi_{\text{dead}} + \psi_{\text{alive}} \,.$$

However, the particles form either a dead cat or a live cat. (Indeed, the configuration Q has probability distribution $|\psi|^2$.)



So, there is a fact about whether the cat is dead or alive.

What does the cat example mean?

- It's often called a "paradox," but that is too weak—that sounds like "get used to it."
- Basically, it's an argument: Cat + atom belong to a quantum system of 10^{25} electrons, protons and neutrons, with a wave function ψ governed by the Schrödinger equation.
  Since the Schrödinger equation is linear, we have that, after 1 hour, the wave function is a "superposition" of the wave function of a dead cat and that of a live cat:

$$\psi = \psi_{\text{dead}} + \psi_{\text{alive}} \,.$$

  However, in reality the cat must be either dead or alive.

John S. Bell: "The problem is: AND is not OR."

Also known as "the measurement problem of quantum mechanics."

Measurement problem

Consider a quantum measurement of the observable $A = \sum_n \alpha_n |n\rangle\langle n|$.

$$|n\rangle \otimes \phi_0 \xrightarrow{t} |n\rangle \otimes \phi_n$$

($\phi_0 =$ ready state of apparatus, $\phi_n =$ state displaying result α_n)

$$\Rightarrow \quad \sum_n c_n |n\rangle \otimes \phi_0 \xrightarrow{t} \sum_n c_n |n\rangle \otimes \phi_n$$

But one would believe that a measurement has an actual, random outcome n_0, so that one can ascribe the "collapsed state" $|n_0\rangle$ to the system and the state ϕ_{n_0} to the apparatus.

Conclusion from this argument:

- Either ψ is not the complete description of the system,
- or the Schrödinger equation is not correct for $N > 10^{20}$ particles,
- or there are many worlds.

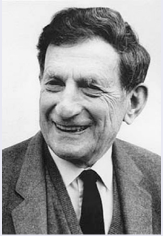
Bell (1982):

"In 1952 I saw the impossible done. It was in papers by David Bohm. Bohm showed explicitly how parameters could indeed be introduced, into non-relativistic wave mechanics, with the help of which the indeterministic description could be transformed into a deterministic one. More importantly, in my opinion, the subjectivity of the orthodox version, the necessary reference to the observer, could be eliminated."

John S. Bell (1928–1990)

History

- 1924: Einstein toys with the idea that photons may have trajectories obeying an equation of motion similar to that of Bohmian mechanics. John Slater joins him.
- 1926: Louis de Broglie discovers Bohmian mechanics, calls it "pilot-wave theory."
- 1945: Nathan Rosen (the R of EPR) independently discovers Bohmian mechanics.
- 1952: David Bohm independently discovers Bohmian mechanics. He is the first to realize that the theory is empirically adequate.



David Bohm (1917–1992)

Is Bohmian mechanics the only realist theory of QM?

No. Other theories that work:

- Other trajectories
  - Nelson's (1968) stochastic mechanics (diffusion paths with drift given by v^{ψ})
- Collapse theories [Ghirardi, Rimini, Weber 1986; Bell 1987; Pearle 1990]
- Many worlds (perhaps) [Schrödinger 1926; Everett 1957]

But Bohmian mechanics is (arguably) the simplest and (in my humble opinion) most convincing one.

Spin

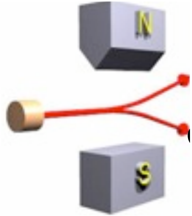
Bohmian mechanics with spin

$\psi(t) : \mathbb{R}^{3N} \to (\mathbb{C}^2)^{\otimes N}$. Equation of motion:

$$\frac{d\mathbf{Q}_i(t)}{dt} = \frac{\mathbf{j}_i}{\rho}(Q(t),t) = \frac{\hbar}{m_i}\mathrm{Im}\frac{\psi^*\nabla_i\psi}{\psi^*\psi}(Q(t),t)$$

where $\phi^*\psi = \sum_{s=1}^{2^N} \phi_s^*\psi_s$ inner product in spin-space

No "actual spin vector" (unlike actual position) needed, no rotational motion needed.



Stern–Gerlach experiment

Wave packet $\psi = \binom{\psi_\uparrow}{\psi_\downarrow}$ splits into two packets, one purely $\uparrow$, the other purely $\downarrow$. Then detect the position of the particle: If it is in the spatial support of the $\uparrow$ packet, say that the outcome is "up."

Identical particles

It may seem essential for identical particles that the particles at time t_1 cannot be identified (matched) with the particles at time t_2, and thus that Bohmian mechanics can't possibly work with identical particles. But it does!

For N identical particles, we assume in Bohmian mechanics the same symmetrization postulate as in standard QM: $\psi(\mathbf{q}_1, \ldots, \mathbf{q}_N)$ is either a *symmetric* or an *anti-symmetric* function.

If we take the particle ontology seriously then

the appropriate configuration space of N identical particles is not the set $\mathbb{R}^{3N}$ of ordered configurations $(\mathbf{Q}_1, \ldots, \mathbf{Q}_N)$ but the set of unordered configurations $\{\mathbf{Q}_1, \ldots, \mathbf{Q}_N\}$,

$$^N\mathbb{R}^3 = \big\{Q \subset \mathbb{R}^3 : \#Q = N\big\} = \big(\mathbb{R}^{3N} \setminus \{\text{collisions}\}\big)/\text{permutations}.$$

And indeed: If $\psi : \mathbb{R}^{3N} \to \mathbb{C}$ is symmetric or anti-symmetric then v^ψ is permutation-covariant and thus projects consistently to a vector field on $^N\mathbb{R}^3$. For general (asymmetric) ψ, this is not the case.

Extending Bohmian mechanics to quantum field theory

Two approaches:

1. "Field ontology":
   Instead of an actual configuration $(\mathbf{Q}_1, \ldots, \mathbf{Q}_N)$ of particles, postulate an actual field configuration $\Phi(\mathbf{x})$; the quantum state is a wave functional $\Psi[\phi]$ on the ∞-dimensional space of all field configurations $\phi = \phi(\mathbf{x})$. Equation of motion

$$\frac{\partial \Phi}{\partial t} = \mathrm{Im}\left[\frac{1}{\Psi[\Phi]}\frac{\delta \Psi}{\delta \phi}\bigg|_{\phi=\Phi}\right]$$

2. "Particle ontology":
   Trajectories for photons, electrons, positrons, etc.
   Particles can be created and annihilated.

Particle creation in Bohmian mechanics

[Bell 1986, Dürr, Goldstein, Tumulka, Zanghì 2003]

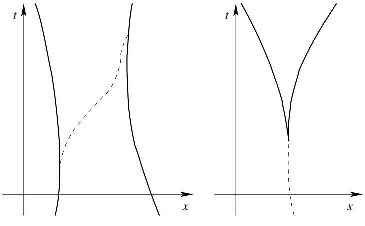
Natural extension of Bohmian mechanics to particle creation:

$$\Psi \in \text{Fock space} = \bigoplus_{N=0}^{\infty} \mathscr{H}_N,$$

configuration space of a variable number of particles

$$= \bigcup_{N=0}^{\infty} \mathbb{R}^{3N}$$

jumps (e.g., n-sector $\to$ $(n+1)$-sector) occur in a stochastic way, with rates governed by a further equation of the theory.



(a) (b)

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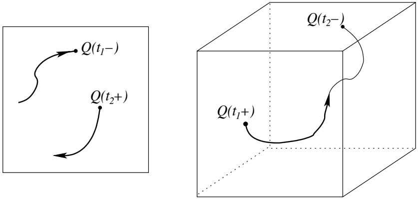


(c) (d)

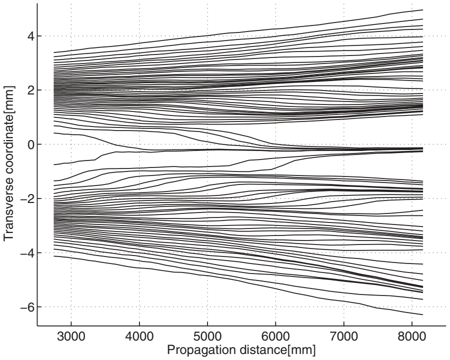
Limitations to knowledge

"Absolute uncertainty" theorem

For the inhabitants of a universe governed by Bohmian mechanics, it is impossible to know the position of a particle more precisely than the $|\varphi|^2$ distribution allows, where φ is the (conditional) wave function of the particle.

Inhabitants of a Bohmian universe cannot measure the trajectory of a particle to arbitrary accuracy without influencing it. That is, when the accuracy is high, the trajectory of the particle is not the same as it would have been without interaction with the measuring apparatus. This is a limitation to knowledge.

And what about this experimental finding?



Sacha Kocsis, . . . , Aephraim Steinberg: Observing the Average Trajectories of Single Photons in a Two-Slit Interferometer, *Science* (2011), realizing a proposal by Howard Wiseman, *New J. Physics* (2007)

- How was this done?
  Weak measurements on many systems with the same wave function.
- Does this prove Bohmian mechanics right?
  No. The experiment would come out the same way in collapse theories or other trajectory theories.

Limitations to knowledge in quantum mechanics

Theorem in Bohmian mechanics, and a "theorem" in ordinary QM

You cannot measure a particle's wave function: There is no experiment that could be applied to any given particle with unknown wave function ψ and would determine ψ (with any useful reliability and accuracy).

For example, in $\mathscr{H} = \mathbb{C}^2$ there is a 2-parameter family of ψs (with $\|\psi\| = 1$ and modulo global phase), but (it can be shown) any experiment yields essentially just 1 bit of outcome.

If you are given $N >> 1$ particles, each with wave fct ψ, then you can determine ψ to arbitrary accuracy and reliability if N is sufficiently large.

If you know that a certain particle has wave fct ψ then you can prove it, in the following sense: You can specify an experiment (with observable $P_{\mathbb{C}\psi}$) that yields outcome "1" with prob. 1 and "0" with prob. 0; if you didn't know ψ the prob. of "0" would be positive.

Upshot: Nature can keep a secret. She knows what the wave function is, but doesn't allow us to measure it.

Limitations to knowledge

Bell (1987):

"To admit things not visible to the gross creatures that we are is, in my opinion, to show a decent humility, and not just a lamentable addiction to metaphysics."

Nonlocality in Bohmian mechanics

$\frac{d\mathbf{Q}_1}{dt}$ depends on $\mathbf{Q}_2(t)$, no matter the distance $|\mathbf{Q}_1(t) - \mathbf{Q}_2(t)|$.

Nonlocality

Bell's nonlocality theorem (1964)

Certain statistics of outcomes (predicted by QM) are possible only if spacelike separated events sometimes influence each other. (No matter which interpretation of QM is right.)

These statistics were confirmed in experiment [Aspect 1982 etc.].

Bell's lemma (1964)

Non-contextual hidden variables are impossible in the sense that they cannot reproduce the statistics predicted by QM for certain experiments.

Upshot of Einstein-Podolsky-Rosen's argument (1935)

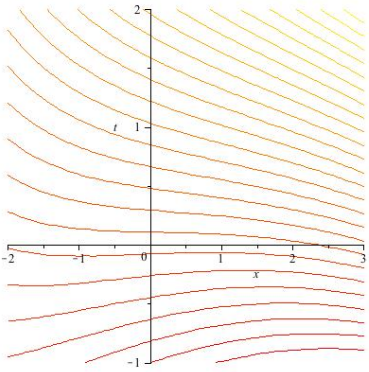
Assume that influences between spacelike separated events are impossible. Then there must be non-contextual hidden variables for all local observables.

Note: EPR + Bell's lemma $\Rightarrow$ Bell's theorem

singlet state $\frac{1}{\sqrt{2}}|{\uparrow\downarrow}\rangle - \frac{1}{\sqrt{2}}|{\downarrow\uparrow}\rangle$

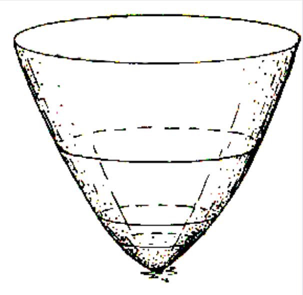
Bohmian mechanics in relativistic space-time

- If a preferred foliation (= slicing) of space-time into spacelike hypersurfaces ("time foliation" $\mathcal{F}$) is permitted, then there is a simple, convincing analog of Bohmian mechanics, $BM_{\mathcal{F}}$. [Dürr et al. 1999] Without a time foliation, no version of Bohmian mechanics is known that would make predictions anywhere near quantum mechanics. (And I have no hope that such a version can be found in the future.)



There is no agreed-upon definition of "relativistic theory." Anyway, the possibility seems worth considering that our universe has a time foliation.

Simplest choice of time foliation $\mathcal{F}$



Let $\mathcal{F}$ be the level sets of the function
T : space-time $\to \mathbb{R}$,
$T(x) =$ timelike-distance(x, big bang).

E.g., T(here-now) = 13.7 billion years

Drawing: R. Penrose

Alternatively, $\mathcal{F}$ might be defined in terms of the quantum state vector ψ, $\mathcal{F} = \mathcal{F}(\psi)$ [Dürr, Goldstein, Norsen, Struyve, Zanghì 2014]

Or, $\mathcal{F}$ might be determined by an evolution law (possibly involving ψ) from an initial time leaf.

Key facts about $\text{BM}_{\mathcal{F}}$

Known in the case of N non-interacting Dirac particles, expected to be true also, say, one day, in full QED with photon trajectories:

Equivariance

Suppose initial configuration is $|\psi|^2$-distributed. Then the configuration of crossing points $Q(\Sigma) = (Q_1 \cap \Sigma, \ldots, Q_N \cap \Sigma)$ is $|\psi_\Sigma|^2$-distributed (in the appropriate sense) on every $\Sigma \in \mathcal{F}$.

Predictions

The detected configuration is $|\psi_\Sigma|^2$-distributed on *every* spacelike Σ.
No superluminal signaling.

As a consequence,

$\mathcal{F}$ is invisible, i.e., experimental results reveal no information about $\mathcal{F}$.
(Another limitation to knowledge)

- Although it may seem to go against the spirit of relativity, I take seriously the possibility that our world might have a time foliation.
- However, there do exist relativistic realist theories of quantum mechanics that do not require a time foliation: A relativistic version [Tumulka 2006] of the Ghirardi-Rimini-Weber (GRW) collapse theory.
- The theory predicts tiny deviations from quantum mechanics that can be tested in principle but not with current technology.
- The theory is somewhat more complicated and less natural than Bohmian mechanics.
- The wave function ψ_Σ on the spacelike hypersurface Σ is random and evolves according to a stochastic modification of the Schrödinger equation.

Bohmian mechanics for a single Dirac particle

No time foliation needed in this case.

Dirac equation:

$$i\hbar\gamma^\mu\partial_\mu\psi = m\psi \qquad \text{or} \qquad i\hbar\frac{\partial\psi}{\partial t} = -i\hbar\boldsymbol{\alpha}\cdot\nabla\psi + m\beta\psi$$

Equation of motion:

$$\frac{dX^\mu}{ds} \propto \overline{\psi}(X^\nu(s))\,\gamma^\mu\,\psi(X^\nu(s))$$

or, equivalently,

$$\frac{d\mathbf{X}}{dt} = \frac{\psi^*\boldsymbol{\alpha}\,\psi}{\psi^*\psi}(\mathbf{X},t) = \frac{\mathbf{j}}{\rho}(\mathbf{X},t)$$

world lines = integral curves of current 4-vector field $j^\mu = \overline{\psi}\gamma^\mu\psi$
world lines are timelike or lightlike at every point

$|\psi|^2$ is conserved in every Lorentz frame.

Foundations of QM come up in cosmology

- The problem of structure formation in the early universe [Sudarsky, Okon 2013]: A slightly non-uniform distribution of matter in space leads, through the effect of gravity, to clumping of matter to galaxies and stars. But the highly symmetrical initial quantum state ψ evolves into a superposition of clumped states. No problem for Bohm.
- The problem of time in quantum gravity: According to the Wheeler-de Witt equation (the central equation of canonical quantum gravity), the wave function of the universe must be an eigenfunction of the Hamiltonian, and thus time-independent. No problem in a Bohm-type theory, as $Q(t)$ still depends on t.
- The Wheeler-de Witt wave function is a superposition of various 3-geometries. But we need to talk about 4-geometries. How do these 4-geometries arise? (Different proposals are provided by the Bohm-type theories, decoherent histories, collapse theories, and perhaps many-worlds, leading to rather different conclusions about the 4-geometry [Struyve, Pinto-Neto 2014; Das et al. 2015].)
- Are there Boltzmann brains in the late universe? Bohm $\Rightarrow$ no.

Thank you for your attention