FUNDAMENTAL PHYSICAL THEORIES: MATHEMATICAL STRUCTURES GROUNDED ON A PRIMITIVE ONTOLOGY

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ABSTRACT OF THE DISSERTATION

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In my dissertation I analyze the structure of fundamental physical theories. I start with an analysis of what an adequate primitive ontology is, discussing the measurement problem in quantum mechanics and theirs solutions. It is commonly said that these theories have little in common. I argue instead that the moral of the measurement problem is that the wave function cannot represent physical objects and a common structure between these solutions can be recognized: each of them is about a clear three-dimensional primitive ontology that evolves according to a law determined by the wave function. The primitive ontology is what matter is made of while the wave function tells the matter how to move. One might think that what is important in the notion of primitive ontology is their three-dimensionality. If so, in a theory like classical electrodynamics electromagnetic fields would be part of the primitive ontology. I argue that, reflecting on what the purpose of a fundamental physical theory is, namely to explain the behavior of objects in three–dimensional space, one can recognize that a fundamental physical theory has a particular architecture. If so, electromagnetic fields play a different role in the theory than the particles and therefore should be considered, like the wave function, as part of the law. Therefore, we can characterize the general structure of a fundamental physical theory as a mathematical structure grounded on a primitive ontology. I explore this idea to better understand theories like classical mechanics and relativity, emphasizing that primitive ontology is crucial in the process of building new theories, being fundamental in identifying the symmetries. Finally, I analyze what it means to explain the word around us in terms of the notion of primitive ontology in the case of regularities of statistical character. Here is where the notion of typicality comes into play: we have explained a phenomenon if the typical histories of the primitive ontology give rise to the statistical regularities we observe.

Acknowledgements

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Dedication

I dedicate this dissertation to my cat, Trizzi.

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Chapter 1

Introduction

There is a basic philosophical question that involves metaphysics, physics and epistemology: Can we explain what the world is like through a fundamental physical theory? This question corresponds to the historic disagreement among scientists and philosophers concerning how to regard physical theories to which people commonly refer as the realist–antirealist debate. The position of the antirealist is the one according to which we should not believe that physics reveals to us something about reality but rather we should be content with physics to be, for example, just empirically adequate. In contrast, the realist is strongly inclined to say not only that physics tells us about reality, but also that it is our only way to actually do metaphysics. I am a realist insofar as I believe that physics actually informs us about the world. That is, I agree on what Tim Maudlin claims in his *Suggestions from Physics for Deep Metaphysics* (Maudlin m.):

> [...] metaphysics, i.e. ontology, is the most generic account of what exists, and since our knowledge of what exists in the physical world rests on empirical evidence, metaphysics must be informed by empirical science.

The main problem I would like to investigate in my dissertation is therefore the following: Granted that physics provides information about the world, what does it mean to explain the world around us in terms of a fundamental physical theory? I believe that this question can be reformulated in this way: Is there a general structure that a fundamental physical theory should have in order to allow us to understand what the world is like?

There are some notions that I believe helped me in finding an answer to this question: the notion of primitive ontology and the one of typicality. The notion of primitive ontology is connected to but not exhausted by another notion, the one of "local beable". This has been introduced by John Stuart Bell in his *Are there Quantum Jumps?*

(Bell 1987). A "beable" is a speculative piece of ontology, it represents what is real according to the theory. It is "local" in the sense that its value can be assigned to a given bounded region of space-time. In the words of Bell:

> These are the mathematical counterparts in the theory to real events at definite places and times in the real world (as distinct from the many purely mathematical constructions that occur in the working out of physical theories, as distinct from things which may be real but not localized, and distinct from the "observables" of other formulations of quantum mechanics, for which we have no use here). (Bell 1987)

It has been suggested that there is some sort of distinction among the objects that are commonly accepted as the ontology of a fundamental physical theory. This is clear in the work of Detlef Dürr, Shelly Goldstein and Nino Zanghì, (DGZ 1992) and (DGZ 1997), in which they talk about a primitive ontology of a theory as opposed to the ontology of the theory in general:

> [...] primitive ontology – the basic kinds of entities that are to be the building blocks of everything else (Except, of course, the wave function)[the parenthetical remark was in a footnote in the original].(DGZ 1992)

I believe that this notion is crucial for understanding what it means for a theory to give an account of the behavior of physical objects in the world we live in. Before fleshing this out, though, it is necessary to characterize the notion of primitive ontology more carefully as a significant ingredient of a fundamental physical theory.

I therefore start from some practical examples in order to try to clarify what the notion of primitive ontology is supposed to be and what is an adequate primitive ontology for a fundamental physical theory. At first I discuss about quantum mechanics. In order to do this properly, I start in Chapter 2 discussing the measurement problem. In that chapter I also analyze the different solutions of the measurement problem: Bohmian mechanics, GRW theory and many worlds quantum mechanics, what are called "quantum theories without observer". I discuss the different possible quantum metaphysics that one can infer from them in Chapter 3.

In Chapters 4 and 5 I show how all the different pictures of the world presented in Chapter 2 can actually be divided in two groups: in the latter group belong theories in which the wave function is a concrete physical object, in the former one there are theories of primitive ontologies in three–dimensional space. After comparing the different views, I argue how the interpretation in terms of a three-dimensional primitive ontology is more satisfactory than the other: there is the need of some three-dimensional primitive ontology as the basic ontology in order for the theory to be able to really solve the measurement problem. That is, an adequate primitive ontology for a quantum theory without observer is the one in which the histories of the primitive ontology are in space-time. In other words, the monism of the wave function, the view according to which the physical world is made of wave functions, is untenable even in theories like GRW or many worlds, in which it seems there is nothing but the wave function.

As noted, there are similarities between the notion of primitive ontology and the notion of local beables introduced by Bell, so one might wonder whether the notion of primitive ontology is "just" the notion of local beable. Chapter 6 will be dedicated to discuss that it is not necessarily the case: there are local beables that are not adequate primitive ontologies. The idea is that if we have a theory in which there are particles and fields such as classical electrodynamics, we might still want to think of the theory having particles as the primitive ontology but not the fields, and not because of their nature of fields, in order to preserve a certain structure of the theory.

What is the primitive ontology, if it is not the notion of local beable? I will pin down the primitive–non primitive distinction more carefully in Chapter 7, in which I will analyze the common structure that emerges from the discussion above: not only quantum theories without observer and classical electrodynamics are mathematical structures grounded on primitive ontology but also any other fundamental physical theories, like classical mechanics relativity have that form. I will explore the application of the notion of primitive ontology to theories we already have, such as classical mechanics and relativity. I will show how this structure is needed for having a genuine and satisfactory explanation of the phenomena in terms of a fundamental physical theory and I will argue how primitive ontology is crucial in the process of building new theories, being fundamental in determining the symmetry properties of a theory.

Once it is established what the microscopic description of the world is, it is necessary to understand how we could (and should) explain the macroscopic regularities we observe. I will turn to this issue in Chapter 8, in particular considering the case in which empirical statistical regularities have to be explained, and here is where the notion of typicality comes into place. Even if the theory is microscopically deterministic, at the macroscopic level we might find statistical regularities. How this is possible has been explained in the context of classical physics by Statistical mechanics, and a similar analysis can be provided in the case of deterministic quantum theories like Bohmian mechanics. The idea is that a statistical regularity is explained if a typical history of the primitive ontology of the theory will produce it. In the case of quantum theories like GRW, which seem "intrinsically" probabilistic, it is often argued that the explanation in this case is different and arguably less problematical (Albert 2001). I will argue instead that the situation is the same: if a typical history of the primitive ontology will produce the same regularity we observed, then the phenomenon has been explained.

Chapter 2

Pandora's Cat and Quantum Theories without Observers

In this chapter I will discuss what is believed to be a most discussed problem of quantum mechanics: the problem of the Schrödinger cat, also called the measurement problem. After having done so, I will argue that it is just a symptom, rather than the cause, of all the mysteries and the paradoxes of quantum mechanics. To anticipate the conclusion, a way to express the moral of the Schr̈odinger cat is to say that quantum mechanics is not a complete theory, that is, the problem of the Schrödinger cat is *the problem of the completeness of quantum mechanics*. One could say that quantum mechanics is not complete in the sense that it is unable to account for the properties we believe macroscopic object should have. This is what I would call *the problem of indefinite properties*. But again, why is quantum mechanics not able to do so? What is the origin of this problem? When we talk about a property, we have in mind the idea of something having that property. But what is that "something" if quantum mechanics is true? I will argue that this problem is parasitic on what we could call *the problem of the lack of a clear ontology*: as clearly Shelly Goldstein (Goldstein 1998) pointed out first, it is not clear what quantum mechanics is about. In the process of trying to figure out what the theory is about we end up with various alternatives: The wave function? The observer? The results of measurement? Particles? Fields? Strings? Only after having answered this question, one can proceed to investigate whether the primitive ontology is an adequate one. The primitive ontology is not adequate if it is not able to represent physical objects and their properties. In that case, we say that the theory is not complete. Some of the primitive ontologies are almost straightforwardly inadequate like, for example, the proposal that quantum is about the observer, as we will see. What about the wave function itself? As we will discuss later, it is usually said that what the problem of the Schrodinger cat is telling us is that the wave function alone, *if it evolves according to Schrödinger equation*, cannot completely describe physical objects. But what about a wave function that does not evolve according to Schrödinger's equation? This is, I believe, the real question that needs to be answered. I will argue that the moral of the problem of the cat is that the description provided by the wave function alone is *never* complete. That is, the wave function cannot be what quantum is fundamentally about, or, in other words, the wave function cannot be the *primitive ontology* of the theory. And the reason for this is that it is a too abstract mathematical object. This is what I will call *the problem of the adequacy of the primitive ontology*. We will see how, once one has a theory with an adequate primitive ontology, one can account for the properties and the behavior of macroscopic objects in three-dimensional space so that the problem of indefinite properties does not arise.

2.1 The Schrödinger Cat: The Measurement Problem

But what is the problem of the Schrödinger cat? It might seem I am being a little bit redundant to explain this problem again: is it not clear already what it is supposed to be, at least in the philosophical community? Nonetheless, I believe that the situation is still quite subtle. Indeed, we will see how it is not so obvious what the problem is and what the solutions are *really* telling us about what the world is like.

Let us start from the beginning. What is "standard" quantum mechanics, the one that is found in physics textbooks? Let us call "state" of a system all that needs to be specified in order to completely describe any physical system. Then the basics assumptions of this theory, that we could call, for reasons that will appear clear later, *bare quantum mechanics*, are the following:

- There is an object, called the wave function and usually written with the Greek letter ψ, that represents the state of the system of any physical system, and
- It evolves in time according to a given differential equation, called the Schrödinger equation.

The wave function $\psi = \psi(q)$ is a function of the configuration $q = (q_1, ..., q_N)$, where

N is the number of "particles". Note that in quantum mechanics so far there are no particles, just the wave function. So the use of the world "particle" in this context will be will be just for convenience. Each $q_i \in \mathbb{R}^3$ with $i = 1, ..., N$ is a degree of freedom of the wave function such that q lives in a space of dimension $d = 3N$. This space, $\mathbb{R}^{3N}$, is called *configuration space*, since it could be identified of the space of the configurations of N particles in three-dimensional space, it there were any particles. Therefore, the wave function is a mathematical object defined on configuration space. In addition, it is complex-valued:

$$\psi : \mathbb{R}^{3N} \to \mathbb{C} \tag{2.1}$$

The space of all wave functions forms an Hilbert space, that is a space that generalizes the notion of Euclidean space. Roughly speaking, this space has a linear structure (it is a vector space) on which an inner product is defined (and therefore it is possible to talk about distance, angles, orthogonality), and such that it is complete (that is, there are not any pathological behavior in taking the limits). In addition, the two wave functions ψ and $c\psi$, where $c \in \mathbb{C}$ is such that $|c|^2 = 1$, are not physically distinct: that is, ψ and $c\psi$ represent the same physical object. Therefore, the wave functions actually form equivalence classes in a projective Hilbert space, defined by the relation: $v \sim w$ when $w = cv$, for any $c \neq 0, c \in \mathbb{C}$. This is what is meant when it is said that wave functions are projective objects. More precisely, they are *rays* in Hilbert space. As anticipated, the wave function ψ evolves in time according to Schrödinger's equation:

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi\,, \tag{2.2}$$

where $\hbar = h/2\pi$, h being the Planck constant $h = 6.63 \cdot 10^{-34}$ m^{3}kg/s, H is the usual nonrelativistic Hamiltonian, that, for spinless particles, is of the form

$$H = -\sum_{k=1}^{N} \frac{\hbar^2}{2m_k} \nabla_k^2 + V, \tag{2.3}$$

containing as parameters the masses m_k of the "particles" as well as the potential energy function V of the system. One can also write the wave function at a time t ψ_t

as evolved according to Schrödinger's equation from the wave function at time $t = 0$ ψ_0 through an operator U, $U = \mathrm{e}^{-\frac{i}{\hbar}Ht}$, that describes the Schrödinger evolution:

$$\psi_t = U\psi_0. \tag{2.4}$$

It is important to emphasize that a crucial feature of Schrödinger's equation is that it is linear: if ψ_1 is a possible description of a physical system at a given time t, and so is ψ_2, then also the sum of the two, namely $\psi_1 + \psi_2$ (up to a normalizing factor), provides a possible description of that physical system at time t. States of this form are called *superposition states*.

Let us now turn to the measurement problem, which has been formulated for the first time by Erwin Schrödinger in his seminal paper (Schrödinger 1983). The situation discussed in the experiment considered by Schrödinger, as shown in Figure 2.1, is as follows: there is a cat in the box, together with a bottle of poison. This bottle is connected to a device that is triggered by the decay of a radioactive nucleus in such a way that if the nucleus decays the poison will be diffused into the box killing the cat. If the nucleus does not decay, nothing happens to the cat. At a given time, the nucleus can or cannot decay. That is, the possible states of the nucleus are: $\psi_{decayed}$ and $\psi_{undecayed}$. Because of linearity, there is another possible quantum state, namely the one described by a wave function of the form $\psi = \psi_{decayed} + \psi_{undecayed}$. If the wave function provides the complete description of the world, and if it evolves according to Schrödinger's equation, then the microscopic superposition of the nucleus amplifies macroscopically to the state that describes cat: A cat alive and dead at the same time? This state is a superposition state, therefore it describes the system being at the same time into two (macroscopically disjoint) states of affair, in this case a dead and an alive cat. This does not correspond to a state that we find in the world, and therefore something wrong is going on. Note that this state is what bare quantum mechanics predicts to happen in any experimental situation. This is why this problem is also called *the measurement problem*. In fact, suppose we want to measure something, take for example the current in a wire. The standard rules of quantum mechanics say that

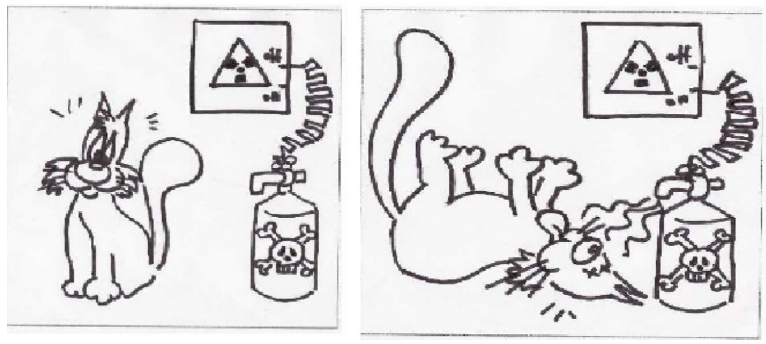

Figure 2.1: The Schrödinger cat experiment.

the results of the experiment will be given, mathematically, by the eigenvalues of some appropriate self-adjoint operator. A vector z is an eigenstate of a given operator A if and only if

$$Az = \xi z,$$

where $\xi \in \mathbb{C}$ is the corresponding eigenvalue. If z is an eigenstate, the operator A just transforms it into a multiple of itself. Because of this reason, it has been considered reasonable for eigenvalues to represent properties. Suppose that the state of the system is an eigenstate ψ_α of the "observable being measured" (in this case, the current, represented by an operator A). That is, $A\psi_\alpha = \alpha\psi_\alpha$. Let the apparatus, the pointer, have an initial wave function ϕ_0 so that the initial total state is $\psi_\alpha\phi_0$. Note that, since we want the measurement to be genuinely such, Schrödinger's evolution should not change the state ψ_α of the system being measured, that is $U\psi_\alpha = \psi_\alpha$. The pointer state, since we want it to be a genuine measurement apparatus, will have to evolve into $\phi_t = U\psi_0 = \phi_\alpha$, such that the information we want to measure about the system will be displayed macroscopically by the position of the pointer. That is, ϕ_α represents the position of the pointer in the direction α corresponding to the state of the system

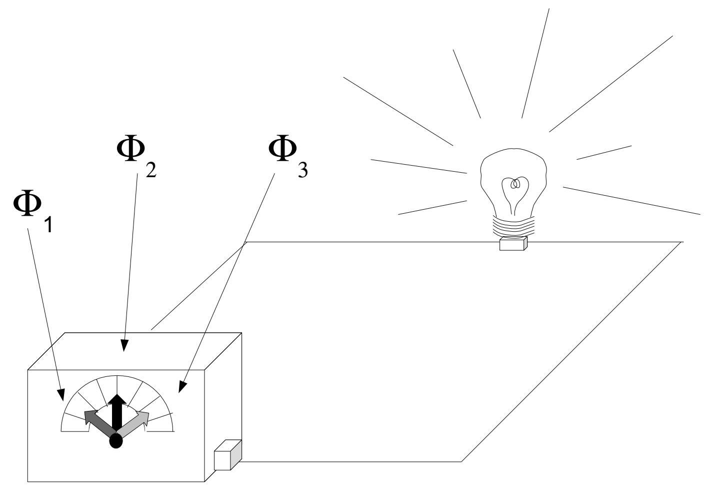

Figure 2.2: Pointers do not point in bare quantum mechanics.

being ψ_α. The final state of the comprehensive system (system+apparatus) is therefore $\Psi_t = \psi_\alpha \phi_\alpha$. Note that α represents a possible results for the experiment: 1 ampere, 2 ampere, 3 ampere, and so on. Now suppose the state of the system is not an eigenstate, but instead it is a superposition of the form $\psi_0 = \sum_\alpha c_\alpha \psi_\alpha$. The initial wave function of the total system is therefore $\Psi_0 = \psi_0 \phi_0 = \sum_\alpha c_\alpha \psi_\alpha \phi_0$. The final wave function will be $\Psi_t = U\Psi_0 = U \sum_\alpha c_\alpha \psi_\alpha \phi_0$ and because of the linearity of the evolution U we will have $\Psi_t = \sum_\alpha c_\alpha \psi_\alpha U \phi_0 = \sum_\alpha c_\alpha \psi_\alpha \phi_\alpha$. This final wave function is analogous to the one of the cat in macroscopic superposition of life and death and describes macroscopic superpositions of pointer positions pointing in different directions. We arrived to a really troublesome conclusion, since the final state corresponds to measurement not having results. In fact, each term of the superposition represents a pointer pointing somewhere (ψ_α corresponds to the pointer pointing to the value α), so that this state describes a pointer not pointing anywhere, as shown in Figure 2.2!

To sum up, then, the three claims:

1. The wave function provides the complete description of any physical system,

2. The wave function evolves according to Schrödinger's equation,

3. Measurements have results

are incompatible. This is, in a nutshell, the problem of the Schrödinger cat as it is usually presented.

2.2 Bell's Alternatives

If we assume that measurements have indeed results, then the lesson we should draw from the measurement problem is that either (1) or (2) must be false. This leads us straightforwardly to the following alternatives:

- Deny (1) and add something to complete the description provided by the wave function, or
- Deny (2) and allow the wave function to evolve according to an equation different from Schrödinger's evolution.

These are the famous alternatives proposed by the physicist John Stuart Bell:

> either the wave function, as given by the Schrödinger equation, is not everything, or is not right (Bell 1987).

This means that (if we assume measurements to have results) there are only two possibilities to solve the measurement problem and to make quantum mechanics a precise (i.e. not ambiguous) fundamental physical theory. In the first possibility the complete description of a physical system is given by the wave function, which still evolves according to Schrödnger's equation, *and* by some additional ("hidden", because it is not "suggested by" the Schrödinger equation) variable. The other only possibility consists in assuming that the wave function provides the complete description of the system but its evolution in time is given by an equation that differs by the one of Schrödnger.

2.3 Some Problematical Attempts

Bell's alternatives leave open the possibility for more than two theories. In fact, depending on what we add to the description of the wave function or how we change

Schrödinger's evolution, we have different theories. A variety of these theories have been proposed, and some are more satisfactory than others. In the following section I will start analyzing some problematical attempts, while in Section 2.4 I will discuss more satisfactory solutions of the measurement problem.

2.3.1 Von Neumann and the Collapse

The first attempt to solve the measurement problem was provided by the famous physicist and mathematician John von Neumann (von Neumann 1932). The basic idea is to postulate that Schrödinger's equation ceases to be valid during a measurement situation. In that case, when a measurement occurs, the evolution is determined by a nonunitary transformation, often called "collapse" or "reduction" of the wave function. This evolution is, of course, incompatible with Schrödinger's evolution since it is random and irreversible: every time there is a measurement, the wave function is not in a superposition state anymore but collapses randomly into one of the terms of the superposition. This evolution is undeniably *ad hoc*, postulated just in order to eliminate all the other terms of the superposition but the one that happens to be the result of the measurement.

We could classify this way of solving the problem as following route number 2: the Schrödinger evolution is not valid all the time, but only in between measurements. Or we could also classify it as a way of completing quantum mechanics: the wave function does not provide the complete description, we need also to add the "observer": she is playing a crucial role in the theory because every time she makes an observation or a measurement, there is a change in the evolution of the wave function. When the measurement occurs, the object is not described by a superpositions state but rather by one of its terms, as shown in Figure 2.3.

Still, here is something to think about: in the defining terms of this theory there is the notion of measurement and observer. But what is an observer? What is a measurement? As Bell has emphasized, any fundamental physical theory should not use in its definition, among its fundamental entities, such vague concepts. Rather, they should be derived by something more fundamental:

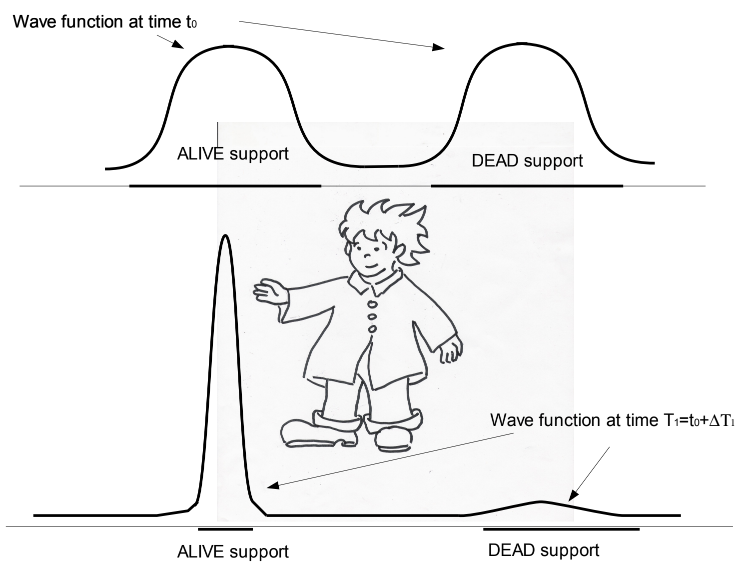

Figure 2.3: Von Neumann and the Schrödinger cat.

> What exactly qualifies some physical systems to play the role of 'measurer'? Was the wavefunction of the world waiting to jump for thousands of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system... with a PhD? (Bell 1987)

After all, is not a measurement a physical process? And is not the observer a physical object as well? In this theory they have a special status, since something distinctive happens when someone "observes" something. But what makes them special? If we want to take this theory seriously, we need to clarify what the observers are and what makes them so fundamentally different from the rest.

2.3.2 Wigner and Consciousness

Fritz London and Edmond Bauer (London and Bauer 1983) proposed that it is human consciousness which defines what an observer is. In the 1960s, the physicist Eugene Wigner followed up on this proposal. In his *Remarks on the Mind-Body Question* (Wigner 1967), Wigner argued that what characterizes the observer is consciousness and that the collapse of the wave function happens because of an interaction of the consciousness on the physical system. In order to make his point, Wigner elaborated an extension of the experiment of the Schrödinger cat, what is called the experiment of Wigner's friend, as shown in Figure 2.4. In this experiment, in addition to the cat, there is also Wigner and one of his scientist friends. The latter performs the experiment of the Schrödinger cat while Wigner is outside the laboratory. According to Wigner, the state of the system (cat+box+scientist) is a superposition state of (the atom having decayed, the cat having died, the friend having seen the dead cat) and (the atom having not decayed, the cat being still alive, the friend having seen the alive cat). But at some point, Wigner comes back and learn the result of the experiment. The idea is to show how consciousness is necessary: if instead of a conscious observer we have some apparatus, as we saw, the linearity of the wave function implies that the wave function is in a linear sum of possible states. In contrast, a conscious observer must be in either one state or another, and this is what makes conscious observations different. Consciousness will then act on the physical state to make it collapse into one of the terms.

As one can see, this is straightforwardly a denial of the closeness of the physical world and, as such, is a very radical position. That implies, also, that in order to construct a quantum mechanics without the measurement problem following this route we need to have a theory of how consciousness works and of how it interacts with the physical world.

2.3.3 Bohr and the Copenhagen Interpretation

Another attempt to solve the measurement problem is the one provided by Niels Bohr, also called the Copenhagen interpretation of quantum mechanics. According to Bohr, quantum mechanics is not a complete theory: it needs classical mechanics for its own foundations. For a detailed account of this view, see (Landau and Lifschitz 1977). So, in this sense, it is a solution that follows route 1. Therefore, there are two fundamental theories: the classical and the quantum theory. But while the classical theory deals with a clear and distinct world, made of cats, planets, tables and chairs, the quantum world is so obscure and far away from our ordinary experiences that it is impossible *in principle* to have a clear picture of it. The idea is that we do not have, intrinsically, the

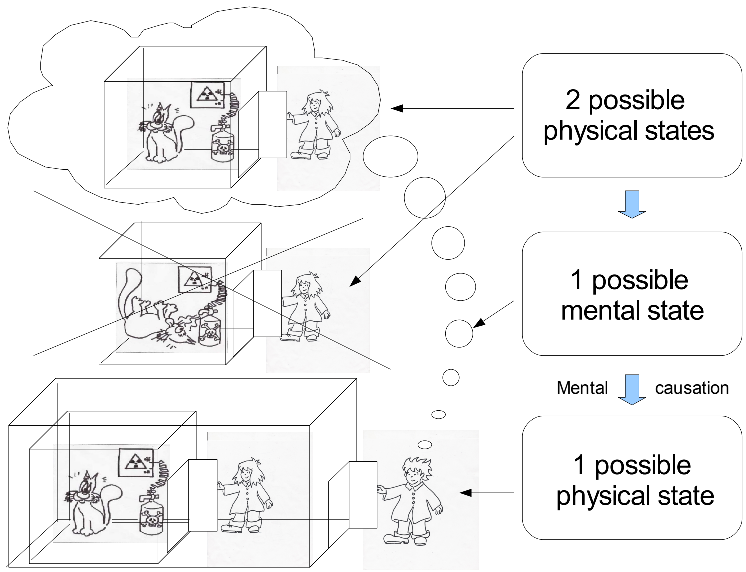

Figure 2.4: Wigner and the Schrödinger cat.

appropriate concepts to be used in the framework of quantum mechanics: the best we can do is to supplement the quantum description with a classical one.

This is where Bohr's (in)famous "wave-particle duality" came from: "wave" and 'particle" are words that we use in ordinary language, but they do not really adequately account for what happens in the microscopic world. They provide at best a partial description of it. This idea came to Bohr probably reflecting on some experiments that at the time shown how what we would regard as particles sometimes would behave like waves and *vice versa*. One of the most famous examples of wave-like behaviors of particles is the two-slits experiments. In this experiment, as shown in Figure 2.5, particles are sent toward a screen with two small slits on it. A second screen is placed behind it to detect the particles. The particles, arriving one by one, hit the second screen, forming a spot. What one would expect is to find on the second screen, after a while, the image of the two slits, corresponding to the arrival of those particles which were not stopped by the first screen. What is found instead is an interference patterns, like a wave passing through the two slits would have produced. Suppose that previous experiments have identified the "entities" sent toward the screen as particles: for example, they showed a track in the bubble chamber. Then how can we explain

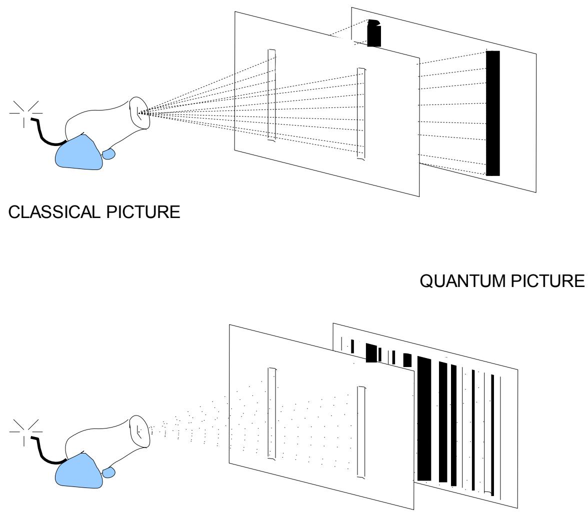

Figure 2.5: The two-silt experiments.

such a wave-like behavior? Bohr thought that we simply cannot: particles and waves are just our inadequate concepts, they do not reflect much about what the quantum world is like. It happens that we do not have, intrinsically, the appropriate concepts to describe quantum reality. Bohr actually went much farther than that. In fact, he concluded that not only in principle it is impossible for us to understand or coherently talk about the quantum world, but also there is no fact of the matter about it. Of course, one is not forced to reach such a radical conclusion from the discussion we have done so far: one thing is that we cannot describe something, another is that there is no realty to it! Nonetheless, nowadays this is what one commonly hears in physics departments all over the world.

Be that as it may, if we assume the quantum world to be real, according to Bohr we have two descriptions, one in which there are superpositions, that would be appropriate for the microscopic world, and one in which there are not, the classical one. Since the cat is a macroscopic object, by definition, she is never in a superposition state. A decaying nucleus instead obeys to the quantum laws, so it can stay in superpositions state. The nucleus, decaying or not, will have influence on what will happen to the cat but there is no paradox: the cat is either dead or alive because she is a classical object,

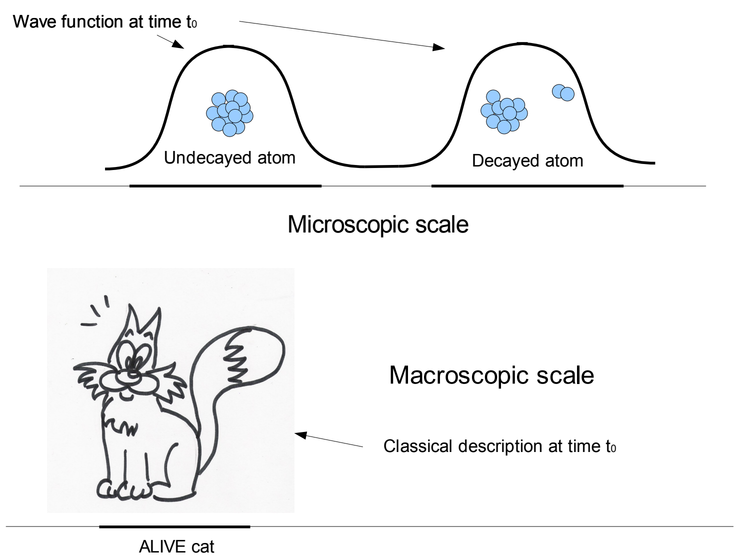

Figure 2.6: Bohr and the Schrödinger cat.

while it is not problematical for the nucleus to be in a superposition state because it is a quantum object, see Figure 2.6.

After a little thought it should be clear how this theory is intrinsically not very satisfactory: in fact, where is the cut between the "solid" classical world and the "wavy" quantum world? How many "particles" must an object have in order to be called macroscopic? This introduces a fundamental ambiguity into the theory, as Bell would put it, that should not be there in a fundamental physical theory:

> Thus in contemporary quantum theory it seems that the world must be divided into a wavy quantum system, and a remainder that is in same sense classical... It introduces a fundamental ambiguity into fundamental physical theory (Bell 1987).

2.4 Quantum Mechanics without Observer

Albert Einstein was really troubled by quantum mechanics. The debate between Einstein and Bohr has been often considered the paradigmatic debate about the foundations of quantum theory. While Einstein insisted on the possibility and the necessity of having a formulation of quantum mechanics in which the observer did not play any fundamental role, and of describing the macroscopic world through such a theory, Bohr solved the measurement problem in his own way, postulating that there are actually two fundamental descriptions, namely quantum and classical mechanics, that are complementary the one to the other. According to most physicists, Bohr was the winner of the debate: "We might not like it, but that is how the world is" is the often repeated slogan. After all, we are the product of evolution and there is nothing that *guarantees* that we will be able to properly describe realty. For example, Colin McGinn (McGinn 1989) has argued that consciousness is something exactly of that sort: we evolved in such a way that we do not actually have the right abilities to grasp what consciousness really is. Bohr is saying the same for the concept we use in physics. Therefore, we should embrace reality: we *thought* we had the appropriate concepts to understand the world but we *do not*. Wigner instead is insisting that consciousness has to play an active role in physics. Either way, the best we can do is to construct such unsatisfactory theories as these ones and we should learn how to live with them.

That might well be the case, but before accepting something like this one should better look at all the alternatives and see whether they work and whether we are really forced to give up any hope of understanding the microscopic world through physics as we know it. Why should we believe that we cannot comprehend the world if there are alternatives in which we actually can? I think that physicists have been a little too hasty in following Bohr or Wigner: before giving up so quickly to the strangenesses of the quantum world and adjusting to the idea that we have to give up the possibility of really understand what the world is like, we should at least analyze whether there are some other alternatives. So, what about them? Are really there theories in which the observer, or consciousness, does not play any crucial role and in which we do not need to postulate a quantum and a classical world? The answer is positive: There are more than one quantum theory that solve the measurement problem, provide a coherent representation of the microscopic world, that do not need the "observer", the specification of the notion of "classical" or consciousness in its formulation. They can be labeled with the name of "quantum theories without observer". The terminology is due to Karl Popper, who first used this expression in his article *Quantum Mechanics Without the Observer* (Popper 1967) and later used in particular by Bell (Bell 1987) and Goldstein (Goldstein 1998). At the end of the day, since there are clear examples of quantum theories without observer, there is nothing that forces us to choose what von Neumann, Wigner and Bohr have proposed. Therefore, I think that we can claim that the real winner of the debate between the two scientists was Einstein, not Bohr: what Bohr believed to be impossible does actually exist, in more than one version. Let us discuss in more detail what these quantum theories without observer are.

2.4.1 Quantum Theory without Observer 1: Additional Variables

This quantum theory without observer solves the measurement problem taking the first of the two routes described above: the wave function does not provide the complete description of the system but something needs to be added to it. Historically, since these additional variables are not suggested by the Schrödinger equation (considered to be the core of quantum mechanics), this theory has also been called "theory of hidden variables". It was first proposed, under the name of "pilot-wave theory", by Louis de Broglie (de Broglie 1928) at the famous Solvay congress of 1927 for a one-particle system. Wolfgang Pauli had some objection to the theory and that discouraged de Broglie so much that he decided not to continue his investigations on the matter any firther. The theory was proposed again, in a more general framework, by David Bohm in 1952 (Bohm 1952). In Bohm's paper, the theory was presented in terms of the so-called "quantum potential" and this was very unfortunate, for a variety of reasons. A better formulation of Bohm's theory is the one provided by Detlef Dürr, Shelly Goldstein and Nino Zanghí in their *Quantum Equilibrium and the Origin of Absolute Uncertainty* (DGZ 1992) under the name of Bohmian mechanics, in which the theory is presented by a system of two coupled first-order differential equations in which there is no mention of the mysterious quantum potential (for a review of the problems of the formulation is terms of the quantum potential, see (DGZ 1992)).

In this theory the complete description of the state of an N point-like particles system is given by the couple (Ψ_t, Q_t), where $\Psi_t = \Psi_t(q)$ is the wave function of the system and $Q_t = (Q_1(t), ..., Q_N(t))$ represents the configuration of the N particles composing the system, q_k being the position of particle k in $\mathbb{R}^3$ [1]. Therefore, the variable Q belongs to the *configuration space* $\mathbb{R}^{3N}$, that is the space of the possible positions that a physical system composed of N particles may have.

The world "particle" in this theory should be taken seriously: there are *really* particles in the world if Bohmian mechanics is true, just like there would have been if classical mechanics were true. Each $Q_k(t)$ is the actual trajectory of the k-th particle in three-dimensional space $\mathbb{R}^3$. This is a very big difference with bare quantum mechanics in which the state of the same system is given only by the wave function and there are no particles with no positions and no trajectories whatsoever.

In Bohmian mechanics, the wave function ψ evolves according to Schrödinger's equation, as in bare quantum theory:

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi\,, \tag{2.5}$$

where H is the usual nonrelativistic Schrödinger Hamiltonian. The particles evolve according to the so-called "guide" or "guiding" equation, which is determined by the wave function:

$$\frac{dQ_k}{dt} = v_k^{\psi}(Q_1, \ldots, Q_N) = \frac{\hbar}{m_k}\mathrm{Im}\frac{\psi^*\nabla_k\psi}{\psi^*\psi}(Q_1 \ldots, Q_N), \tag{2.6}$$

where m_k, $k = 1, \ldots, N$, are the masses of the particles. That is, the wave function defines a velocity field v^ψ for the particles. This is the sense in which the wave function "guides" the motion of the particles, and this is where the original name of the theory, "pilot-wave theory", comes from. Note, though, that this is a velocity field in configuration space, not in three-dimensional space.

It should be noted that in Bohmian mechanics the wave function Ψ is the wave function of the entire universe. But the fact that we have configurations in Bohmian mechanics allows us to define the wave function of a given physical system (smaller than the universe): it is the *conditional* wave function, as described in (DGZ 1992), in which

[1] A remark about notations: usually capital letters are used for the *actual* values. Otherwise variables are written in lower case. Roughly speaking, where there is a Q_k (and not a q_k), there is particle k.

we simply plug in the actual configurations of the particles that constitutes everything but the system. If Y represents the actual configuration of all the particles but those that compose the system of interest, which are described by the configuration x, then the wave function of the system is given by $\psi(x) = \Psi(x, Y)$.

Equations (2.5) and (2.6) form a complete specification of the theory. What we have in Bohmian mechanics is a dynamical system for the variables (Ψ, Q). Without any other axiom (about properties assignment, for example), all the results obtained in the framework of non relativistic quantum mechanics follow from the analysis of this system.

As a consequence of Schrödinger's equation and of Bohm's law of motion, we have an important consequence: the distribution $|\psi(q)|^2$ is "equivariant" (see (DGZ 1992)). This means that if the configuration $Q_t = (Q_1(t), \ldots, Q_N(t))$ of a system is random with distribution $|\psi_t|^2$ at some time t, then this will be true also for any other time t. Thus, we can consistently assume the *quantum equilibrium hypothesis*, which asserts that whenever a system has wave function ψ_t, its configuration Q_t is random with distribution $|\psi_t|^2$ [2]. This hypothesis is not as hypothetical as its name may suggest: it follows in fact from the law of large numbers under the assumption that the initial configuration of the universe is typical (i.e., not-too-special) for the $|\Psi|^2$ distribution, with Ψ the initial wave function of the universe. As a consequence of the quantum equilibrium hypothesis, a Bohmian universe, even if deterministic, appears random to its inhabitants. In fact, the probability distributions observed by the inhabitants agree exactly with those of the quantum formalism (see (DGZ 1992) for details). We will come back on this issue in Chapter 8.

Here is a rough idea of why in Bohmian mechanics the problem of the cat does not arise (see Figure 2.7). Even if the wave function is in a superposition, the complete description of the system is given by the wave function and by the positions of particles. Therefore, as far as they are concerned, they are either "here" or "not here". The particles composing the cat are initially in a certain configuration that corresponds to

[2] The word "equilibrium" here is used as Bolzmann would have used it: the equilibrium state is the state in phase space that has the biggest size.

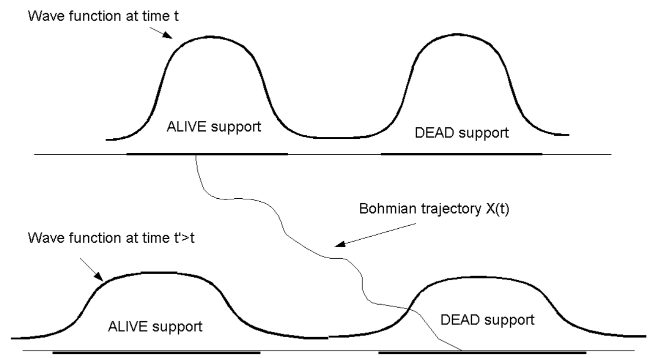

Figure 2.7: Bohmian mechanics and the Schrödinger cat.

an alive cat. With that we mean simply (and roughly) that the particles are so arranged that they form molecules that interact with each other as they do in living things. Call the set of configuration corresponding to an alive cat L and the one corresponding to a dead cat D. Note that the state of a dead cat is macroscopically distinct from that of an alive cat: it is possible to specify macroscopic quantities to differentiate the two states, like for example the temperature of the cat. Due to the fact that the wave function is what it is, the configuration of particles will evolve into a given final configuration. At the final time (which is enough to assume that the experiment is over) the particles will end up, again, either "here" or "not here". In particular, they will end up either in that set L of configurations corresponding to an alive cat or in the set D corresponding to a dead cat. The conditional wave function of the cat will make it the case that the bump whose support does not contain the particles of the cat will not influence the other bump, so that we can consider it to evolve *as if* there was only just the bump containing the particle.

2.4.2 Quantum Theory without Observer 2: Nonlinear Evolution of the Wave Function

We have discussed already how von Neumann's approach can be regarded as a theory that takes route 2 (that is, the denial that the wave function evolves according to Schrödinger's equation) to solve the measurement problem. This theory had the problem that it needed a definition of "observer" in order to establish when we have the collapse of the wave function. A much more satisfactory realization of this second possibility that does not involve the notion of observer at any level is the theory called "spontaneous collapse" or "spontaneous localization" theory. The project was initiated by Philip Pearl (Pearl 1976) in the 70s and developed further by Gian Carlo Ghirardi, Alberto Rimini and Tulio Weber in the 80s (GRW 1986). Other names under which this theory is known are "dynamical reduction" theory and, more simply, "GRW" theory, from the initials of the names of the developers of the theory. In this theory the wave function does not evolve according to Schrödinger's equation. Rather, it evolves according to a different equation in which the superposition wave function spontaneously "collapses" to one of its terms.

We can imagine that the deterministic evolution is for some time undisturbed and then, at an entirely random moment, it is interrupted by the stochastic one, after which the deterministic evolution again prevails. These "jumps" happen at random times with an average frequency λ that in the original GRW model is of the order of $\lambda \sim 10^{-15}\mathrm{s}^{-1}$ (that roughly means that the stochastic evolution prevails every 300 millions years). This parameter should be intended as a new constant of nature.

In von Neumann's theory, the collapse rule tells us that observation changes randomly the state of an object from the initial wave function ψ to one of the possible results states. In GRW instead such a rule is a fundamental law of nature. When nature acts on the wave function, it localizes it in a neighborhood of a given position $x \in \mathbb{R}^3$. But what is the wave function transformed to? One simple possibility is to assume that the initial wave function gets multiplied by a Gaussian with a given dispersion, σ. The parameter σ should to be considered an additional constant of nature: The empirical predictions of the theory decide what is the most suitable value for it, which is $\sigma \sim 10^{-7}$ m. With these values for λ and σ, the predictions of GRW theory, for suitably short times, are indistinguishable from those of standard quantum mechanics.

More technically, the situation is the following. Consider a quantum system described by an N-"particle". [3] wave function $\psi = \psi(q_1, ..., q_N)$, $q_k \in \mathbb{R}^3$, $k = 1, \ldots, N$; for any point x in $\mathbb{R}^3$ (the "center" of the collapse that will be defined next), define the *collapse operator*

$$\mathsf{L}_i(x) = \frac{1}{(2\pi\sigma^2)^{3/2}}\, e^{-\frac{(\widehat{Q}_i - x)^2}{2\sigma^2}}\,, \tag{2.7}$$

where $\widehat{Q}_i$ is the position operator of "particle" i. Let ψ_{t_0} be the initial wave function. Then ψ evolves in the following way:

1. It evolves unitarily, according to Schrödinger's equation, until a random time $T_1 = t_0 + \Delta T_1$, so that

$$\psi_{T_1} = U_{\Delta T_1} \psi_{t_0}, \tag{2.8}$$

   where ΔT_1 is a random time distributed according to the exponential distribution with rate $N\lambda$.

2. At time T_1 it undergoes an instantaneous collapse with random center X_1 and random label I_1 according to

$$\psi_{T_1} \mapsto \psi_{T_1+} = \frac{\mathsf{L}_{I_1}(X_1)^{1/2} \psi_{T_1}}{\|\mathsf{L}_{I_1}(X_1)^{1/2} \psi_{T_1}\|}. \tag{2.9}$$

   I_1 is chosen at random in the set $\{1, \ldots, N\}$ with uniform distribution. Different labels identify different particles. The center of the collapse X_1 is chosen randomly with probability distribution.

$$\mathbb{P}(X_1 \in dx_1 | \psi_{T_1}, I_1 = i_1)\, \langle \psi_{T_1} | \mathsf{L}_{i_1}(x_1) \psi_{T_1} \rangle\, dx_1 \|\mathsf{L}_{i_1}(x_1)^{1/2} \psi_{T_1}\|^2 dx_1. \tag{2.10}$$

3. Then the algorithm is iterated: ψ_{T_1+} evolves unitarily until a random time $T_2 =$

[3]Note that there are no real particles in this theory: the word "particle" is used only for convenience in order to be able to use the standard notation and terminology.

$T_1+\Delta T_2$, where ΔT_2 is a random time (independent of ΔT_1) distributed according to the exponential distribution with rate $N\lambda$, and so on.

In other words again, the evolution of the wave function is the Schrödinger evolution interrupted by collapses. When the wave function is ψ, a collapse with center x and label i occurs at rate

$$r(x,i|\psi) = \lambda \langle \psi \,|\, \mathsf{L}_i(x)\psi\rangle \,, \tag{2.11}$$

and when this happens, the wave function changes to $\mathsf{L}_i(x)^{1/2}\psi/\|\mathsf{L}_i(x)^{1/2}\psi\|$.

Thus, if between time t_0 and any time $t > t_0$, n collapses have occurred at the times $t_0 < T_1 < T_2 < \ldots < T_n < t$, with centers $X_1,\ldots,X_n$ and labels $I_1,\ldots,I_n$, the wave function at time t will be

$$\psi_t = \frac{L^{F_n}_{t,t_0}\psi_{t_0}}{\|L^{F_n}_{t,t_0}\psi_{t_0}\|} \tag{2.12}$$

where $F_n = \{(X_1,T_1,I_1),\ldots,(X_n,T_n,I_n)\}$ and

$$L^{F_n}_{t,t_0} = U_{t-T_n}\mathsf{L}_{I_n}(X_n)^{1/2}\,U_{T_n-T_{n-1}}\mathsf{L}_{I_{n-1}}(X_{n-1})^{1/2}\,U_{T_{n-1}-T_{n-2}}\cdots\mathsf{L}_{I_1}(X_1)^{1/2}\,U_{T_1-t_0}. \tag{2.13}$$

Since T_i, X_i, I_i and n are random, ψ_t is also random. It should be observed that (unless t_0 is the initial time of the universe) also ψ_{t_0} should be regarded as random, being determined by the collapses that occurred at times earlier that t_0. However, *given* ψ_{t_0}, the statistics of the future evolution of the wave function is completely determined; for example, the joint distribution of the first n collapses after t_0, with particle labels $I_1,\ldots,I_n \in \{1,\ldots,N\}$, is

$$\mathbb{P}\big(X_1\in dx_1, T_1\in dt_1, I_1=i_1,\ldots,X_n\in dx_n, T_n\in dt_n, I_n=i_n|\psi_{t_0}\big) = \\ \lambda^n \mathrm{e}^{-N\lambda(t_n-t_0)}\|L^{f_n}_{t_n,t_0}\psi_{t_0}\|^2\,dx_1dt_1\cdots dx_ndt_n\,, \tag{2.14}$$

with $f_n = \{(x_1,t_1,i_1),\ldots,(x_n,t_n,i_n)\}$ and $L^{f_n}_{t_n,t_0}$ given, *mutatis mutandis*, by (2.13).

The rate of collapses is given by $N\lambda$, where N is the numbers of "particles" of the system, so that in the case of a macroscopic object in which $N\sim 10^{23}$, we have 10^8 collapses per seconds. That is, using the words of Bell (also, see Figure 2.4.2):

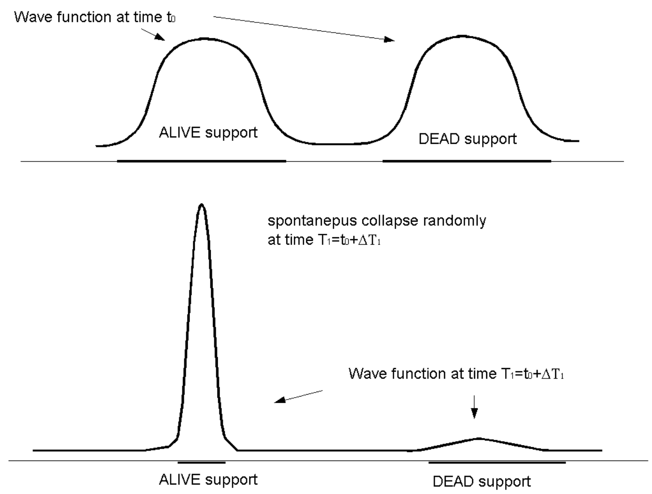

Figure 2.8: GRW theory and the Schrödinger cat.

> any embarrassing macroscopic ambiguity in the usual theory is only momentary in the GRW theory. The cat is not both dead and alive for more than a split second (Bell 1987).

2.4.3 Quantum Theory without Observer 3: Measurement have no Results

Following Bell's alternative presented in Section 2.2, it seems that in Bohmian mechanics we add some entity to complete the description provided by the wave function, while in GRW theory we change the evolution of the wave function. One might be willing to keep both the completeness of the description of the wave function and the simplicity of the linearity of the Schrödinger evolution. Then, the only option is to reject the idea that measurement have results: in that case, option (3) listed in Section 2.2 would be false. This is the approach first proposed by Hugh Everett in his PhD thesis *Relative State Formulation of Quantum Mechanics* (Everett 1950), even if he framed his intent in slightly different terms.

The idea is that the wave function, even if it can stay in superposition, provides the complete description of the universe even if it does not seem to be the case. This is

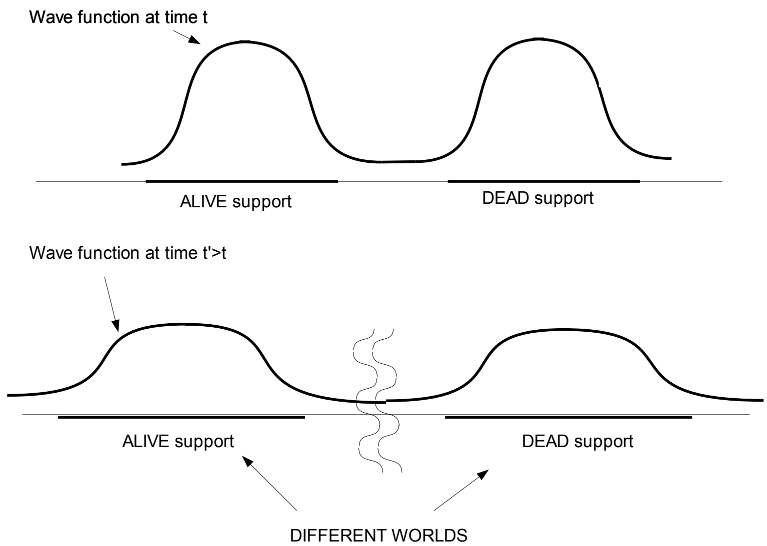

Figure 2.9: The many worlds theory and the Schrödinger cat.

due to the fact that the universe is very different from what we think it is. Each term of the superposition wave function represents different states of affair, corresponding to different measurement results. If the wave function is in a superposition state, we come to the conclusion that measurements do not have results if we think as the results being realized all in *this* space-time. If instead we have a more "liberal" view of what the universe is, we might interpret each term as living in a different space-time, in a different world. In this respect, it is not that measurement do not have results, they do not have results in this space-time. Rather, all the possible measurement results indeed are realized in the *multiverse*, the space of all space-times in which each result can be though of realizing, see Figure 2.9. For this reason this theory is also called "many worlds" quantum mechanics.

This is a rather wild idea, and, as presented, it is also very vague. In fact, stated like this, it is not so obvious how it should be intended. First of all, when a measurement happens it seems we are supposed to think that there is a "splitting" of the world in a number of other words, one for any possible result of the measurement. But how exactly should we intend this splitting? One should specify what these "words" are: Are they really different space-times or are in the same space-time but they are superimposed into the same space-time but "transparent" the one to the other? There is no right or wrong answer: the point is that depending on what the answer to this question is, we have a different theory.

Attempts have been made to make precise this theory, focusing on the specification of what the worlds are in terms of *subjective* experiences of the observer. In this way, we have solutions of the measurement problem which are in some sense very close to Wigner's solution. This is the original proposal of Everett, that called his theory "relative state" formulation of quantum mechanics, since the states are relative to the observer. A possible solution, also based on the attempt to account for the observer's experiences and proposed by David Albert and Barry Loewer in their *Interpreting the Many-Worlds Interpretation* (Albert and Loewer 1988), is the so called "many minds" theory: while physical states described by the wave function linearly evolve, mental states are not in superpositions but they collapse. Also in this theory, as in Wigner's one, we explicitly mention the mind. But the two theories are different, since in Wigner's picture consciousness acts on physical bodies to make them collapse, while here we just have collapse of the mental states without any mental causation.

Be that as it may, as soon as we talk about the observer's subjective experiences, we need to invoke a theory of mind, as in Wigner's theory. And therefore the same objections raised for that theory will apply also in this case. The challenge is therefore to figure out whether a many worlds theory in which consciousness or mental states do not play any role can be developed. And the conclusion is that there does not seem to be any. Let us in fact ask: What are the differences between bare quantum mechanics and the many worlds theory? After all, they both have the same ingredients: there is just the wave function ψ, and it evolves according to Schrödinger's equation. We have seen that the bare theory has the measurement problem, so why this one does not? The strategy of a many world theory is to show that our perceptions of the macroscopic world are, somehow, not in superposition. And in order to succeed in this project they will have to provide a theory of perception (on the recognition of this, see David Wallace in his *Everett and Structure* (Wallace 2003)).

To conclude, I wish to notice that there is a huge amount of people that find the many worlds approach rather fascinating. I think there are at least three reasons for this. The first is the need of preserving the mathematical beauty and the simplicity of the bare theory, which is accomplished directly in many worlds, since in its framework the wave function provides the complete description and it evolves linearly. It should be noted, though, that it is not so obvious what counts as simpler: completeness of the wave functions, linearity of Schrödinger's equation, and a theory of the mind or one of the alternatives listed above? Another reason that is usually provided in favor of the many worlds account is that the reconciliation of quantum mechanics with relativity seems to be easier in the case of many worlds theory than in the other alternatives. So many worlds really seems to be the most promising of all: no additional variables, no ugly dynamics, maybe a little of philosophy of mind, but a more or less natural relativistic invariance. In this respect, no surprise that most of the physicists or philosophers that declare their position in this matter claim their favorite theory to be Everett. When we will discuss what symmetry properties are in Chapter 6, it will be evident that exactly the opposite is the case. Another, not so noble, reason for being fascinated by many worlds quantum mechanics is that, somehow, this theory seems to solve all the problems but also keep some of the paradoxical flavor of quantum mechanics. People are fascinated by strange and "mysterical" consequences and this was one of the appeal of quantum mechanics at some point. But maybe it went too far, as the Schrödinger's cat paradox has shown. Now, a theory like the many worlds theory, which is less extreme but not so much seems just perfect: it allows for parallel universes (in some respect), to time travel (in some respect, see (Deutsch and Lockwood 1994)), and some weird theories of personal identity (see (Barrett 2003)), so we can write a lot of articles about it! Bell (Bell 1987) talked about "romantic" pictures of the world provided by quantum mechanics as opposed to "unromantic" ones. To the former group belong Bohmian mechanics and GRW theory (at least in some version of it, as we will see), and many worlds theory seems to fit very naturally among the romantic ones (see also (Tumulka 2007) for a nice comparison between romantic and unromantic views of quantum mechanics).

2.5 The Tails of the Schrödinger Cat: the Problem of Indefinite Properties

In the previous presentation of the measurement problem, I have been a bit vague: when I said that if the wave function is in a superposition state then the pointer is not pointing anywhere, I did not exactly say what was wrong with it.

Let try to spell this out. One first way of doing it is the following: we believe that macroscopic objects have definite properties. Localization, for example, is one of the properties that we commonly ascribe to macroscopic objects. So, what are the mathematical entities in the theory that represent properties? In bare quantum mechanics, in in which all there is is the wave function evolving to according to Schrödinger's equation, it is not that straightforward: only later von Neumann (von Neumann 1932) provided a clear axiomatization of the theory, already in the framework of his "observer" theory. This is how Jeffrey Barrett (Barrett 2003) discusses the formalization of quantum mechanics provided by von Neumann (von Neumann 1932):

> The standard von Neumann-Dirac theory is based on the following principles:
>
> 1. *Representation of States*: The possible physical states of a system S are represented by the unit-length vectors in a Hilbert space (which for present purposes one may regard as a vector space with an inner product). [...]
> 2. *Representation of Properties*: For each physical property P that one might observe of a system S there is a linear (so-called projection) operator P (on the vectors that represent the possible states of S) that represents the property.
> 3. *Eigenvalue-Eigenstate Link*: A system S determinately has physical property P if and only if P operating on s (the vector representing S's state) yields s. We say then that s is in an eigenstate of P with eigenvalue 1. [...]
> 4. *Dynamics*: (a) If no measurement is made, then a system S evolves continuously according to the linear, deterministic dynamics, which depends only on the energy properties of the system. (b) If a measurement is made, then the system S instantaneously and randomly jumps to a state where it either determinately has or determinately does not have the property being measured. The probability of each possible post-measurement state is determined by the system's initial state. More specifically, the probability of ending up in a particular final state is equal to the norm squared of the projection of the initial state on the final state.

That is, (2) and (3) say that the eigenvalues of self-adjoint operators represent properties. And here is the reason why the experiment of the cat shows there is a problem: the pointer points, meaning that at that time it is localized in a definite region of space, for the pointer to be not pointing would amount to say that the object is either delocalized or maybe not even in any state at all. What state is that? It is not an eigenstate of the position operator (as it would be if it was not in superposition), so what should I say? So the problem is that we cannot ascribe properties to macroscopic objects as we would like to do so if the wave function is all there is, if it evolves according to Schrödinger's evolution, and if the rule for assigning properties to objects is the one we have specified.

Here is the way in which Albert and Loewer in their *Tails of the Schrödinger Cat* (Albert and Loewer 1996) present the problem of the Schrödinger cat:

> What are the physical properties of a cat whose state is CAT [the superposition state]? The answer to this question depends on the connection between quantum states and physical properties. On the standard textbook understanding of quantum theory that connection is EER [eigenstate-eigenvalue rule]: an observable (i.e. any genuine physical property) has a well defined value for a system S when and only when S's quantum state is an eigenstate of that observable. Since CAT is not an eigenstate of the observable corresponding to the observable Aliveness (whose values are alive and dead) it follows from EER that the unfortunate cat that finds itself in CAT is neither alive nor dead. This struck Schrödinger and most others who have thought about it subsequently as absurd. Avoiding this absurdity has come to be known as "the measurement problem".
> The measurement problem is a special instance of what Philip Pearle calls the reality problem. What he means by this is that if all physical processes evolve in conformity with Schrödinger's law then in all sorts of commonplace physical interactions - not only measurements - states of systems are produced that are not eigenstates of ordinary physical properties. Given EER the ordinary properties of these systems will almost invariably fail to possess determinate values. We have seen that the following three propositions are incompatible:
>
> 1. All physical processes are governed by Schrödinger's law.
> 2. The eigenstate-eigenvalue rule.
> 3. Ordinary properties (pointer position, aliveness, etc.) always possess determinate values.
>
> Solving the reality (and measurement) problems will mean giving up or modifying one or more of these three (Albert and Loewer 1996).

Presenting the measurement problem in terms of properties and operators in fact gives rise to the so-called *problem of the tails*, discussed for the first time by Albert and Loewer: the claim is that GRW cannot solve the measurement problem. It is not sufficient to say that GRW has to modify (1), it ha also to modify (2) the EE rule to assign properties. Albert and Loewer point out that GRW modifies Schrödinger's linear dynamics but still has the problems of indefinite properties. In fact, GRW is completely specified once the evolution equation for the wave function is specified. This evolution is constructed modifying the Scrödinger equation in such a way that every now and then the wave function of the macroscopic object collapses to a Gaussian bell curve. Now notice that, being a Gaussian, the wave function has tails that are never vanishing, not even at infinity. These are the "tails" of the Schrödinger cat.

In their words:

> while most of GRW(ALIVE)'s amplitude is indeed (as mentioned above) concentrated in the 'alive' region of the state, it also has non-zero tails which extends into the 'dead' region. And so it follows from the eigenstate-eigenvalue rule that the cat is as a matter of fact not determinately alive (or dead), when GRW(ALIVE) obtains, after all. So the GRW theory, as we have stated above, patently fails to solve the Schrödinger's paradox. (Albert and Loewer 1996) [(GRW)ALIVE and GRW(DEAD) are the element of the superposition describing the state CAT mentioned above.]

What they are saying is obvious: the wave function after the collapse is not *exactly* an eigenstate of "aliveness" or "deadness" (whatever that operator could be!), so the problem is still to be solved! So why did not we realize it before? Here is maybe one of the reasons: when we were thinking to the measurement problem, it seemed impossible to find a way for the superposition wave function to be representing an ordinary property because there are, in the case of measurement with two possible outcomes, two "bumps" in the wave function of equal height. How could possibly one assign a property to only one of the bumps? In the case of GRW theory instead, at least one of the bumps is so tiny that we can consider it as gone, and we are allowed to assign the property to the bigger of the bumps. That is, we might say that the big bump is *approximately* an eigenvalue of the operator representing the property that we want to describe. But of course, to be *approximately* an eigenvalue is not to be an eigenvalue, and therefore the problem is still there. To fix this, Albert and Loewer have provided a

> [. . .] new proposal, an alternate proposal, about the relation between

> position talk and wave function talk; a new proposed supervenience rule (Albert and Loewer 1990).

They suggest to change the EE rule with a different one:

> "particle x is in region R" if and only if the proportion of the total square amplitude of x's wave function which is associated with points in R is greater than or equal to $1 - p$ (Albert and Loewer 1990).

where the parameter p is a conventional matter. This rule is deliberately vague because the parameter p inside the rule is vague such that "talk about the location of particles turns out to be vague" (Albert and Loewer 1990). But this is not a problem for Albert and Loewer, since it just means that

> our everyday language will supervene only vaguely (just as it always has) on the micro-language of particle positions, and that that language will itself supervene only vaguely (and this is something new, and something which other attempts at solving the quantum-mechanical measurement problem - Bohm's, for example - can avoid) on the fundamental language of physics (Albert and Loewer 1990).

Therefore, in this formulation of the measurement problem, in order for GRW to solve the problem, we need to change the EE rule with a different rule, that has been named later in the literature *the fuzzy link* (Clifton and Monton 1999). It is called "link" because it is supposed just to give a rule of correspondence between the language of fundamental physics and ordinary language. We will discuss this approach in full in Chapter 5 as opposed to the approach in which those "links" are actually considered as new ontologies, that we will discus in Chapter 5.

The problem of the tails arises because in assigning properties to macroscopic objects, given their microscopic description, we end up with objects that do not have definite properties, like an alive and a dead cat. I will argue in Chapter 4 that this indefiniteness is due to the kind of mathematical objects we are taking as representing physical objects. In particular, if we take the wave function as what tables and chairs are made of, we end up with indefinite properties. Macroscopic properties should be reducible in terms of the best fundamental physical theory we have. Albert and Loewer want to reduce macroscopic properties to microscopic ones, and they do so with their rule to connect macroscopic properties to the fundamental physics. I will argue that they do not succeed in doing so in Chapter 5, in which I will comment on the status of these connecting rules, in a discussion that will involve the issue of reductive explanation.

2.6 The Problem of the Lack of a Clear Ontology

In all this chapter we have talked about wave functions, operators, measurement results, particles... but what is "out there" in the world? The question that should be asked now is the following: *What is quantum mechanics about*? This question is a tricky one: when I say "about", what do I really mean? One might think that the question above could be paraphrased as follows: *What is there in the world if quantum mechanics is true*? That is, what is the ontology of the theory? If we do not specify which one, among the mathematical variables that appear into the specification of the theory, correspond to what is "out there", the theory remains just empty mathematics.

But the question above could also mean the following: *What are tables and chairs made of?* In fact, there seems to be no reason to restrict what exists to what exists *in the physical world*, since, for example, numbers and laws could exists without being physical. Since quantum mechanics is a fundamental *physical* theory, the question: "What is it about?", should be intended in my opinion as: "What are tables and chairs made of?', and not as the more general question: "What is there if the theory is true?"

There are a lot of variables in a fundamental physical theory. If we want to be realist about it, we need to decide which of the different variables represent physical objects and therefore we should distinguish them from the other variables that appear into the theory as well but do not represent physical objects. Some of the variables of the theory are mathematical constructs so they exists in the same sense as numbers exist: the operators in quantum mechanics, the Hamiltonian, the potential energy, and so on. Some of the variables may correspond to something physically real: positions of particles, field values, strings, the wave function and so on. I will call (the mathematical representation of) what constitutes physical objects at the fundamental level *primitive ontology* of the theory, as suggested by Dürr, Goldstein and Zanghí, (DGZ 1992).

2.7 The Problem of the Adequacy of the Primitive Ontology

Are all mathematical variables adequate primitive ontology? To answer this question one should determine, among the variables in the theories, which ones are suitable to represent physical objects. We have already seen how some of the primitive ontology proposed for quantum theories without observer are, more or less obviously, inadequate: von Neunamm's theory, and the Copenhagen interpretation include in their very formulation some notions that are intrinsically vague, and this is unacceptable for a fundamental physical theory. And when we try to make precise what an observer, say, means, we end up with the need of consciousness to explain the physical world. For this reason, people have considered the quantum theories without observer, Bohmian mechanics, the GRW theory, and many worlds. Let us now see in more detail what we have learned so far about them. Suppose we start from a realistic attitude toward quantum mechanics and we want to know what tables and chairs are. Our first guess would be that they are made of wave functions, what is called *monism* about the wave function: after all, isn't it *the* object of quantum mechanics? The entity that evolves in time according to Schrödinger's equation? The possibility of regarding quantum mechanics as a theory about the wave function is something that also Schrödinger would have liked very much, of course: he was one of the first who tried to allow for the wave function to be the complete description of a system. But, as we have seen in the cat experiment, the wave function of the entire system, evolving to the Schrödinger equation, had in it

> the living and the dead cat (pardon the expression) mixed or smeared out in equal parts (Schrödinger 1983).

Schrödinger therefore dismissed the idea of the wave function representing reality in a complete way because of the presence of these macroscopic superpositions. Now, it seems that in GRW we can avoid this problem allowing for quantum jumps. That is, it seems that we can describe the world completely just with the wave function, if we allow it to have a nonlinear evolution. As Bell pointed out, Schrödinger would have probably liked those quantum jumps: at the same time they do not allow for macroscopic superpositions and it seems we can still think about the wave function as providing the complete description, without the need of adding anything to the theory. This is not exactly right: also in GRW we need to add something to the description provided by the wave function. As we saw in the problem of the tails, we need to add some rule to make a connection between our macroscopic talk and the talk of fundamental physics. I will argue that in GRW we have to add much more than that, since these practical rules are needed to define the primitive ontology of the theory.

So, the question is the following: Can the wave function be an adequate primitive ontology? Can the wave function represent physical objects? That is, is it possible for the wave function to be what the theory is about? I will argue in Chapter 5 that it does not: it is intrinsically an inadequate primitive ontology. Not so much because it can be in superposition (otherwise with fields we will be in trouble all the time), but rather because it lives in $\mathbb{R}^{3N}$, configuration space, which is a space of very large number of dimensions: if we think that the world is made of, just to play it safe, an Avogadro number of "particles", then the dimension of configuration is $M = 3N = 3 \times 10^{23}$! And, as such, it does not have any possibility of representing, by itself, an object in three-dimensional space, unless we already assume there are particles in $\mathbb{R}^3$. If that is right, then, not even in GRW tables and chairs are made of wave functions.

Before dealing with this problem, in the next chapter we will analyze the different interpretation of the different solutions of the measurement problem in more detail. This will allow me to divide them into two groups and to compare them in Chapter 4 and 5, arguing that the one in which the wave function is taken as the primitive ontology of the theory is very problematical, at best, and should be abandoned.

The terminology "local beables" has been introduced by Bell in the framework of GRW theory:

> These are the mathematical counterparts in the theory to real events at definite places and times in the real world (as distinct from the many purely mathematical constructions that occur in the working out of physical theories, as distinct from things which may be real but not localized, and distinct from the "observables" of other formulations of quantum mechanics, for which we have no use here). A piece of matter then is a galaxy of such events (Bell 1987).

“Beable” is short for ”maybeable”, things that might exist if the theory is true. Among the variables in the theory there are certain entities that can mathematically represent localizable objects and some that cannot. If with localized we mean that they can be attached to a region of space-time, then, for example, positions, fields and strings are local beables, while the wave function is not. Therefore, if a local beable is what can represent physical objects, then the wave function cannot do that. The notion of local beable seems similar to the notion of primitive ontology. But, for reasons that will be clear later, not all local beables (such as the electric and magnetic fields in classical electrodynamics) need to be regarded as part of the primitive ontology. If so, then in classical electrodynamics fields do not constitute physical objects. We will come back on the differences between local beables and primitive ontology in Chapter 6.

Chapter 3

On the Plurality of Quantum Metaphysics

In the previous chapter we briefly presented Bohmian mechanics, GRW, and the many worlds theory. Each of them can be considered to be an answer to the problem of constructing a quantum theory without observer, and each of themy can be considered to be a reaction to what Bell considered the obvious moral of the problem of the cat: either Schrödinger's evolution is not always correct or the state of the system is not described only by the wave function. Then we have the many worlds theory, that corresponds, as we saw, to the attempt of making sense of the idea that all measurement results are realized.

In this and in the following chapters I will discuss these theories from two different perspectives and discuss what are the possible metaphysical morals that we can get from these theories.

3.1 Bohmian Mechanics

As we discussed in the previous chapter, in Bohmian mechanics the state of a physical system at a given time t is given by the couple (Ψ_t, Q_t). The wave function Ψ evolves according to Schrödinger equation, and the particles evolve according to the guide equation determined by the wave function, see equations (2.5) and (2.6).

Now, let us try to infer something about metaphysics from the hypothesis that this theory is true. What is there if Bohmian mechanics is true? We have already answered to this question: particles and the wave function constitute the state, which provides the ontology of the theory. But if one instead asks the question: "What are tables and chairs made of if Bohmian mechanics is true?", then we can provide different answers. In fact, it is not obvious that all there is gets to constitute physical objects. Only if we assume so, the two questions coincide. But if, for example, we believe in the existence of numbers, of laws of nature, of consciousness, then of course they are part of the ontology even if they do not constitute any physical object. So let us now to analyze what are the possible alternative answers to the question: What is the primitive ontology of Bohmian mechanics?

3.1.1 Particle Bohmian Mechanics

The most natural, at least to me, interpretation of Bohmian mechanics is the one provided by Bohm and Hiley (Bohm and Hiley 1993), by Bell (Bell 1987) and by Dürr, Goldstein, Tumulka, Zanghí and myself (DGZ 1992), (DGZ 1997), (DGZ 2004) (Allori et al. 2007): it is a theory of particles in motion. That is, tables and chairs are made of particles: they are the primitive ontology of the theory. In the usual terminology, the actual positions $Q_1, ..., Q_N$ in $\mathbb{R}^3$ of the particles are considered the *hidden variables* of the theory: the variables which, together with the wave function, provide a complete description of the system, the wave function alone providing only a partial, incomplete, description. This terminology suggests that the "main" object of the theory is the wave function, with the additional information provided by the particles' positions, which seem to play in some respect some sort of secondary role: we added them, it seems, just to "recover" the properties of the macroscopic objects. The situation is instead very much the opposite: if Bohmian mechanics is a theory of particles, their positions are the *primitive* variables, and the description in terms of them must be completed by specifying the wave function to define their dynamics. In other words, according to this view, Bohmian mechanics is a theory is about particles in three-dimensional space, while the wave function is *not* to be considered as a material field. In this regard, Dürr, Goldstein and Zanghí write:

> if one focuses directly on the question as to what the theory is about, one is naturally led to the view that Bohmian mechanics is fundamentally about the behavior of particles, described by their positions - or fields, described by field configurations, or strings, described by string configurations - and only secondarily about the behavior of wave functions (DGZ 1997).

A similar attitude is the one of David Bohm and Basil Hiley in their book *The Undivided Universe*:

> In our interpretation, however, we do not assume that the basic reality is thus described primarily by the wavefunction. Rather [...] we begin with the assumption that there are particles following definite trajectories [...] We then assume that the wavefunction, describes a qualitatively new kind of quantum field which determines the guidance conditions and the quantum potential acting on the particle. We are not denying the reality of this field, but we are saying that its significance is relatively subtle in the sense that it contains active information that 'guides the particle in its self-movement under its own energy [...] So ultimately all manifestations of the quantum fields are through the particles (Bohm and Hiley 1993).

Therefore, in Bohmian mechanics particles are the primitive ontology of the theory. Particles constitute what tables and chairs are made of, they are the little lego bricks that build up every physical object. We will call this approach to Bohmian mechanics, "particle Bohmian mechanics" or BMp, to differentiate to the approach that we will see in the next section.

If Bohmian mechanics is a theory of particles, what is the wave function in this view? There can be more than one choice, as we will discuss further in Section 4.6. According to Dürr, Goldstein and Zanghí and to Bohm and Hiley, the wave function is part of the furniture of the world, it is real. But, contrarily to what happens in the case of the primitive ontology, which constitutes matter, it does not constitute matter but, rather, it tells matter how to move. It provides the law for the motion of the particles, just as the potential in classical mechanics is the generator of the motion of the positions of the particles. Dürr, Goldstein and Zanghí [1] have a Platonic view of law, then we have no problem to regard the wave function as real insofar as laws are real. But there is nothing that forces us to do so, especially if one has a Humean view of laws. In that case, one might be less committed to the existence of the wave function, since it would just turn out to be our best and most informative way of writing down the theory of the world. Or if one is a Nominalist with respect to laws, one would naturally regard the wave function as nonexisting.

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[1] Private communication.

3.1.2 Configuration Bohmian Mechanics

In the previous section we discussed Bohmian mechanics as a theory of particles, in which the wave function was not something that constitutes matter. In contrast, in this section I will analyze Bohmian mechanics as a theory according to which tables and chairs are indeed made of particles but also of wave functions.

From the fact that in the theory there is the wave function, it might be natural to conclude that it has to be describing a physical object. Not just real, but real *and* physical. And if we start taking the wave function so seriously as a physical entity, then we should indeed realize that the space in which it lives should be real too. So we end up having, in this view, both three-dimensional space, where the particles live, and configuration space, where the wave function lives. One might say that both of these spaces are indeed part of physical space. This is what Bradley Monton in his *Wave Function Ontology* (Monton 2002) calls the "mixed ontology":

> A proponent of a mixed ontology would hold that, in addition to the 3N-dimensional space, there also exists a three-dimensional space.

Dürr, Goldstein and Zanghí would opt to say instead that physical space is $\mathbb{R}^3$, while $\mathbb{R}^{3N}$ is just a space constructed from physical space for mathematical convenience. Another, maybe more radical move is to say exactly the opposite: physical space *is* configuration space, while three-dimensional space is just an illusion. This is David Albert's idea, that he expresses in *Elementary Quantum Metaphysics* (Albert 1996). He notes that, given that the wave function is an object in $\mathbb{R}^M$, with $M = 3N$, it is physical space in the case of Bohmian mechanics, as well as in any theory in which the wave function is taken to represent physical objects. As a consequence, $Q \in \mathbb{R}^{3N}$ is actually not describing N particles in three dimensional space $\mathbb{R}^3$. That is, Q is not the n-tuple $(Q_1, ..., Q_N)$, where Q_k is a triplet in $\mathbb{R}^3$. Rather, the whole universe is just a single point Q in the highly dimensional $\mathbb{R}^M$ together with the wave function Ψ, that should be intended as well as a concrete, physical field in such a space. In Albert's words:

> it has been essential [...] to the project of quantum-mechanical *realism* (in *whatever* particular form it takes - Bohm's theory, or modal theories, or Everettish

> theories or theories of spontaneous localization), to learn to think of the wave functions as physical objects *in and of themselves*. And of course the space those sorts of objects *live* in, and (therefore) the space *we* live in, the space in which any realistic understanding of quantum mechanics is necessarily going to depict the history of the world as *playing itself out* (if space is the right name for it - of which more later) is *configuration space*. And whatever impression we have of the contrary (whatever impression we have, say, of living in a three-dimensional space, or in a four-dimensional space-time) is somehow flatly illusory. [...] In Bohm's theory, the world will consist of exactly two physical objects. One of those is the universal wave function and the other is the universal *particle* (Albert 1996).

In contrast with the view presented earlier, here configuration space plays a very fundamental role: it is the fundamental physical space in which we live. For this reason we might call this theory configuration Bohmian mechanics or cBM, as opposed to BMp presented above. The name "configuration" is misleading, since this space is primitive, it is not constructed from the configuration of particles in three-dimensional space. In fact there are no three-dimensional particles, and there is no three-dimensional space.

Now, it seems clear that any complete, fundamental physical theory must account for the behavior of macroscopic objects of three-dimensional space. In other words, we could say that what physics ought to be able to do is to account for what appears to be happening around us. If we take configuration space to be physical space, then we need to explain why it seems as if we live in a three-dimensional space. Given that the wave function is in configuration space and it is all there is, and given that this space is primitive, we do not have enough resources to recover or deduce three-dimensional space without making use of the very definition of configuration. In fact, if the theory talks about the behavior of one single point Q and a field Ψ in this highly dimensional space (call this dimension M and forget that $M = 3N$), then we have no more that that. We might be tempted to regard the coordinates of Q as grouped into triples Q_k, such that $Q = (Q_1, ..., Q_N)$, representing the three spatial coordinates of the "particles". And, accordingly, to consider the variables of the wave function as $\Psi(Q) = \Psi(Q_1, ..., Q_N)$, where a single coordinate Q_k is a triplet of terms, each of them is in $\mathbb{R}^3$ and represents a spatial coordinate of a "particle". But we simply do not have any justification to do that: there are no three-dimensional particles in this theory, just one single particle in configuration space and a field in that space. The only way we could make the identification of Q with $(Q_1, ..., Q_N)$, $Q_k \in \mathbb{R}^3$, is to *already* know that the configuration can be divided as such. That is, we can interpret the word "configuration" as the N-tuple of the positions of N three-dimensional particles if we assume there are N particles in $\mathbb{R}^3$. As Monton also emphasized (Monton 2002), one could think that $\mathbb{R}^3$ supervenes on $\mathbb{R}^{3N}$ so that one does not really need to postulate the existence of a three-dimensional space. But that it is simply false, since there are many ways in which N particles could evolve in three-dimensional space and give rise to the same configuration space. The point is that we simply have not enough structure to read off only from configuration space the three-dimensional world. In short, there is an explanatory gap between the behavior of something on a highly dimensional abstract space and the behavior of objects in three-dimensional space. Therefore, if one wants the theory to be a satisfactory fundamental physical theory, one has to add to the specification of the theory some rule in order to recover three-dimensional space from it.

Albert agrees that we need some map from $\mathbb{R}^M$ to $\mathbb{R}^3$, and in particular he claims that it is the Hamiltonian that gives us such a rule (Albert 1996). His reasoning goes as follows. Physical space is $\mathbb{R}^M$, where it happens to be the case that $M = 3N$, even if this does not mean that there is some *a priori* reason to group the components of Q in triples $Q_k \in \mathbb{R}^3$, as we have seen before. The total Hamiltonian of the world is something like

$$H = \nabla_q^2 + V(q), \tag{3.1}$$

where $q \in \mathbb{R}^M$. Without any further restrictions, this Hamiltonian could apply to a space in which we have different groupings of the coordinates of q. But, Albert claims, it is an *empirical fact* of the world that the potentials V should be written as

$$V(q) = \sum_{i<j} V(|q_i - q_j|),$$

where $q = (q_1, ..., q_N)$, $q_k \in \mathbb{R}^3$, for any $k = 1, ..., N$. And this is what insures us of the appearances of the world as three-dimensional. The structure of the actual Hamiltonian, Albert says, is the one that picks as natural the grouping in terms of triplets and therefore explains why we think we live in a three-dimensional space. Note that there is no further explanation of why the Hamiltonian is the way it is or the dimensionality M of physical space is what it is: for example, there is no explanation of why M cannot be a prime number [2].

In addition, a theory should be able to account of the assignment of properties to macroscopic objects, as we do in terms of our ordinary language. For example, it will need to explain what do we mean when we say: "the table is localized in the middle of the room" in the terms of the fundamental physical theory. What does it mean to say that there is a table in the middle of the room, in cBM? As we have anticipated at the end of the previous chapter, in order to answer this question we need to add a rule of correspondence between the macroscopic language and the microscopic one.

3.2 GRW Theory

Spontaneous collapse theories are usually characterized a theories in which the wave function provides the complete description of the system but does not evolve according to Schrödinger's equation [3]. It evolves according to a stochastic nonlinear equation that makes it the case that the linear evolution of Schrödinger equation is interrupted randomly by collapses. As we did for Bohmian mechanics, let us ask ourselves: What are tables and chairs made of if GRW is true? What is the primitive ontology of GRW theory?

3.2.1 Bare GRW

As it was somewhat natural to regard Bohmian mechanics as a theory of particles, it seems natural to regard GRW as a theory about the wave function. After all, is not it the object that appear in the modified Schrödinger's equation? This is the view, entertained for example by people like David Albert in the previosly mentioned

[2]This has been suggested by Tim Maudlin, private communication.

[3]As anticipated, in Chapter 4 I will challenge this characterization.

(Albert 1996), Peter Lewis in his *Life in Configuration Space* (P. Lewis 2004) and Alberto Rimini and Oreste Nicrosini in their *Relativistic Spontaneous Localization: a Proposal* (Rimini and Nicrosini 2003). This position, that we will call GRW∅ to emphasize its minimality, could be summarized as follows: the wave function is what GRW theory is about, the wave function is a real, physical field,

> just like electromagnetic fields in classical electrodynamics (Albert 1996).

With the difference that the wave function now lives in a much bigger space: configuration space, the space of all the positions of all the particles in the universe.

Therefore, since in GRW∅ all there is is the wave function, and it lives in configuration space, physical space is configuration space, as in cBM. Also as in cBM, we have the problem of explaining how it is the case that, even if physical space has a dimension much bigger than 3, it looks as if it were three-dimensional. As emphasized in Section 3.1.2, there is an explanatory gap between the behavior of something on a highly dimensional abstract space and the behavior of objects in three-dimensional space so that it is necessary to add some rule to GRW∅ in order to recover the three-dimensional space of our experience. As in cBM, it is the Hamiltonian that provides the connection.

As underlined in Section 2.5, in GRW∅ we have the problem of the tails, the problem of assigning properties to macroscopic physical objects given their microscopic description. As we have mentioned earlier, we might run into trouble, as Albert and Loewer point out in (Albert and Loewer 1996), if we assume that the usual eigenstate-eigenvalue link of orthodox quantum mechanics holds also in GRW∅. In fact this, as already discussed in the second chapter, would cause certain properties, like the localization of a macroscopic object for example, which we think are defined to remain undefined instead. Therefore, we need to have a different rule to connect ordinary macroscopic appearances with what the physics tells us in terms of wave functions. Albert and Loewer have proposed the fuzzy link. In this way, they say, it is possible to recover what we usually mean when we talk about localizable objects on the macroscopic level and the appearances of those objects to be localized while they are not.

3.2.2 Flashy GRW

What we discussed in 2.4.2 is, more or less, all there is to say about the mathematical formulation of the GRW theory according to most people. The linear evolution of the wave function is interrupted by collapses. When the wave function is ψ, a collapse of center x and label k occurs at rate given by equation (2.11), and when this happens, the wave function changes to $\mathsf{L}_i(x)^{1/2}\psi/\|\mathsf{L}_i(x)^{1/2}\psi\|$. Contrarily to the idea that it is natural to consider GRW as a theory about the wave function Gian Carlo Ghirardi, the "G" in GRW and the leading supporter of the theory, and Bell believe that the description in terms of the wave function is not the whole story. Concerning this possibility, Bell noted the following:

> But the wave function as a whole lives in a much bigger space, of $3-N$ dimensions. It makes no sense to ask for the amplitude or phase or whatever of the wave function at a point in ordinary space. It has neither amplitude nor phase nor anything else until a multitude of points in ordinary three-space are specified (Bell 1987).

Therefore, the wave function alone does not seem to be able to describe physical objects. He then adds:

> However GRW jumps (which are part of the wave function, not something else) are well localized in ordinary space. Indeed each is centered on a particular space time point (x, t). So we can propose these events as the basis of the"local beable" of the theory (Bell 1987).

The idea is that GRW can account for the structure of events in three-dimensional space in terms of the points $(X_k, T_k), k = 1, ..., N$ of space-time that correspond to the localization events of the wave function. Once a history of the wave function is given, from an initial time $t = 0$ to a given time t, also the set of points in space-time, representing the localization events happened between the initial time an the time t,

$$F = \{(X_1, T_1), (X_2, T_2), \ldots (X_i, T_i), \ldots (X_K, T_K), \ldots\}, \tag{3.2}$$

with $T_1 < T_2 < \ldots < T_k < \ldots$, is determined. Bell's proposal is to consider F as the image of the world that is given us by the GRW theory: a history of the world is

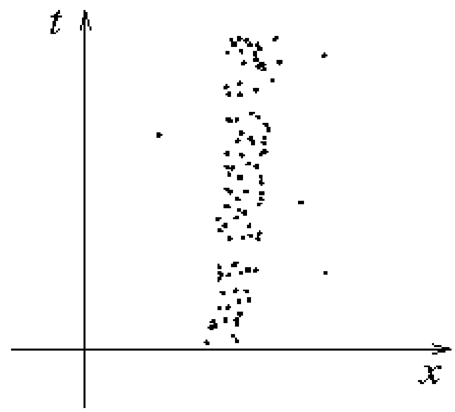

Figure 3.1: A typical pattern of flashes in space-time, and thus a possible world according to the GRWf theory.

given by a discrete set of flashes in space-time. Therefore, we should add something to the wave function also in the case of GRW theory. But this is not just an addition. Rather, these "flashes", the space-time points in which localizations occur are, in the terminology I prefer, the primitive ontology of the theory. They are what exists in space-time according to this GRW theory, that we will call GRWf or flashy GRW. This view of GRW has been adopted in (Tumulka 2006), (Maudlin 2007), (Allori et al. 2005), (Allori et al. 2007).

To sum up, Bell's idea is that GRW can account for objective reality in three-dimensional space in terms of these space-time points (X_k, T_k) that happen to correspond to the localization events (collapses) of the wave function. Note that if the number N of the degrees of freedom in the wave function is large, as in the case of a macroscopic object, the number of flashes is also large. That is to say that large numbers of flashes can form macroscopic shapes, such as tables and chairs. In Figure 3.1 a possible image of the world in this theory is shown. Note however that, at almost every time, space is in fact empty, containing no flashes and thus no matter. Thus, while classical atomic theory of matter or BMp entail that space is not everywhere continuously filled with matter but rather is largely void, this theory entails that at most times space is entirely void. As a consequence, tables and chairs are really very different from what we think they are, not being there most of the time. This, of course, applies also to ourselves.

What is the wave function in GRWf? In GRWf the space-time locations of the flashes can be read off from the history of the wave function given by (2.12) and (2.13): every flash corresponds to one of the spontaneous collapses of the wave function, and its space-time location is just the space-time location of that collapse. Accordingly, equation (2.14) gives the joint distribution of the first n flashes, after some initial time t_0. According to this theory, the world is made of flashes and the wave function serves as the tool to generate the "law of evolution" for the flashes: equation (2.11) gives the rate of the flash process, that is, the probability per unit time of the flash of label i occurring at the point x. For this reason, Roderich Tumulka, who after Bell first took seriously GRWf in his *A Relativistic Version of the Ghirardi-Rimini-Weber Model*, has preferred the word "flash" to "hitting" or "collapse center", or "jump": the latter words suggest that the role of these events is to affect the wave function, or that they are not more than certain facts about the wave function, whereas "flash" suggests rather something like an elementary, primitive event.

A final remark: It has been argued (see, e.g., (P. Lewis 2004)) that the first proposal of GRW$\varnothing$ is due to Bell, on the basis of the following passage:

> No one can understand this theory until he is willing to think of [the wavefunction] as a real objective field rather than just a 'probability amplitude'. Even though it propagates not in 3-space but in $3N$-space (Bell 1987).

But the situation seems to be more complicated than that. In fact Bell subsequently proposed GRWf, as we have seen above. One could interpret that as a change of heart but, interestingly enough, after having underlined the importance of (in his terminology) local beables for a fundamental physical theory, Bell proposed GRW to be about "stuff" in configuration ($3N$-dimensional) space. Bell wrote:

> The GRW-type theories have nothing in their kinematics but the wave function. It gives the density (in a multidimensional configuration space!) of *stuff*. To account for the narrowness of that stuff in macroscopic dimensions, the linear Schrödinger

> equation has to be modified, in the GRW picture by a mathematically prescribed spontaneous collapse mechanism [Emphasis in the original] (Bell 1987).

He made a similar remark personally to Ghirardi (quoted by the latter in (Bassi and Ghirardi 2003)) in a letter dated October 3, 1989:

> As regards ψ and the density of stuff, I think it is important that this density is in the $3N$-dimensional configuration space. So I have not thought of relating it to ordinary matter or charge density in 3-space. Even for one particle I think one would have problems with the latter. So I am inclined to the view you mention 'as it is sufficient for an objective interpretation' [...] And it has to be stressed that the 'stuff' is in $3N$-space - or whatever corresponds in field theory.

It is very difficult to figure out what Bell had in mind in this respect: maybe he thought to both GRWf and GRW∅ as tenable possibilities. But this interpretation seems to be in contrast with his view of symmetry properties of the theory that led him to propose GRWf to start with, as we will see in Section 7.9.

3.2.3 Mass Density GRW

Gian Carlo Ghirardi endorsed Bell's view in spirit but, probably motivated by the development of the continuous collapse models and by the weirdness of the choice of Bell, he preferred to introduce a different proposal for his theory. Ghirardi's choice is GRW to be about a field: We have a variable $m(x,t)$ for every point $x \in \mathbb{R}^3$ in space and every time t, defined by

$$m(x,t) = \sum_{k=1}^{N} m_k \int\limits_{\mathbb{R}^{3N}} dq_1 \cdots dq_N \, \delta(q_k - x) \left|\psi(q_1, \ldots, q_N, t)\right|^2 . \tag{3.3}$$

In words, one starts with the $|\psi|^2$-distribution in configuration space $\mathbb{R}^{3N}$, then obtains the marginal distribution of the i-th degree of freedom $q_k \in \mathbb{R}^3$ by integrating out all other variables q_j, $j \neq k$, multiplies by the mass associated with q_k, and sums over k. We will call this theory GRWm. This theory was essentially proposed by Fabio Benatti, Gian Carlo Ghirardi, and Renata Grassi in their *Describing the Macroscopic World: Closing the Circle within the Dynamical Reduction Program* (Benatti et al. 1995)[4].

[4]They first proposed (for a model slightly more complicated than the one considered here) that the matter density be given by an expression similar to (3.3) but this difference is not relevant for our

The field $m(x,t)$ is supposed to be understood as the density of matter in space at time t. As shown in (Benatti et al. 1995), this field can be identified, on a macroscopic scale, with the usual mass density of physical objects. Since GRWm is a theory about the behavior of a field $m(x,t)$ on three-dimensional space, the microscopic description of reality provided by the mass density field, it is continuous rather than corpuscular. For example according to this description, the cat is not made of particles but she corresponds to a given configuration of the field $m(x,t)$ that, on the macroscopic scale, resembles her familiar shape. In particular, the wave function is in a superposition of the states of life and death, there is a practically instantaneous evolution of the field toward the particular distribution of masses, corresponding either to the state of life or to the state of death. This is reminiscent of Schrödinger's early view of the wave function as representing a continuous matter field. But while Schrödinger was obliged to abandon his early view because of the tendency of the wave function to spread, the spontaneous wave function collapses built into the GRW theory tend to localize the wave function, thus counteracting this tendency and overcoming the problem. Actually, given that all there is in the world is the mass density field and all the rest is determined by it, the GRW theory might even be interpreted as the only example that realized Einstein's program of developing a pure field theory that takes into account also of the quantum phenomena.

Note that the wave function in GRWm, similarly to what happens in GRWf, allows for the law of motion for the mass density field.

3.3 Many Worlds

This is the proposal according to which the wave function provides the complete description *and* it evolves according to Schrödinger's equation. If a theory like that is true, then what is the theory about?

---

purposes.

3.3.1 Bare Many Worlds

There are a growing number of people that try to make sense of the many worlds theory as a theory about the wave function. The strategy is to show that the macroscopic description we provide through our everyday language somehow emerges from a description in terms of wave functions. We will call this approach MW∅.

As in cBM and GRW∅, since MW∅ is about the wave function, the first problem of this theory is to recover the perception of three-dimensional space, since physical space is configuration space, the space in which the wave function lives. This is related to the so-called *the problem of the preferred basis*: since the wave function can be written in many basis, why the one in terms of $x \in \mathbb{R}^3$ is privileged with respect to, say, the one in terms of the impulse p or the one in terms of $y \in \mathbb{R}^{124359324}$? Proponents of this theory rely on the effect of the interaction of the system with its environment. But how this is supposed to be achieved is still a work in progress (see, again, (Wallace 2003) in this respect).

Moreover, this theory needs to explain why we assign the properties we assign to macroscopic objects if they are made of wave functions. In MW∅ properties of macroscopic objects can be determined by the eigenstate-eigenvalue (EE) rule, or what was called the fuzzy link. At the microscopic level, we have superpositions, so that microscopic objects have properties that seem contradictory to us. This is not considered problematical since, it is said, we do not have direct access to the microscopic world and therefore we cannot guarantee that the properties that we think there are are indeed there. At the macroscopic level, instead, it is argued that the different terms of the superpositions, due to the interaction with the environment, lose coherence. They "decohere", such that they are not able to interact with each other anymore so that each of them can be considered *as if* in an independent world. In other words, the idea is that each term of the superposition describes a different macroscopic object. After a given time, called decoherence time, it is practically impossible for them to interact with one another. For this reason, they are in the same space-time but they are "transparent" the one to the other.

At first, one might complain about the fact that decoherence is an approximate process (Kent 1990): for all practical purposes, the interference terms can be neglected. But the problem here is just to account for the properties of macroscopic objects in terms of the language of the wave function. This is what decoherence should be doing for us, and this is exactly what it does, since the notion of a macroscopic object is vague. For all practical purposes, we can say that a table is in the middle of the room if one of the terms of the superposition wave function is peaked in a region $R \in \mathbb{R}^3$ that corresponds to the middle of the room and the other terms will not interact with it after a while.

3.3.2 Bell Many Worlds

Bell had his own take on what the many worlds theory is supposed to be: it is a theory about configurations of particles in $\mathbb{R}^3$ that are randomly distributed in space-time according to a probability distribution give by $|\psi|^2$:

> [I]nstantaneous classical configurations [...] are supposed to exist, and to be distributed [...] with probability $|\psi|^2$. But no pairing of configurations at different times, as would be effected by the existence of trajectories, is supposed (Bell 1987).

The wave function evolves according to Schrödinger's equation as in Bohmian mechanics but the configurations at different times are *not* connected the one to the other with trajectories. I will call this theory BMW, Bell many worlds.

To better understand this theory, consider stochastic mechanics. It is a theory of particles that evolve according to a diffusion process generated by a Schrödinger wave function. BMW can be considered as the limit of stochastic mechanics when the diffusion coefficient goes to infinity. In contrast, Bohmian mechanics is the limit of stochastic mechanics when the coefficient goes to zero. As is evident, the role of the wave function in BMW is to generate the probability distribution for the configuration.

Therefore, the theory looks like both GRWf and BMp: there are events in space-time, like in GRWf, even if they do not corresponding to jumps of the wave function, and there are configurations, as in Bohmian mechanics, even if no connections between two configurations at different times.

Therefore, BMW can be understood as suggesting that the configurations at different times are not connected by any law. It could also be regarded as suggesting that configurations at different times are (statistically) independent. If one considers the configurations corresponding to a macroscopic superposition, they form a family of correlated configurations associated with the terms of the superpositions, with no interaction between the families. These families can be regarded as comprising many worlds, superimposed in a single space-time. Metaphorically speaking, the universe according to BMW resembles the situation of a TV set that is not correctly tuned, so that one always sees a mixture of two channels. In principle, one might watch two movies at the same time in this way, with each movie conveying its own story composed of temporally and spatially correlated events [5]

Let us also note that BMW provides a very strange picture of the world. It is a theory in which the distribution of the (macroscopic) pointer positions is given by $|\psi|^2$ at any time but there is no correlation whatsoever between what there is at a given instant of time and what there is at the previous or following instant. In any theory in which we have a wave function evolving according to Schrödinger's equation, the wave function of the universe must presumably be thought of as consisting of several packets that are very far apart in configuration space that correspond to unrealized outcomes of quantum measurements. Some of the packets will have support in events that did not take place in our time, such as for example the dinosaurs have never become extinct. In Bohmian mechanics, where there the diffusion coefficient is zero, it is not possible for the configuration to jump, in an instant, from the support of one wave packet to a macroscopically distinct one. In the case we are considering instead, because of the large diffusion coefficient, the configuration will very probably visit in every second those distant regions supporting the other packets: therefore, at time t there can be dinosaurs and at time $t + dt$ they could have disappeared. In other words, if this theory is correct, the fact that right now there are books full of stories of the world in the past, everyone remembers what (she thinks) happened to her or to somebody she knows, the media transmitting what has happened in the previous hours in the world does not

[5] This example is due to Roderich Tumulka, private communication.

guarantee that all these records are actually reliable. Rather, according to this theory, the records are most likely to be false: at one instant there is a set of records that do not reflect in any way truthfully what has happened. That means, plainly, that they are not records at all! The world described by this theory is so radically different from what we think of it that it is hard to take this theory seriously, since, in this theory, the actual past will typically entirely disagree with what is suggested by (what we call) the past.

Note that in BMW, the particle positions play a crucial role: the represent the primitive ontology of the theory, just as in BMp, but they are not connected by any deterministic law.

Chapter 4

Quantum Theories without Observer as Theories about a Primitive Ontology in Three-Dimensional Space

In the previous chapters I presented not only a variety of quantum theories without observer but also a variety of possible different metaphysics for each of them. Bohmian mechanics, GRW and the many worlds theory have been usually presented as alternative solutions of the measurement problem. But they can be interpreted very differently, depending on what the theory is considered to be about. The main suggestion here is that the different metaphysical approaches we described in the last chapter can be roughly grouped into two main categories that reflect a particular approach to physics: on one hand, we have the view according to which Bohmian mechanics, GRW and Everett theory are about some primitive ontology in three-dimensional space, while on the other hand we have the view according to which those theories should be interpreted as being about the wave function. In this chapter and in the next I will explore and compare the two approaches, and I will argue that the approach that considers the wave function as the primitive ontology of quantum theories without observer is at best problematical and that the primitive ontology of the theory, in order to be adequate, should be in three-dimensional space.

The particle trajectories in BMp (the view that Bohmian mechanics is a theory of particles), the mass density in GRWm (the viewthat GRW is about the mass density field), the flashes in GRWf (the view that GRW is about events in space-time), the particle positions in BMW (the view according to which the many worlds theory is about particles) have something in common: they are what Bell (Bell 1987) the local beables of these theories and what we have called the primitive ontology of the theory:

the stuff that things are made of [1]. *These theories are about the behavior of a primitive ontology in $\mathbb{R}^3$, not about the behavior of the wave function.*

The wave function also belongs to the ontology of BMp, GRWm, GRWf, BMW but it is not part of the primitive ontology: according to these theories, physical objects are not made of wave functions. The wave function is part of the ontology insofar as it can be regarded as a real entity, but it is not physical in nature. Some scholars, like Fay Dowker (Dowker and Herbauts 2006), Edward Nelson (Nelson 1985) and Bradley Monton (Monton 2002), (Monton 2006) have suggested that the wave function does not exist at all. Others, like Dürr, Goldstein and Zanghí, regard the wave function as nomological in nature. The two positions are not irreconcilable, if one is a Nominalist with respect to laws. We will come back to the issue of the nature of the wave function in Section 4.6.

Be that as it may, one of the the main *motivations* for this approach is that it seems that in each of these theories the only reason the wave function is of any interest at all is that it is relevant to the behavior of the primitive ontology. Roughly speaking, the primitive ontology tells us what matter is and the wave function tells us how the matter is moving. In BMp the wave function determines the motion of the particles *via* the guide equation, in GRWm the wave function determines the distribution of matter in the most immediate way, in GRWf the wave function determines the probability distribution of the future flashes, in BMW the wave function at a given time determines the probability distribution of the configurations at a later time.

The histories of the primitive ontology in space-time have been called by Goldstein [2] "decorations" of space-time. Each theory involves a dual structure $(\mathscr{X}, \Psi)$: the primitive ontology $\mathscr{X}$ providing the decoration, and the wave function Ψ governing its motion. The wave function in each of these theories, which has the role of generating the dynamics for the primitive ontology, has a nomological character utterly absent in the primitive ontology.

[1] The concepts of primitive ontology and local beables could be considered as conflating for now. We will see in Chapter 6 how they differ.

[2] Private communication.

Let us note that even the Copenhagen interpretation involves a similar dual structure: what might be regarded as its primitive ontology is the classical description of macroscopic objects which Bohr insisted was indispensable (including in particular pointer orientations conveying the outcomes of experiments) with the wave function serving to determine the probability relations between the successive states of these objects. In this way, the wave function governs a primitive ontology, even for Bohr's view. An important difference, however, between the Copenhagen interpretation on the one hand and BMp, GRWm, GRWf, and BMW on the other is that the latter are fully precise about what the primitive ontology is (respectively, particle positions, continuous matter density, flashes, and positions again), whereas the Copenhagen interpretation is rather vague, even noncommittal, since the primitive ontology is macroscopic and the notion of "macroscopic" is an intrinsically vague one. Therefore, if on the one hand we acknowledge that the Copenhagen interpretation has a clear primitive ontology, on the other hand we tend to regard it as an unsatisfactory one, since it is defined on the macroscopic scale and the notion of macroscopic is ill-defined. So the natural move in order to construct a quantum theory with a clear and not vague primitive ontology would be to define the primitive ontology directly on the microscopic scale. And this is exactly what BMp, GRWf, GRWm, and BMW are doing.

4.1 Physical Equivalence

To better appreciate the concept of primitive ontology, it might be useful to regard the positions of particles, the mass density and the flashes, respectively, as the *output* of the theories presented with the wave function, in contrast, serving as part of an *algorithm that generates this output*, as emphasized in (Allori et al. 2007). Suppose we want to write a computer program for simulating a system (or a universe) according to a certain theory. For writing the program, we have to face the question: Which among the many variables to compute should be the output of the program? All other variables are internal variables of the program: they may be necessary for doing the computation, but they are not what the user is interested in. In the way we propose to understand these theories, the output of the program, the result of the simulation, should be the particle world-lines, the mass density field, respectively the flashes; the output should look like Figure 3.1. The wave function, in contrast, is one of the internal variables and its role is to implement the evolution for the output, the primitive ontology of the theory.

The proposal is that two theories be regarded as *physically equivalent* when they lead to the same history of the primitive ontology. Conversely, one could define the notion of primitive ontology in terms of physical equivalence: The primitive ontology is described by those variables which remain invariant under all physical equivalences.

4.2 The Flexible Wave Function

As a consequence of the view that the various quantum theories without observer are ultimately not about wave functions but about histories of a primitive ontology in space-time (either particles, or flashes, or matter density), the law of evolution of the wave function should no longer be regarded as playing the central role in the theory.

There might be different ways of producing the same output, using different internal variables. For example, two wave functions that differ by a gauge transformation generate the same law for the primitive ontology. In more detail, when (external) magnetic fields are incorporated into BMp by replacing all derivatives ∇_k in the defining equations of Bohmian mechanics by $\nabla_k - ie_k A(q_k)$, where A is the vector potential and e_k is the electric charge of particle k, then the gauge transformation

$$\psi \mapsto \mathrm{e}^{i\sum_k e_k f(x_k)}\psi\,, \quad A \mapsto A + \nabla f \tag{4.1}$$

does not change the trajectories nor the quantum equilibrium distribution. As another example, one can write the law for the primitive ontology in either the Schrödinger or the Heisenberg picture. As a consequence, the same law for the primitive ontology is generated by either an evolving wave function and static operators or a static wave function and evolving operators. In more detail, BMp can be reformulated in the

Heisenberg picture by rewriting the law of motion as follows:

$$\frac{dQ_k}{dt} = -\frac{1}{\hbar}\mathrm{Im}\frac{\langle\psi|P(dq,t)[H,\widehat{Q}_k(t)]|\psi\rangle}{\langle\psi|P(dq,t)|\psi\rangle}(q = Q(t)), \tag{4.2}$$

where H is the Hamiltonian (e.g., for N spinless particles given by (2.3)), $\widehat{Q}_k(t)$ is the (Heisenberg-evolved) position operator (or, more precisely, triple of operators corresponding to the three dimensions of physical space) for particle k and $P(dq,t)$ is the projection-valued measure (PVM) defined by the joint spectral decomposition of all (Heisenberg-evolved) position operators (DGTZ 2005).

Maybe more interestingly, in GRWf, for example, the fact that the wave function is collapsing does not seem to be crucial. Instead, the central role is played by the random set F of flashes for GRWf, respectively by the random matter density function $m(x,t)$ for GRWm. From this understanding of GRWf as being fundamentally about flashes, we obtain a lot of flexibility as to how we should regard the wave function and prescribe its behavior. As we will point out in the next section, it is not necessary to regard the wave function in GRWf as undergoing collapse; instead, one can formulate GRWf in such a way that it involves a wave function ψ that evolves linearly (i.e., following the usual Schrödinger evolution). The same can be done in the case of BMp, in which one can rewrite the theory in terms of a "collapsed" wave function, as we will see in Section 4.2.2.

4.2.1 GRWf Without Collapse

Suppose the wave function at time t is ψ_t. Then according to equation (2.11), for GRWf the rate for the next flash is given by

$$r(x,i|\psi_t) = \lambda \|\mathsf{L}_i(x)^{1/2}\psi_t\|^2. \tag{4.3}$$

Observe that ψ_t, given by equation (2.12), is determined by ψ_{t_0} and the flashes (X_k, T_k) that occur between the times t_0 and t; it can be rewritten as follows:

$$\psi_t = \frac{\mathsf{L}_{I_n}(X_n, T_n; t)^{1/2} \, \cdots \, \mathsf{L}_{I_1}(X_1, T_1; t)^{1/2} \psi_t^L}{\|\mathsf{L}_{I_n}(X_n, T_n; t)^{1/2} \, \cdots \, \mathsf{L}_{I_1}(X_1, T_1; t)^{1/2} \psi_t^L\|} \tag{4.4}$$

where we have introduced the Heisenberg-evolved operators (with respect to time t)

$$\mathsf{L}_{I_k}(X_k, T_k; t)^{1/2} = U_{t-T_k} \mathsf{L}_{I_k}(X_k)^{1/2} U_{T_k - t} = U_{t-T_k} \mathsf{L}_{I_k}(X_k)^{1/2} U_{t-T_k}^{-1} \tag{4.5}$$

and the linearly evolved wave function

$$\psi_t^L = U_{t-t_0} \psi_{t_0} \, , \tag{4.6}$$

where t_0 is the initial (universal) time. By inserting ψ_t given by equation (4.4) in (4.3) one obtains that

$$r(x, i|\psi_t) = \lambda \frac{\|\mathsf{L}_i(x)^{1/2} \mathsf{L}_{I_n}(X_n, T_n; t)^{1/2} \cdots \, \mathsf{L}_{I_1}(X_1, T_1; t)^{1/2} \psi_t^L\|^2}{\|\mathsf{L}_{I_n}(X_n, T_n; t)^{1/2} \cdots \, \mathsf{L}_{I_1}(X_1, T_1; t)^{1/2} \psi_t^L\|^2}. \tag{4.7}$$

Suppose that the initial wave function is ψ_{t_0}, i.e., that the linearly evolved wave function at time t is ψ_t^L. Then the right hand side of equation (4.7) defines the conditional rate for the next flash after time t, given the flashes in the past of t. Note that this conditional rate thus defines precisely the same flash process as GRWf. In particular, we have that

$$\mathbb{P}_{\psi_t^L}(\text{future flashes}|\text{past flashes}) = \mathbb{P}(\text{future flashes}|\psi_t). \tag{4.8}$$

The collapsed wave function ψ_t provides precisely the same information as the linearly evolving wave function ψ_t^L together with all the flashes. Thus, one arrives at the surprising conclusion that the Schrödinger wave function can be regarded as governing the evolution of the space-time point process of GRWf, so that GRWf can indeed be regarded as a *no-collapse* theory involving flashes. I say "no-collapse" to underline that the dynamics of the primitive ontology is then governed by a wave function evolving according to the standard, linear Schrödinger equation. However, while the probability distribution of the future flashes, given the collapsing wave function ψ_t, does not depend on the past flashes, given only ψ_t^L it does.

A notable difference between the two presentations of GRWf is that while the GRW collapse process ψ_t is a Markov process,[3] the point-process F of flashes is generically non-Markovian. In more detail, we regard a point process in space-time as Markovian if for all $t_1 < t_2$,

$$\mathbb{P}\big(\text{future of } t_2 \big| \text{past of } t_2\big) = \mathbb{P}\big(\text{future of } t_2 \big| \text{strip between } t_1 \text{ and } t_2\big)\,, \tag{4.9}$$

where "future of t_2" refers to the configuration of points after time t_2, etc.. To see that F is non-Markovian, note that the distribution of the flashes in the future of t_2 depends on what happened between time 0 and time t_2, while the strip in space-time between t_1 and t_2 may provide little or no useful information, as it may, for small duration $t_2 - t_1$, contain no flashes at all.

The two versions of GRWf, one using the collapsing wave function ψ_t and the other using the non-collapsing wave function ψ_t^L, should be regarded not as two different theories but rather as two formulations of the same theory, GRWf, because they lead to the same distribution of the flashes.

4.2.2 Particle Bohmian Mechanics With Collapse

In the previous section we showed that GRWf can be reformulated in terms of a linearly evolving wave function. Conversely, Bohmian machanics can be reformulated so that it involves a "collapsed" wave function. In this formulation the evolution of the wave function depends on the actual configuration. The state at time t is described by the pair (Q_t, ψ_t^C), where $Q = (Q_1, \ldots, Q_N)$ is the (usual) configuration but $\psi_t^C : \mathbb{R}^{3N} \to \mathbb{C}$ is a different wave function than usual, a collapsed wave function. Instead of usual equations of Bohmian mechanics (the guidance equation and Schrödinger's equation),

[3] This means that $\mathbb{P}\big(\text{future}\big|\text{past \& present}\big) = \mathbb{P}\big(\text{future}\big|\text{present}\big)$. In more detail, the distribution of the ψ_t for all $t > t_0$ conditional on the ψ_t for all $t \leq t_0$ coincides with the distribution of the future conditional on ψ_{t_0}.

the state evolves according to

$$\frac{dQ_k}{dt} = \frac{\hbar}{m_i} \mathrm{Im} \frac{\psi^{C*} \nabla_k \psi^C}{\psi^{C*} \psi^C} (Q_1, \dots, Q_N) \,, \tag{4.10}$$

which is the same as the guide equation with ψ replaced by ψ^C, and

$$i\hbar \frac{\partial \psi^C}{\partial t} = -\sum_{k=1}^{N} \frac{\hbar^2}{2m_k} (\nabla_k - i\tilde{A}_i)^2 \psi^C + (V + \tilde{V}) \psi^C \tag{4.11}$$

which is the same as Schrödinger's equation except for the imaginary pseudo-potentials ($\sigma \approx 10^{-7}$ m is the same constant as in GRW)

$$\tilde{A}_k = \frac{i}{\sigma^2} (q_k - Q_k) \,, \quad \tilde{V} = -\frac{i}{\sigma^2} \sum_{k=1}^{N} \frac{\hbar^2}{m_i} (q_k - Q_k) \cdot \mathrm{Im} \frac{\psi^{C*} \nabla_k \psi^C}{\psi^{C*} \psi^C} \tag{4.12}$$

making equation (4.11) nonlinear and Q-dependent. A solution (Q_t, ψ_t^C) of equations (4.10) and (4.11) can be obtained from a solution (Q_t, ψ_t) of usual equations of Bohmian mechanics by setting

$$\psi^C(q_1, \dots, q_N) = \exp\left(-\sum_{k=1}^{N} \frac{(q_k - Q_k)^2}{2\sigma^2} \right) \psi(q_1, \dots, q_N) \,. \tag{4.13}$$

This is readily checked by inserting (4.13) into equations (4.10) and (4.11). The ensemble of trajectories with distribution $|\psi|^2$ cannot be expressed in a simple way in terms of ψ^C. Nonetheless, for given initial configuration Q_0, we obtain from equations (4.10) and (4.11), with given initial ψ_0^C, the same trajectory Q_t as from the usual equations of Bohmian mechanics with the corresponding ψ_0. This may be enough to speak of physical equivalence.

One can read off from (4.13) that ψ^C is a *collapsed* wave function: Whenever ψ is a superposition (such as for Schrödinger's cat) of macroscopically different states with disjoint supports in configuration space, then in ψ^C all contributions except the one containing the actual configuration Q_t are damped down to near zero. (Still, the evolution is such that when two disjoint packets again overlap, the trajectories display an interference pattern.)

	Particles	Fields	Flashes
Deterministic	BMp	BQFT, Sm	
Indeterministic	SM, BTQFT, BMW, GRWp	GRWm	GRWf, Sf

Table 4.1: Different possibilities for the primitive ontology of a theory are presented: particles, fields and flashes. These different primitive ontologies can evolve according to either deterministic or stochastic laws. Corresponding to these possibilities we have a variety of physical theories: (particle) Bohmian mechanics (BMp), Bohmian quantum field theory (BQFT), a mass density field theory with Schrödinger evolving wave function (Sm), stochastic mechanics (SM), Bell-type quantum field theory (BTQFT), Bell's version of many-worlds (BMW), a particle GRW theory (GRWp), GRW theory with mass density (GRWm), GRW theory with flashes (GRWf), and two theories with flashes governed by Schrödinger wave functions (Sf). For a detailed description of these theories, see the text.

Of course, the unitarily evolving ψ_t is much more natural than ψ_t^C as a mathematical tool for defining the trajectory Q_t; Schrödinger's equation is a simpler equation than (4.11). Still, the example shows that we have the choice in Bohmian mechanics between using a collapsed wave function ψ^C or a spread out wave function ψ.

I conclude from this discussion that what many have considered to be the crucial, irreducible difference between Bohmian mechanics and GRW theory, namely that the wave function collapses in GRW but does not in Bohmian mechanics, is not in fact an objective difference at all, but rather a matter of how GRW of Bohmian mechanics are presented.

4.3 A Plethora of Theories

One may wonder whether some primitive ontology (flashes and continuous matter density) work only theories with stochastic evolutions for the wave function while others (particle trajectories) work only with theories in which the wave function evolves linearly. This is not the case: as we have already emphasized, the evolution of the wave function is not crucial for the specification of the theory, which is identified by the histories of the primitive ontology. Let us analyze, with the aid of Table 4.3, several possibilities, also discussed in (Allori et al. 2007): there can be at least three different kinds of primitive ontology for a fundamental physical theory, namely particles, fields, and flashes. Those primitive ontologies can evolve either according to a deterministic or to a stochastic law and their evolution is what determines the theory. This law for the primitive ontology can be implemented with the aid of a wave function evolving either stochastically or deterministically. By definition, theories with the same histories for the primitive ontology but different evolution for the wave function are physically equivalent theories.

Note that once the structure $(\mathscr{X}, \Psi)$ is defined, we have different possibilities regarding the choices of:

- The primitive ontology $\mathscr{X}$ (particles, fields, strings,...),
- The kind of law of evolution for the primitive ontology (deterministic vs. stochastic),
- The form of the law of evolution for the primitive ontology (e.g., in the case of a deterministic theory, there are possible different orders of differential equations),
- The kind of law for the wave function Ψ (deterministic vs. stochastic),
- The form of the law of evolution for the wave function.

What is crucial to define a theory is that the histories $\mathscr{X}_t$ of the primitive ontology are the same, no matter what the wave function is doing. Bohmian mechanics (BMp) is the prototype of a theory in which we have a particle primitive ontology that evolves deterministically according to a law specified by a wave function that also evolves deterministically. Bohmian mechanics with collapse presented above is a theory which is physically equivalent to BMp. In Figure 4.1 is represented the image of the world provided by BMp.

The natural analog for a theory with particle ontology with indeterministic evolution is stochastic mechanics (SM), in which the law of evolution of the particles (the primitive ontology) is given by a diffusion process while the evolution of the wave function, the usual Schrödinger evolution, remains deterministic (see (Nelson 1985), (Goldstein 1987) for details). Even if the primitive ontology of BMp and SM are the same, since their trajectories differ are different theories. In Figure 4.2 is represented the image of the world provided by SM.

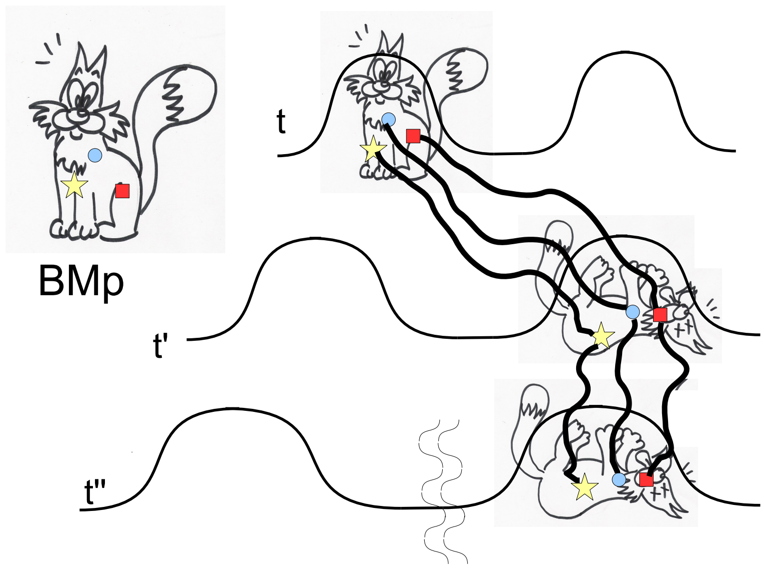

Figure 4.1: The image of the world provided by BMp.

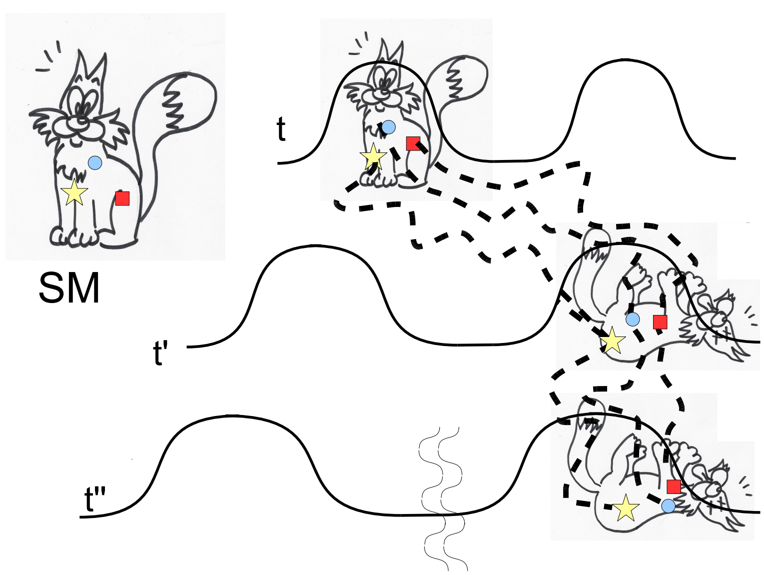

Figure 4.2: The image of the world provided by SM.

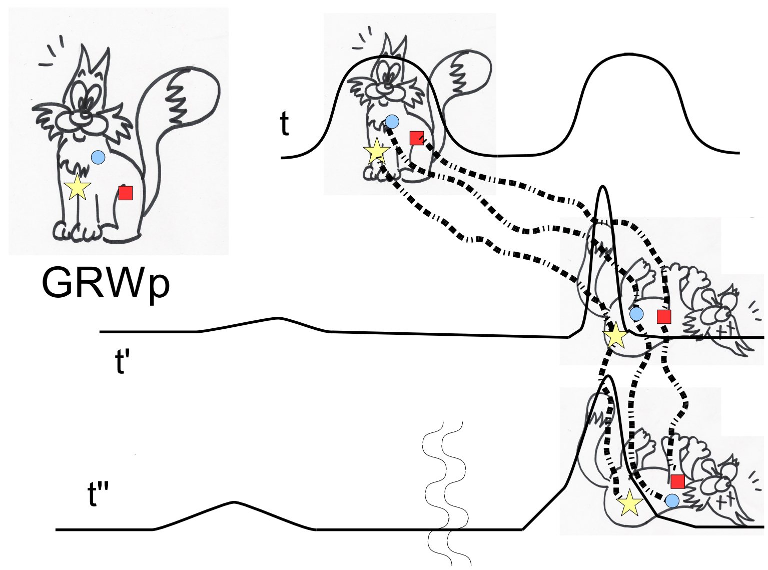

Figure 4.3: The image of the world provided by GRWp.

Another example involving stochastically evolving primitive ontology of particles with a deterministically evolving wave function is provided by a Bell-type quantum field theory (BTQFT) in which, despite the name, the primitive ontology is given by particles evolving indeterministically to allow for creation and annihilation (for a description, see (DGTZ 2004), (Bell 1987), (DGTZ 2005)).

Another possibility for a stochastic theory of primitive ontology of particles is a theory, GRWp, in which the particle motion is governed by the guide equation but with a wave function that obeys a GRW-like evolution in which the collapses occur exactly as in usual GRW theories except that, once the time and label for the collapse has been chosen, the collapse is centered at the actual position of the particle with the chosen label, rather than at random according to equation (2.10). A garbled formulation of this theory is presented in (Bohm and Hiley 1993). Again, this theory is not physically equivalent to BMp or SM because, even if the primitive ontology is the same, the trajectories are different. In Figure 4.3 is represented the image of the world provided by GRWp.

What in Table 4.3 we call a Bohmian quantum field theory (BQFT) involves a primitive ontology of fields, evolving deterministically (Bohm 1952),

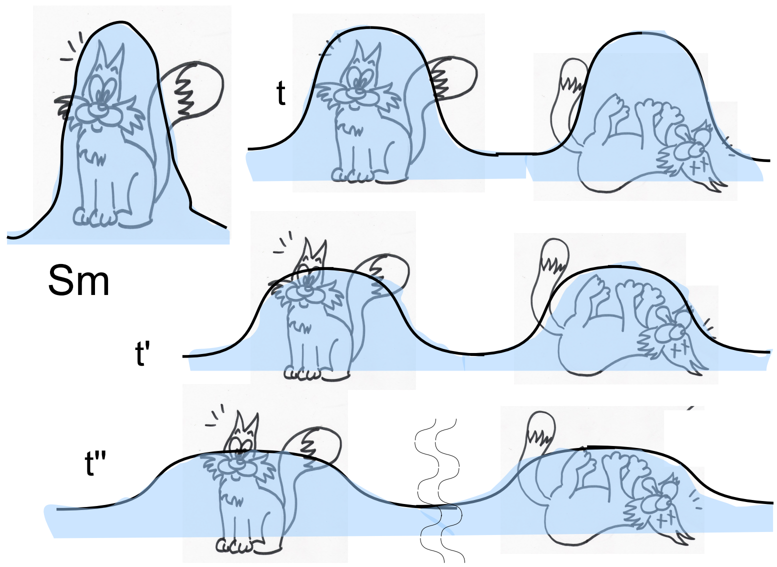

Figure 4.4: The image of the world provided by Sm.

(Struyve and Westman 2006). In the model the wave function evolves according to Schrödinger's equation but, arguably, the same trajectories could be generated by a nonlinearly evolving wave function.

Another example is provided by the theory, that I can be called Sm (S for Schrödinger evolution), in which the primitive ontology is given by the mass density field but evolving with a Schrödinger wave function, always evolving according to Schrödinger's equation, with no collapses. In contrast, GRWm provides an example of a theory of fields that evolve stochastically. In Figures 4.4, and 4.5 are represented the images of the world provided by Sm and GRWm.

Concerning theories with flashes, these are inevitably stochastic, and GRWf, in which the flashes track the collapses of the wave function, is the prototype. As seen previously, GRWf without collapse is physically equivalent to GRWf, in which the wave function collapses. In Figure 4.6 is represented the image of the world provided by GRWf.

There are also theories with flashes in which the wave function never collapses. Such theories are closer to BMp than to GRWf in this respect, but we will see also in which other respect they are close to BMW. Consider the following example, denoted by Sf.

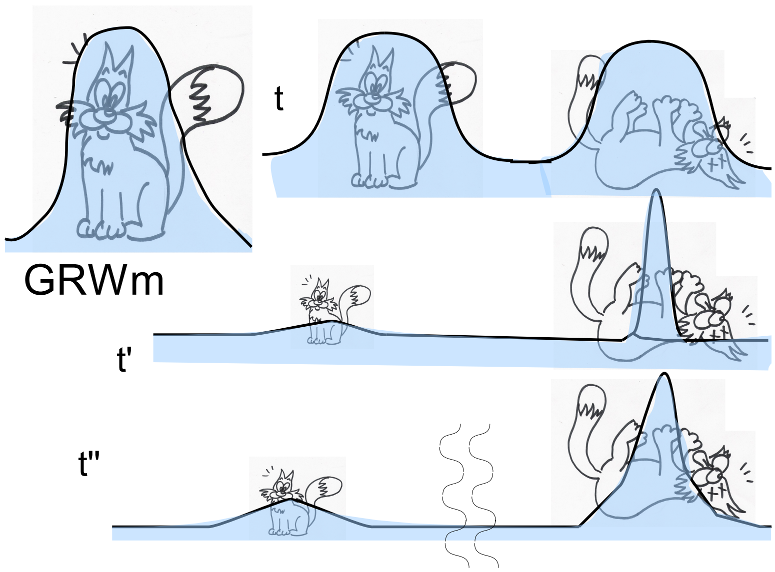

Figure 4.5: The image of the world provided by GRWm.

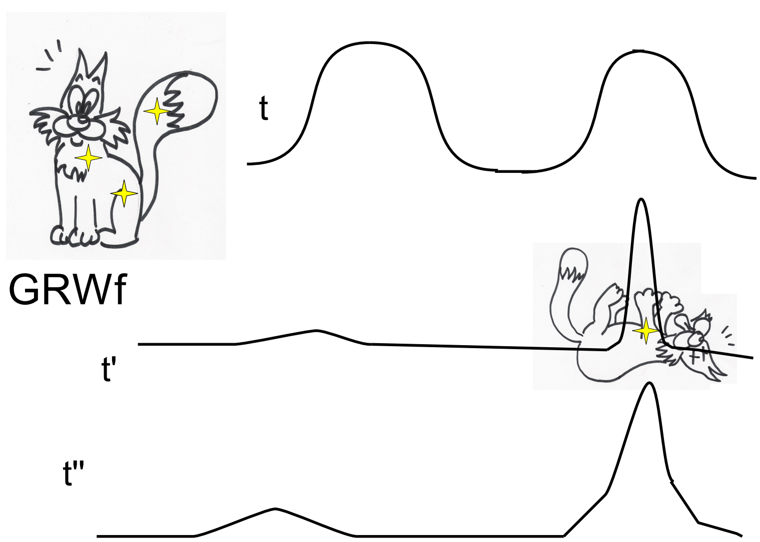

Figure 4.6: The image of the world provided by GRWf.

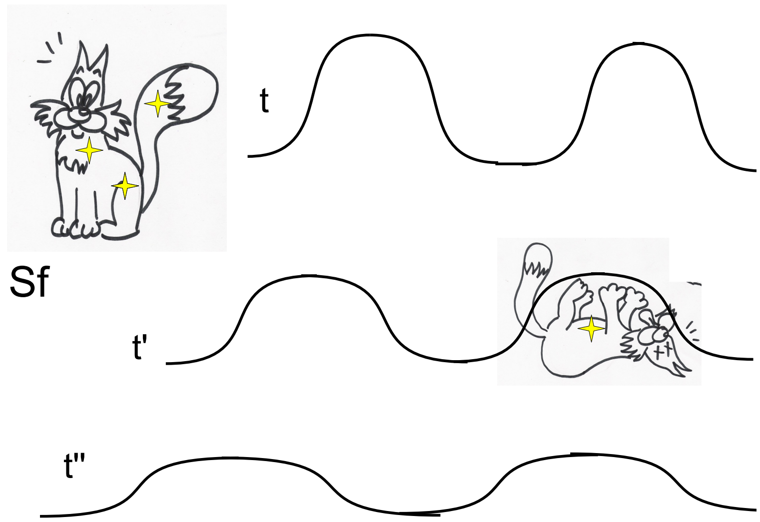

Figure 4.7: The image of the world provided by Sf.

Here S stands for Schrödinger (evolution). Using this notation we have that BMp = Sp, SM=Sp', and BTQFT=Sp". In this theory, the primitive ontology consists of flashes with their distribution determined by a Schrödinger wave function $\psi = \psi(q_1, \ldots, q_N)$, that evolves always unitarily, as in BMp, according to the N-"particle" Schrödinger evolution. The flashes are generated by the wave function exactly as in GRWf. Thus, the algorithm, whose output is the flashes, is the same as the one described for GRW, with steps 1., 2. and 3., with the following difference: the first sentence in step 2. is dropped, since no collapse takes place. In other words, in Sf flashes occur with rate (2.11) but are accompanied by no changes in the wave function. Accordingly, equation (2.14) is replaced by

$$\mathbb{P}\big(X_1 \in dx_1, T_1 \in dt_1, I_1 = i_1, \ldots, X_n \in dx_n, T_n \in dt_n, I_n = i_n | \psi_{t_0}\big)$$
$$= \lambda^n \mathrm{e}^{-N\lambda(t_n - t_0)} \prod_{k=1}^{n} \langle \psi_{t_k} | \mathsf{L}_{i_k}(x_k) \psi_{t_k} \rangle \, dx_1 dt_1 \cdots dx_n dt_n \, ,$$

where $\mathsf{L}_i(x)$ is the collapse operator given by (2.7).

In Figure 4.7 is represented the image of the world provided by Sf.

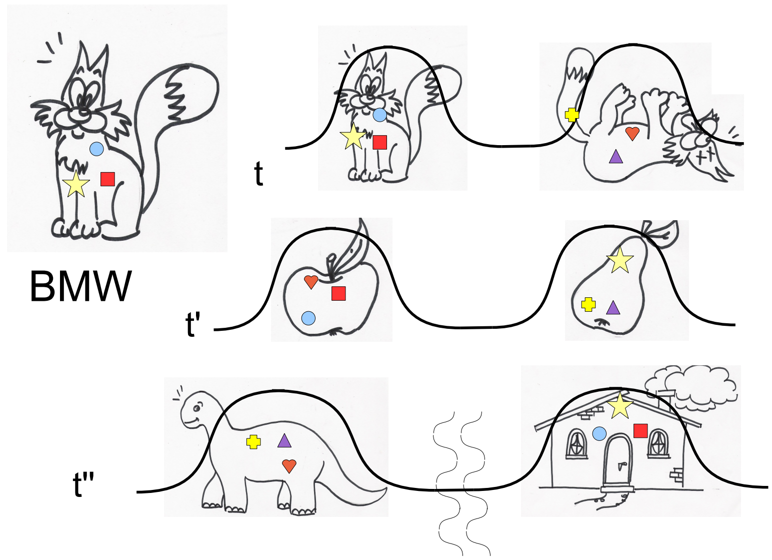

Figure 4.8: The image of the world provided by BMW.

As we have seen, in BMW the wave function ψ evolves according to Schrödinger's equation and instantaneous classical configurations the primitive ontology of the theory, are distributed with probability $|\psi|^2$. See Figure 4.8 a picture of the world according to BMW is shown. In the notation used in the table, we would have BMW=Sp*, to differentiate it from the others theories of particles guided by a Schrödinger wave function. Even if the primitive ontology is the same, it evolves according to a different law, so the theories are not physically equivalent. In BMW there are no pairing of configurations at different times. Because of this, as we observed earlier, the world described by BMW is radically different from what we are accustomed to. In fact, in this theory, the actual past will typically entirely disagree with what is suggested by our memories, by history books, by photographs and by other records of (what we call) the past. In this respect it is even weirder than other many worlds theory, as we will see in the next section.

Let me note that the notation used so far can be misleading, since it focuses on the evolution of the wave function, which is not really fundamental. Maybe a better notation would have been one which focuses on what the theory does to its primitive ontology $\mathscr{X}$, specifying in a subscript the law u of evolution of the primitive ontology

	Primitive Ontology	Theory	physical equivalent to
$p^{(\text{Schr.})}_{\text{determ.traj.}}$	particles	BMp	collapsed BMp
$p^{(\text{Schr.})}_{\text{random}}$	particles	BMW	
$p^{(\text{GRW})}_{\text{random}}$	particles	GRWp	
$p^{(\text{Schr.})}_{\text{random}'}$	particles	SM	
$p^{(\text{Schr.})}_{\text{random}''}$	particles	BTQFT	
$m^{(\text{Schr.})}_{\text{determ.}}$	mass density field	Sm	
$m^{(\text{GRW})}_{\text{random}'}$	mass density field	GRWm	
$\phi^{(\text{Schr.})}_{\text{determ.}}$	fields	BQFT	
$f^{(\text{Schr.})}_{\text{random}}$	flashes	Sf	
$f^{(\text{GRW})}_{\text{random}'}$	flashes	GRWf	linear GRWf

Table 4.2: The plethora of theories in the notation $\mathscr{X}_u^{(f)}$.

$\mathscr{X}$ and as a superscript, between parenthesis to underline its side role, the law f of evolution of the wave function Ψ:

$$\mathscr{X}_u^{(f)}$$

Table 4.3 summarizes all the theories in the new notation.

4.4 Schrödinger Wave Functions and Many Worlds

At first glance, in an Sf or Sm world, the after-measurement state of the apparatus seems only to suggest that matter is very spread out. However, if one considers the flashes, governed by the rate (2.11), or the mass density (3.3), that correspond to macroscopic superpositions, one sees that they form independent families of correlated flashes or mass density associated with the terms of the superposition, with no interaction between the families. The families can indeed be regarded as comprising many worlds, superimposed on a single space-time. (Note that in order to preserve the stability of the macroscopic world, the different worlds in Sm and Sf need not to interact among themselves so as to remain reciprocally transparent.)

Therefore, Sf and Sm, though they are simple mathematical modifications of GRWf and GRWm respectively, provide very different pictures of reality. In Sf and Sm, pointers never point in a specific direction (except when a certain direction in standard quantum mechanics would have probability more or less one), but rather all directions are, so to speak, realized at once. In Sf and Sm, like BMW and MW∅, because they have superimposed space-times, reality is very different from what we usually believe it is to be like. It is populated with ghosts we do not perceive, or rather, with what are like ghosts from our perspective, because the ghosts are as real as we are, and from their perspective, we are the ghosts.

Let us note that the theory Sm is closely related to, if not precisely the same as, the version of quantum mechanics proposed by Schrödinger in 1926 (Schrödinger 1926). After all, Schrödinger originally regarded his theory as describing a continuous distribution of matter (or charge) spread out in *physical space* in accord with the wave function on *configuration space* (Schrödinger 1926). He soon rejected this theory because he thought that it rather clearly conflicted with experiment. Schrödinger's rejection of this theory was perhaps a bit hasty. Be that as it may, according to what we have said above, Schrödinger did in fact create the first many-worlds theory, though he probably was not aware that he had done so. However, Schrödinger did write that:

> $\psi\bar{\psi}$ is a kind of weight-function in the system's configuration space. The wave-mechanical configuration of the system is a superposition of many, strictly speaking of all, point-mechanical configurations kinematically possible. Thus, each point-mechanical configuration contributes to the true wave-mechanical configuration with a certain weight, which is given precisely by $\psi\bar{\psi}$. If we like paradoxes, we may say that the system exists, as it were, simultaneously in all the positions kinematically imaginable, but not 'equally strongly' in all (Schrödinger 1927).

4.5 Common Structure

To summarize, up to now, here are some features that the theories analyzed in this chapter have in common:

- There is a clear primitive ontology $\mathscr{X}$ that it describes matter in space and time;

- There is a wave function Ψ that evolves according to a given (possibly stochastic) dynamical evolution (either according to Schrödinger's equation or, at least, for microscopic systems very probably for a long time approximately so).

- The wave function governs the behavior of the primitive ontology by means of (possibly stochastic) laws.

In addition, as we will see more in detail in Chapter 8, we have:

- The theory provides a notion of a typical history of the primitive ontology (of the universe), for example by a probability distribution on the space of all possible histories; from this notion of typicality the probabilistic predictions emerge.
- The predicted probability distribution of the macroscopic configuration at time t determined by the primitive ontology (usually) agrees (at least approximately) with that of the quantum formalism.

4.6 The Role of the Wave Function

Let us now clarify one issue: If the primitive ontology of the theory are the building block of the physical world, they are the stuff in three-dimensional space physical objects are made of, what is the wave function if not a material object?

4.6.1 The Wave Function as a Property?

One way of interpreting the wave function if it is not part of the primitive ontology is to say that the wave function is a property of the particles. Monton seems to have this view in some of his writings:

> the wave function doesn't exist on its own, but it corresponds to a property possessed by the system of all the particles in the universe (Monton 2006).

If it is the case, then the wave function is not physical but it is instead an abstract entity. It is not really clear to me what "the wave function is a property" is supposed to mean, given that it is not clear to me what a property is supposed to be.

Be that as it may, what kind of property is the wave function supposed to be? Categorical or dispositional? In my understanding, a dispositional property is a property that is what it is in virtue of the laws of physics. For example, the mass of an object can be considered a dispositional property in the sense that it expresses the resistance of the body to be accelerated by external forces. In contrast, the mass can be thought as a categorical property of the body as it specifies its own nature. It seems difficult to regard the wave function as property *of the particle* in any case, since the wave function is *of the universe*, the biggest system of all. We have seen that in Bohmian mechanics, since we have particles, we can define a wave function for the subsystem of configuration x using the conditional wave function $\psi(x) = \Psi(x, Y)$, where Y is the actual configuration of the environment of the x-system. In any case, it does not seem right to consider the wave function (not even the conditional one) as a categorical property of the particles: in fact, it does not in any ways determine its nature. It might seem a little less far fetched to think to the conditional wave function as a dispositional property but actually it is difficult since it might happen not: the conditional wave function might not evolve according to Schrödinger's equation. It would do that only in particular situations like the one in which the wave function has a particular form, the so called effective wave function. In any case, independently of whether one can make sense of the wave function being a property of the particle or one has to assume that the wave function is an holistic property, a property of the universe as a whole, I do not really see any advantage in saying that the wave function is a property, unless what one means is, at the end of the day, that it is a law. Laws and properties seems to share the feature of being abstract entities, so one would not worry about the wave function being physically real either way. But saying that the wave function is a law gives a clearer role to the wave function than saying it is a property. Talking of physical theories in terms of properties rather than in terms of laws does not seem to do any good. One might think that there are categorical properties that determine the very nature of the physical objects. My intuition is just the other way round: a particle is considered to be an electron rather than a proton because, say, in the bubble chamber it will rotate one way or another under the magnetic field imposed on it. That is, physical objects and their properties are parasitic on the theory in which they are described, rather the other way round. Therefore, it seems to proceed backwards to say that the wave function is a property rather than saying that it is a law.

4.6.2 The Wave Function as a Useful Mathematical Tool

Another proposal, or maybe a different way of intending the former proposal, is to consider the wave function just as a useful mathematical tool. Monton writes:

> I think that the wave function is a useful mathematical tool; it is useful to describe systems as having quantum states, represented by wave functions. But as a matter of ontology, the wave function doesn't exist or at least, the wave function is no more real than the numbers (like 2, or B) that go into the equations used to describe quantum systems. The wave function is, at best, an abstract entity and if you're a nominalist about abstract entities like I am, then you should be happy to say that the wave function doesn't exist (Monton 2006).

A similar view has been discussed by Fay Dowker and Isabelle Herbauts (Dowker and Herbauts 2006) in the framework of GRW theory: They provide a model of spontaneous collapse theories on a lattice in which

> the state $|\psi_0\rangle$ can be deduced FAPP [for all practical purposes] from the field configuration [...] and becomes an"executive summary" of the past reality containing no independent information.

A number of objections have been raised to the view that the wave function is not existing as a physical object, which are similar to the objections that have been raised against the next proposal that we will analyze. For this reason, we will deal with them in the next sections.

4.6.3 The Wave Function as a Law

A slightly different but related approach has been proposed by Dürr, Goldstein and Zanghì who have analyzed the nature of the wave function in the framework of Bohmian mechanics in their *Bohmian Mechanics and the Meaning of the Wave Function* (DGZ 1997), and that I have taken as more or less implicitly up to now.

In their view the wave function has to be intended as the object that allows for the generation of the law of motion for the particles rather than a field on configuration space or just a mathematical tool. As they say:

> the role of the wave function in this theory, expressed by the association $\psi \to v^{\psi}$, is to generate the vector field, given by the right hand side of (3) [the guide equation], that defines the motion (DGZ 1997).

In this theory, the wave function has to be intended not as a part of physical reality but as a law:

> We propose that the wave function belongs to an altogether different category of existence than that of substantive physical entities, and that its existence is nomological rather than material (DGZ 1997).

The wave function expresses a law for the motion of the particles, just as the Hamiltonian in classical mechanics is the generator of the motion of the positions of the particles. In fact, in classical mechanics, the complete description of any physical system is provided by $X = (q_1, ..., q_N, p_1, ..., p_N)$, the set of the configurations and the momenta of all the particles. Then the classical Hamiltonian H_{class} is a function of the space of the Xs, the phase space, and it is the generator of the time evolution of that state:

$$\frac{dq}{dt} = \frac{\partial H_{class}}{\partial p} \tag{4.14}$$

$$\frac{dp}{dt} = -\frac{\partial H_{class}}{\partial q}, \tag{4.15}$$

or, more compactly,

$$\frac{dX}{dt} = \mathrm{Der}H_{class}, \tag{4.16}$$

where Der is a suitable operation of derivation. In the same way $\log\Psi$ is the generator for the motion of particles:

$$\frac{dQ}{dt} = \ \mathrm{Der}[\log\Psi]. \tag{4.17}$$

In this framework, Bohmian mechanics is a theory about particles that evolve according to a law of motion depending on the wave function, that has to be intended, according to Dürr, Goldstein and Zanghí, as part of the law for the particles. As in Newtonian mechanics it is necessary to specify the momentum p and the potential V in order to generate the trajectories $q(t)$ of the particles, so in BMp it is necessary to specify the wave function ψ and the Hamiltonian H in order to generate the trajectories $q(t)$ of the particles. The situation is very similar in the plethora of theories discussed in 4.3:

In Bohmian mechanics the primitive ontology (particles positions) evolves deterministically according to a law generated by a wave function that evolves deterministically; in BMW the primitive ontology (particles positions) evolves stochastically according to a law generated by a wave function that evolves deterministically; in BTQFT, and in in stochastic mechanics the primitive ontology (particles positions) evolves stochastically according to a law generated by a wave function that evolves deterministically; in GRWp the primitive ontology (particles positions) evolves stochastically according to a law generated by a wave function that evolves stochastically; in Sm the primitive ontology (the mass density field) evolves deterministically according to a law generated by a wave function that evolves deterministically; in GRWm the primitive ontology (the mass density field) evolves stochastically according to a law generated by a wave function that evolves stochastically; in Sf the primitive ontology (the flashes) evolves stochastically according to a law generated by a wave function that evolves deterministically; in GRWf the primitive ontology (the flashes) evolves stochastically according to a law generated by a wave function that evolves stochastically; in BQFT the primitive ontology (fields) evolves deterministically according to a law generated by a wave function that evolves deterministically. In all these cases, the primitive ontology $\mathscr{X}$ evolves according to a given equation governed by the wave function. Therefore, again, it is necessary to specify the wave function ψ in order to generate the trajectories of the primitive ontology. Therefore, according to Dürr, Goldstein and Zanghí we can conclude that the wave function in all those theories has a sort of nomological status that differs from the one of the primitive ontology: the wave function has the role of generating the dynamics for the primitive ontology, whatever it is.

One complaint might be that this view suggests that there are different degrees of reality, as suggested by Harvey Brown and David Wallace in their *Solving the Measurement Problem: de Broglie-Bohm Loses out to Everett* (Brown and Wallace 2005) and by David Albert [4]: since the primitive ontology composes physical objects while the wave function does not, either we deny the existence of the wave function or we have to admit that something is more real than something else.

[4]Private communication.

This seems to me an unfair criticism. Saying that the wave function is real but not physical does not imply there are different degrees of reality: in fact, they might be two kind of substances, or entities, or different way of existing. After all, the very same objections could be raised (but they are not) to a Platonist in the philosophy of mathematics, a dualist in the philosophy of mind, and a realist with respect to laws in ethics or in philosophy of science: Numbers, consciousness, moral and physical laws can exist even if they are non-physical.

Another objection to this view is the following, as expressed by Harvey Brown and David Wallace in the framework of Bohmian mechanics:

> [...] reality is not some property which we can grant or withhold in an arbitrary way from the components of our mathematical formalism. The wave-function evolves; it dynamically influences the corpuscles; in interference experiments its existence is explanatorily central to the observed phenomena. On what grounds could we just dismiss it as a mathematical fiction? (Brown and Wallace 2005)

There are several components to this idea: first, the wave function has an explanatory role; second, the wave function interacts with the particles; third, the wave function evolves in time. For these reasons, they argue, we should consider the wave function as physically real.

As already stressed, I do not think that the claim that the wave function should be physically real because it is explanatory is really a serious objection: even if something has to be postulated existing in order to explain the behavior of matter it does not follow that that entity has to be *physically* real. In fact, if one has a realistic view of laws, laws are explanatory but they do not exist in physical space. Of course the problem would be to explain the somehow mysterious interaction between laws and matter, but this is a general problem for the view, not a particular one for quantum theories without observer.

Concerning the second objection, that since the wave function interacts with the particles in Bohmian mechanics it should be considered physical, one could respond saying that the situation is similar to the one in classical mechanics: as in Newton's theory the potential can interact with the particles and no one would be tempted to say that the potential is physically existing, so in quantum mechanics the wave function interacts with the particles but it is not necessary to assume it to be a physical field. Wallace and Brown anticipate this reply:

> [...] there is a bad analogy to resist here. From the corpuscles perspective, the wave-function is just a (time-dependent) function on their configuration space, telling them how to behave; it superficially appears similar to the Newtonian or Coulomb potential field, which is again a function on configuration space. No-one was tempted to reify the Newtonian potential; why, then, reify the wave-function? (Brown and Wallace 2005)

and respond emphasizing that the analogy between the wave function and the potential is a bad one:

> Because the wave-function is a very different sort of entity. It is contingent (equivalently, it has dynamical degrees of freedom independent of the corpuscles); it evolves over time; it is structurally overwhelmingly more complex (the Newtonian potential can be written in closed form in a line; there is not the slightest possibility of writing a closed form for the wave-function of the Universe.) (Brown and Wallace 2005)

That is, the wave function evolves in time in a way which is independent of the particles, while the potential in classical mechanics does not. In addition, the wave function cannot be written, contrarily to what can be done with the potential, in a closed form in a line. Therefore, the wave function and the potential are not even similar mathematically. The problem is therefore that the analogy between the wave function and the Hamiltonian is not that straightforward. Brown and Wallace continue making a contrast between the potential in classical mechanics on one hand and the electromagnetic fields in classical electrodynamics and the wave function in quantum theories without observer on the other hand. The wave function should be regarded to be similar to the electromagnetic fields since they are both evolving in time according to an independent equation, and should be contrasted with the potential that does not. While the potential was not considered as physically real, the electromagnetic fields have been. As a consequence, also the wave function should be considered as physically real. In their words:

> Historically, it was exactly when the gravitational and electric fields began to be attributed independent dynamics and degrees of freedom that they were reified: the Coulomb or Newtonian fields may be convenient mathematical fictions, but

> the Maxwell field and the dynamical spacetime metric are almost universally accepted as part of the ontology of modern physics (Brown and Wallace 2005).

I will come back to the status of the electromagnetic fields in Chapter 6. Be that as it may, let me notice that John Wheeler and Richard Feynman in their *Interaction with the Absorber as the Mechanism of Radiation* (Wheeler and Feynman 1945) tried to eliminate the electromagnetic field in their theory, rewriting the dynamical equations such that the electromagnetic field entirely disappears. One could try the same in the case of the wave function in order to be able to say that the wave function is not physical. Goldstein and Teufel (Goldstein and Teufel 2000) try to find a way to eliminate it from the theory and recover it as an effective, phenomenological object just as Wheeler and Feynman attempted to do for the electromagnetic field. The idea starts from some observations in quantum cosmology. In that theory, the basic equation for the wave function is the Wheeler-De Witt equation. This is an equation for the wave function of the universe Ψ that in this case is static and can be written as follows:

$$H\Psi = 0 \tag{4.18}$$

for a suitable Hamiltonian H. Similarly to what happens in Bohmian mechanics, also in quantum cosmology it would be possible to regard Ψ as the generator of the velocity field v^Ψ for the primitive ontology $\mathscr{X}$ of quantum cosmology:

$$\frac{d\mathscr{X}}{dt} = v^\Psi \tag{4.19}$$

where in this case $\mathscr{X}$ would include also the configurations of the gravitational field. The field form would be different from the one of Bohmian mechanics and, again differently from Bohmian mechanics the wave function Ψ would not evolve in time. The basic idea is that one could recover the Schrödinger's equation describing a time dependent wave function phenomenologically, being the Wheeler-De Witt equation the fundamental law.

This is connected with the third criticism that the wave function evolves in time, while we tend to think of laws as constant in time. While at the present state of development Bohmian mechanics it might be uncomfortable to regard the wave function as a law because of this time dependence, as suggested by Brown and Wallace, in a future quantum cosmology physical reality would be described by the relevant primitive ontology $\mathscr{X}$ and the wave function would be static and easily considerable a law.

To summarize, the present situation is that the wave function evolves in time but in a more general theory the hope is that this would not be the case anymore. As Shelly Goldstein often stresses [5], here is a nice graphical summary of how the situation has evolved historically in quantum mechanics:

$$(\psi) \rightarrow (\mathscr{X}, \psi) \rightarrow (\mathscr{X}).$$

We started from standard quantum mechanics, in we just had the wave function; we moved to Bohmian mechanics, in which matter is made of particles and the wave function is a time dependent object that tells matter how to move; and we will (probably, or hopefully) end up with a theory of quantum cosmology with a suitable primitive ontology $\mathscr{X}$ in which it would be straightforwardly possible to regard the wave function as a law, it being static.

In any case, it is argued, even if we might have a static wave function, the two of the three features that distinguish the wave function from the potentials are still there:

> As for contingency, it is at most an article of faith with some physicists that the Wheeler-de Witt equation has a unique solution. And as for complexity (in our view perhaps the most important criterion) the structure encoded in the cosmological wavefunction will if anything be richer than that encoded in the nonrelativistic wavefunction (Brown and Wallace 2005).

As far as the lack of simplicity of the wave function, I wish to observe that, as soon as one writes the equation for the wave function as (4.18), some sort of relevant simplicity is already recovered, that equation expressing the fact that there exists an Hamiltonian H such that the wave function Ψ is static. In the case of the worry about the uniqueness of the solution of the Wheeler-de Witt equation, I do not see any reason to be that pessimistic as to believe that there will not be a unique solution: there are no reason to believe the solution will be unique, but there might be no reason also to

[5] Private communication.

indicate that it will not be it as well.

Another objection, raised again by Brown and Wallace, to the idea that the wave function could be seen as a law in perspective of the development of a theory of quantum cosmology is that this project is purely speculative and, in any case, in the theory that we have right now the situation is different: the wave function does evolve in time and this makes it very different from what we would consider a law. In this regard, I would like to observe that we should always take theories in perspective, keeping in mind that in a sense they are always speculative and provisional, since they are not logical deduction from experimental data. So, the charge does not seem to be that devastating:. In the present theory we cannot so naturally or straightforwardly identify the wave function as a law because it evolves in time. But why do we worry so much? What we do when we construct our fundamental physical theories is try to get a grasp on what there is, and the way in which we proceed is by little steps forward, one at the time. What the theory that we have right now is telling us is that the wave function seems to play a certain role, namely the generator of the motion of matter. Given that in the current theory it evolves in time, the wave function might not fit perfectly with our intuition that laws are static, but the fact that the wave function is not static in the present theory is telling us is not so much that the wave function is a material object but rather that we should look for a theory in which the wave function is static. In fact, what seems to be the lesson of the quantum theories without observer is that the wave function in the theory has a role which is different from the one of the primitive ontology.

A last complaint could be that the wave function is controllable [6]: we can prepare physical systems in the state that we want. If so, it is difficult to regard the wave function as a law, since we do not seem to have control over them. This objection is easily taken care of if one remembers that the wave function that we can have control over is the wave function of the system, while the one that should be intended as nomological is rather the wave function of the universe, and we do not have any control

[6] Shelly Goldstein mentioned this objection to me, together with its response.

over it.

4.7 State, Primitive Ontology and Bell's Alternatives

Usually people talk about states, while I have drawn my attention to the primitive ontology. It is crucial to contrast the two notions. The state of the system is what provides a complete description of the world to any instant in time. It can be said to be the ontology. In the language I used, it is therefore composed of the variable characterizing the primitive ontology supplemented by all those variables that allow for the closure of its dynamics. As we have seen above, in order to determine the evolution of the primitive ontology in time, we need to specify its law of evolution, that in quantum theories without observer depends on the wave function, which also evolves in time.

Consider BMp: the primitive ontology is the configuration Q of particles in space. In contrast, the state is given by the couple (Q, Ψ) of the positions of all the particles and the wave function of the universe. Similarly, in BMW the primitive ontology is the one of configurations and the state is the couple (Q, Ψ). In the case of GRWp, the particles have trajectories determined by an evolution generated by a GRW-evolving wave function. In SM, the particles have stochastic trajectories generated by a Schrïodinger evolving wave function. That is also true for BTQFT, for a different stochastic law for the trajectories. In BQFT, Schrödinger wave function generates deterministic trajectories for the fields. In the case of GRWm, the primitive ontology is given by the mass density field $m(x, t)$, whose evolution is governed by a stochastic wave function. Note that in this case the mass density, as it is defined, is a functional of the wave function: $m = F(\Psi)$, with F given by equation (3.3). The same can be said in the case of GRWf: the probability distribution of the flashes, that are the primitive ontology of the theory, are defined in terms of the wave function by equation (2.14). The situation is the same in the case of Sm and Sf, that differ from GRWm and GRWf respectively only in the fact that the wave function evolves linearly.

Therefore, this is the crucial difference between GRWm, GRWf, Sm and Sf on one hand and BMp, SM, GRWp, BMW and BTQFT on the other: while in the latter the state there is composed of the primitive ontology and by the wave function, in the former there is a sense in which this is not the case.

Therefore, on one hand the state by definition gives us everything. But on the other hand also in these theories of mass density and flashes the state tells us less than the primitive ontology because it does not specify *per se* which of the variables has a nomological role (the wave function ψ) and which constitute physical objects (the primitive ontology).

In this framework, the measurement problem is the problem of the inadequacy of the wave function as the primitive ontology of quantum mechanics. The measurement problem is caused by the fact that the wave function cannot represent physical objects. So possible solutions of the measurement differ not in the fact that we either add something or we let the wave function evolve to an equation that is different form Schrödinger. Rather, different solutions are characterized by what they take as the histories of the primitive ontology of the theory. Therefore, we could label a theory a "Bohmian" solution of the measurement problem if the wave function and the primitive ontology are independent. According to this definition, SM, GRWp, BMW, and BTQFT are, together with BMp, are "Bohmian solutions" of the measurement problem. Instead, we can say that a "GRW" strategy to solve the measurement problem is the one in which the primitive ontology depends on the wave function: this is the case of GRWm, GRWf, Sm and Sf, that all have in common the fact that their primitive ontology is defined in terms of the wave function. Both the flashes and the mass density in these theories are functionals of the wave function. Therefore in BMp, SM, BMW and BTQFT the state is given by $(\mathscr{X}, \Psi)$. This is also the case in GRWf, GRWm, Sf and Sm, where now $\mathscr{X} = F(\Psi)$. In fact, in the latter theories the wave function, strictly speaking, is not the state of the system anymore, if we have to intend "state" as what we need to specify in order to provide a complete description. In fact, there is nothing in Ψ that allows to determine the function F that specifies the primitive ontology.

Note that, because of the similarity between GRWm and GRWf on one hand and Sm and Sf on the other, also GRWm and GRWf can be regarded as many worlds theories, in which the worlds have different "sizes" corresponding to the fact that the collapsed wave function is much bigger than the other terms of the superpositions.

4.8 Supervenience: Logical and Natural

In philosophical jargon, when there is dependence between two variables it is said that the dependent one supervenes of the other. The template for the definition of supervenience is the following: Y supervene on X if no two possible situations are indiscernible with respect to X while differing in Y. For instance, chemical properties supervene on physical properties insofar as any two possible situations that are physically indistinguishable are chemically indistinguishable. One could notice that the mass density and the flashes supervene on the wave function: there cannot be a difference in the mass density or in the flashes without a difference in the wave function. As we saw, this is very different to what happens in Bohmian mechanics, in which positions are specified independently from the wave function. The mass density and the flashes, unlike the positions in BMp, are not be specified in addition to the wave function, but rather are determined by it. Therefore, they are in some respect "hidden variables" of the theories.

Peter Lewis (P. Lewis 2006) has argued that this is an indication of the fact that there is no mass density or flashes ontologically "added" to the wave function, they are just rules of translation from the language of the wave function to ordinary language. In the case of the flashes, he writes:

> the structure of flashes is already present in the wavefunction; flashes are discontinuous localizations in the wavefunction, projected into three-dimensional space. So again, there is no need to postulate the flashes as an addition to our fundamental ontology (P. Lewis 2006).

But this unspecified supervenience it is not enough to arrive to Lewis' conclusion, as we will now see. More precise notions of supervenience can be obtained by filling in this template.

A specification of supervenience that is relevant for our purpose is connected with the distinction between logical (or conceptual) and natural (or nomic) supervenience. Y supervenes logically on X if no two logically possible situations are identical with respect to X but distinct with respect to Y. In its *The Conscious Mind* David Chalmers writes:

> One can think of it loosely as possibility in the broadest sense, corresponding roughly to conceivability, quite unconstrained by the laws of our world. It is useful to think of a logically possible world as a world that it would have been in God's power (hypothetically!) to create, had he so chosen. [...] In determining whether it is logically possible that some statement is true, the constraints are largely conceptual. [...] It should be stressed that the logical supervenience is not defined in terms of deducibility in any system of formal logic. Rather, logical supervenience is defined in terms of logically possible worlds (and individuals), where the notion of a logically possible world is independent of these formal considerations. [...] biological properties supervene logically on physical properties. Even God could not have created a world that was physically identical to ours but biologically distinct. There is simply no logical space for the biological facts to independently vary. When we fix all the physical facts about the world including the facts about the distribution of every last particle across space and time we will in effect also fix the macroscopic shape of all the objects in the world, the way they move and function, the way they physically interact (Chalmers 1996).

In general, when we have logical supervenience between Y and X, we say that X entails or implicates Y, i.e. $Y = f(X)$. To provide an example, the description "table" supervenes logically on the configuration of the particles composing the table: the table *is just* a bunch of particles.

We can also have a supervenience which is not logical: this is the case in which X is always correlated with Y in the natural world. Here is the example that Chalmers provide:

> the pressure exerted by one mole of a gas systematically depends on its temperature and volume according to the law $pV = KT$, where K is a constant. [...] In the actual world, whenever there is a mole of gas at a given temperature and volume, its pressure will be determined: it is empirically impossible that two distinct moles of gas could have the same temperature and volume, but different pressure. It follows that the pressure of a mole of gas supervenes on its temperature and volume in a certain sense. [...] But this supervenience is weaker than logical supervenience. It is logically possible that a mole of gas with a given temperature and volume might have a different pressure; imagine a world in which the gas constant K is larger or smaller, for example. Rather, it is just a fact about nature that there is this correlation. This is an example of natural supervenience of one property on others: in this instance, pressure properties supervene naturally on temperature, volume, and the property of being a mole of gas (Chalmers 1996).

Another example of natural supervenience is this case with respect to entities in a fundamental physical theory, is the one between the charge density ρ and the electromagnetic fields, as pointed out by Tim Maudlin in his *Completeness, Supervenience, and Ontology* (Maudlin 2006). The relation between the two is

$$\rho = \frac{1}{\epsilon_0} \nabla \cdot E \tag{4.20}$$

and it is a law of nature. In general, Y supervenes naturally on X if any two naturally possible situations with indiscernible X have indiscernible Y. A naturally possible situation is one that could actually occur in nature, without violating any natural laws: this is a much stronger constraint than logical possibility. We can think of this kind of possibility as nomic possibility: possibility subject to the laws of nature.

The distinction between logical and natural supervenience can be summarized as follows: if Y supervenes logically on X, then once God has created a world with certain X, the Y comes along for free; if Ysupervenes naturally on X, then after making the X, God had to do more work in order to make the Y: he had to make a law relating the X and the Y. Once the law is defined, X will automatically bring along the Y. But one could, in principle, have had a situation where they did not.

As we have seen above, for example in BMp the state does not supervene on the primitive ontology. In the case of GRWm, GRWf, Sm and Sf insetad the primitive ontology is determined by, supervenes on, the wave function. But what kind of supervenience is that? The primitive ontology naturally, and not logically, supervene on the wave function. That is, it is an additional law of nature. There is nothing that forces us conceptually to choose one law or another, one needs to *posit* one. We will discuss this issue more in Chapter 5.

4.9 Primitive Ontology and Macroscopic Properties

We usually talk in our everyday language about macroscopic bodies and their properties. But what are macroscopic bodies and what are their properties? Our concepts are vague and this is sensible because we want a physical object to be not-so sharply defined in terms of the number of molecules they possess (also because, when we started using the terms we did not know about molecules): "table" still represent a table even if the latter has fewer molecules. This is because macroscopic objects are identified through what they do: "table" refers to a four-legged object with something flat on top that is able to support the weight of other objects on it. In this sense, our definitions of macroscopic objects are also conventional.

As far as their macroscopic properties are concerned, what is the situation? Everybody tends to assign to macroscopic objects peculiar features that seem proper, intrinsic of the object itself. They are what we usually call properties. Common everyday examples of features that are considered properties are mass, weight, color, shape, and so on. But notice that also these properties reflect a particular way of carving up reality, one that is useful to us in some respect: they are also, in this sense, vague and conventional. That is, it is for our convenience that we talk, using vague concepts, in terms of "such and such" an object having "such and such" a property.

What are macroscopic objects and what are their properties in this framework? One of the theses of David Chalmers and Frank Jackson in their *Conceptual Analysis and Reductive Explanation* (Chalmers and Jackson 2001) is that if there is no *a priori* entailment from X to Y (no logical supervenience), then we cannot have a reductive explanation of Y in terms of X. The contrary is true: *a priori* entailment allows for reductive explanation. With *a priori* entailment they mean implication: P implicates Q when $P \supset Q$ is *a priori*. They claim, and I agree with them, that a reductive explanation of statements about macroscopic objects like tables and chairs in terms of the underlying microscopic physical theory of a primitive ontology in three-dimensional space is possible because the microscopic description implicates the macroscopic one.

Chalmers and Jackson argue that conceptual analysis suffices for *a priori* entailment and therefore for reductive explanation. Conceptual analysis is the attempt to figure out the extension of our concepts, including those regarding macroscopic objects. When sufficient information is provided, only reflection is needed to determine the extension of our concepts. This is why they talk about *a priori* entailment. Once we know about atoms, we can determine, by pure reflection, that water is H_2O, that it is liquid at a temperature of 30^o C, that "table" is just a bunch of particles arranged in a particular way, and that it can be vaporized if put at a temperature of 300^oC. As we saw in Section 4.8, since the theory is about the primitive ontology $\mathscr{X}$, macroscopic objects and their properties are functions of it, they logically supervene on the primitive ontology. And therefore they are entailed by the primitive ontology. As such, they are reducible to the description provided in terms of the microscopic fundamental physical theory.

4.9.1 Empirical Equivalence

As we saw above, GRWf can be reformulated so that the wave function evolves linearly, in the usual manner according to Schrödinger's equation, and BMp can be reformulated in terms of a collapsed wave function. These facts indicate that the disagreement between the predictions of the two theories should not be regarded as arising merely from the fact that they involve different wave function evolutions as usually assumed. So where is the empirical disagreement is coming from? As we will see in Chapter 8, the idea is that the source of the empirical disagreement between Bohmian mechanics and GRWf can be regarded as lying, neither in their having different evolutions for the wave function, nor in their having different ontologies, but rather in the presence or absence of equivariance with respect to the Schrödinger evolution. More explicitly, the idea is that a theory is empirically equivalent to the quantum formalism (i.e., that its predictions agree with those of the quantum formalism) if it yields an equivariant distribution (defining typicality) relative to the Schrödinger evolution that can be regarded as "effectively $|\psi|^2$." Let me explain: In this framework, the role of the wave function has immediate consequences for the criteria for the *empirical equivalence* of two theories, i.e., the statement that they make (exactly and always) the same predictions for the outcomes of experiments. In this section I would like to better define what it means for one theory to be empirically equivalent to another.

Before entering into this, I wish to note a couple of remarkable aspects of the notion of empirical equivalence. One is that, despite the difficulty of formulating the empirical content of a theory precisely (a difficulty mainly owed to the vagueness of the notion "macroscopic"), one can sometimes establish the empirical equivalence of theories. Another remarkable aspect is that empirical equivalence occurs at all. One might have expected instead that different theories typically make different predictions, and indeed the theories of classical physics would provide plenty examples. But in quantum mechanics empirical equivalence is a widespread phenomenon; see (GTTZ 2005) for discussion of this point.

Let me turn to the criteria for empirical equivalence. The empirical equivalence of two theories basically amounts to the assertion that the two worlds, governed by the two theories, share the same macroscopic appearance. Therefore, we have to focus on how to read off the macroscopic appearance of a possible world according to a theory. And according to our view about primitive ontology, the macroscopic appearance is a function of the primitive ontology, but not directly a function of the wave function. In cases in which one can deduce the macroscopic appearance of a system from its wave function, this is so only by virtue of a law of the theory implying that this wave function is accompanied by a primitive ontology with a certain macroscopic appearance. In short, empirical equivalence amounts to a statement about the primitive ontology $\mathscr{X}$: $Z = f(\mathscr{X})$. In more detail, the position Z_t of, say, a pointer at time (circa) t is a function of the primitive ontology: In BMp and GRWm it can be regarded as a function $Z_t = Z(Q_t)$ of the configuration, respectively as a function $Z_t = Z(m(x,t))$ of the m field, at time t, whereas in GRWf it is best regarded as a function of the history of flashes over the past millisecond or so.

Note that GRWm and GRWf are empirically equivalent, i.e., they make always and exactly the same predictions for the outcomes of experiments. In other words, there is no experiment we could possibly perform that would tell us whether we are in a GRWm world or in a GRWf world, assuming we are in one of the two. This should be contrasted with the fact that there are possible experiments (though we cannot perform any with present technology) that decide whether we are in a Bohmian world or in a GRW world. The reason is simple. Consider any experiment, which is finished at time t. Consider the same realization of the wave function on the time interval $[0, t]$, but associated with different primitive ontologies in the two worlds. At time t, the result gets written down, encoded in the shape of the ink; more abstractly, the result gets encoded in the position of some macroscopic amount of matter. If in the GRWf ontology, this matter is in position 1, then the flashes must be located in position 1; thus, the collapses are centered at position 1; thus, the wave function is near zero at position 2; thus, the density of matter is low at position 2 and high at position 1; thus, in GRWm the matter is also in position 1, displaying the same result as in the GRWf world.

Concerning the empirical equivalence between a theory and standard quantum mechanics, we need to ask whether the probability of the event $Z_t = z$ agrees with the distribution predicted by standard quantum mechanics. The latter can be obtained from the Schrödinger wave function ψ_t for a sufficiently big system containing the pointer by integrating $|\psi_t|^2$ over all configurations in which the pointer points to z. To understand what that means, we need to understand what it means for a theory to account for certain empirical statistical distributions. This will have to do with the notion of typicality, that I will discuss in Chapter 8.

4.9.2 Macroscopic Properties and The Classical Limit

Macroscopic properties are classical properties, the problem of recovering macroscopic properties from the microscopic fundamental physical theory is, at the end of the day, the problem of the classical limit. In fact, the theory has to explain not only that macroscopic objects have *some* properties, but also that they have the *familiar classical* properties. That is, it is necessary not only to recover from the microphysics that there is a table right here, but it is also necessary to explain why it is, say, solid. In other words, it is important that the theory is able to provide, in the appropriate classical limit, the properties we would assign to macroscopic objects *as if* classical mechanics was true. But also it is important to show that these properties are not destroyed after some time. That is, it is not sufficient to show that there is a solid table at time t, in fact we also need to ensure that this solidity will preserve through time, since this is what actually happens. In other words, not only should we explain the presence of classical properties at a given time, but we also need to explain their stability through time.

Now, let us note that if we are in the framework of theories in which we have histories of primitive ontology in space-time, it is somewhat straightforward what we should intend when we talk about the classical limit: quantum histories and classical histories are "close" to one another (with suitable definition of closeness, see (Allori 2002) for a discussion of the classical limit in the framework of Bohmian mechanics). If instead we think of quantum theories without observers as theories about the wave function, it is much more difficult even to formulate the problem: Where do we start? What is it that is converging to classical trajectories?

The classical limit has been analyzed in the framework of Bohmian mechanics though as a theory about particles. It emerges when the histories of the primitive ontology are generated by a wave function that is a *local plane wave*, see Figure 4.9. This is a wave function with a particular structure of the form

$$\psi(x,t) = R(x,t)\mathrm{e}^{\frac{i}{\hbar}S(x,t)}, \tag{4.21}$$

for suitable R and S, such that it can be thought as a sum of "virtual" wave packets, each of (local) wavelength $\lambda(x,t) \simeq \frac{\hbar}{\nabla S(x,t)}$ which is also equal to its support. Each packet will guide the primitive ontology in such a way that we get the correct classical limit, that is the quantum trajectories converge to the classical trajectories. One can also show that local plane waves get quickly formed in the conditions in which we expect to find a classical limit (namely, when the length of variation of the potential is much bigger that the wavelength of the local plane wave). In addition, in order to prove classical behavior to be stable, one needs to show that the local plane wave structure remain preserved in time and this is where decoherence, the interaction of the system with its environment, comes to help. I think that, arguably, similar kinds of arguments can be used to discuss the classical limit in the framework of quantum theories of histories of primitive ontology in space-time so that one can guarantee that as soon as the local plane wave is formed, it will guide classically the primitive ontology of the theory.

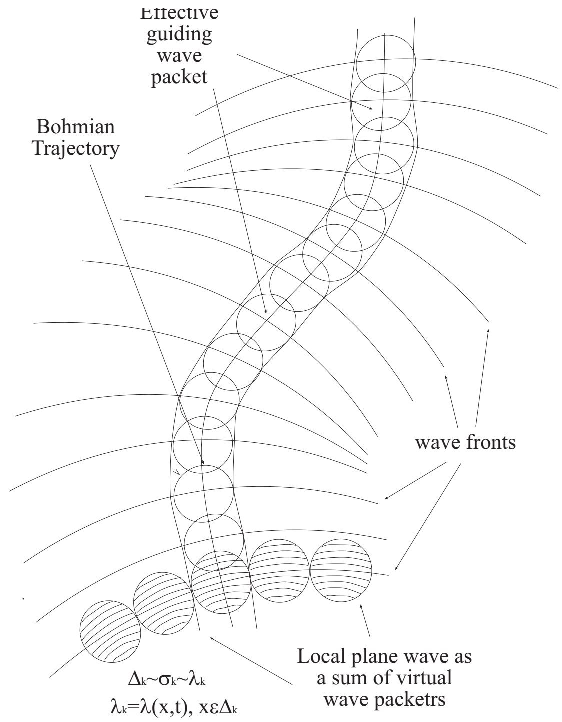

Figure 4.9: The local plane wave structure.

4.10 What is the Wave Function?

Before concluding this chapter, I wish to add a further remark. As we have seen previously, in quantum mechanics the wave function is a ray. That is, ψ and $c\psi$, where c is a nonzero complex number, represent the same physical state. Why we should believe the wave function to be such a mathematical object? As emphasized by Wigner (Wigner 1939), in ordinary quantum mechanics both ψ and by $c\psi$ generate the same transition probability, and therefore, they do not represent physically distinct states. If one considers these probabilities as fundamental, like the Copenhagen interpretation does, then it makes sense that both ψ and $c\psi$ describe the same physical state. In Bohmian mechanics the two wave functions generate the same velocity field for the particles, and in GRW, the same can be noticed (see (Allori et al. 2007)). In all cases, ψ and $c\psi$ describe the same physics, and this justifies the mathematical nature of the wave function as a ray.

This suggests also that the wave function is like a gauge potential. It is similar in this respect to the A and ϕ gauge fields that appear in classical electrodynamics. That is, the wave function is what generates the correct histories of the primitive ontology, just like A and ψ generates in classical electrodynamics. Indeed, since the wave function a ray, it is defined up to a time gauge: ψ and $c\psi$ represents the same state and c can depend on time. This expresses a sort of temporal gauge invariance. From quantum mechanics we know that the time evolution of the wave function is given by a linear equation in Hilbert space, namely Schrödinger's equation. But this is only a matter of convenience and not justified by the geometrical nature of the entity that represents the state. In other words, Schrödinger's equation gives the evolution of the wave function only in a particular time gauge, the one in which the time evolution is linear. To put it differently, the wave function does not necessarily have a deterministic behavior, being it similar to a classical gauge field, until one fixes the time gauge. As in classical electrodynamics one chooses a gauge in order to fix the potentials, the same has to be done in quantum mechanics such that it is possible to write a deterministic (linear) equation for the wave function. One difference between quantum mechanics and classical electrodynamics is that while in classical electrodynamics Maxwell's equations of motion (in addition to Newton's law) are for the electromagnetic fields E and B and not in terms of their potentials A and ϕ, in quantum mechanics instead just the opposite is true. The fundamental equation (the only one there is) is in terms of the gauge field ψ. The gauge invariant objects are defined in term of the wave function: in ordinary quantum mechanics this gauge invariant quantity is the transition probability, in Bohmian mechanics instead the invariance quantity is velocity field. In the other theories it would be the quantity that defines the histories of the primitive ontology.

Chapter 5

Quantum Theories without Observer as Theories about the Wave Function

In contrast to what we have seen in the previous chapter, cBM (the view that Bohmian mechanics is about a particle and the wave function in configuration space), GRW∅ (the view that GRW is about the wave function), MW∅ (the view that many worlds is a theory about the wave function) share the following feature: the wave function has not some fancy or weird status, as suggested by what we have seen in the previous chapter. Rather, it is part of the ontology at most fundamental level. That is, f it is the case, it exists as a physical, material field. In cBM, the wave function is part of what constitutes ourselves and the rest of physical objects, together with a particle in configuration space; in GRW∅, and in MW∅, the metaphysics is simpler, since there is only the wave function.

In all these theories, physical space is configuration space and physical objects are made of wave functions. In this framework, three-dimensional space and all the macroscopic world are emergent from the description provided by the wave function. In the case of GRW, also the mass density and the flashes in GRW are "emergent": GRWf and GRWm are not two different theories. Rather, as stressed by Lewis in his *GRW: A Case Study in Quantum Ontology* (P. Lewis 2006), they provide different ways of describing what happens using different languages. Let us analyze how this emergence is supposed to take place.

5.1 Three-Dimensional Space as Emergent

In the framework of the approach according to which the wave function is a matter field, physical space is configuration space, we need some map to recover the appearance of three dimensional space. As we pointed out in Chapter 3, an entity in configuration space does not specify an arrangement of objects in ordinary three-dimensional space. An entity in configuration space is given by specifying the values of $3N$ parameters, but nothing intrinsic to the space specifies which parameters correspond to which particles in three-dimensional space. In order to specify a configuration of particles in three-dimensional space, a particular correspondence between parameters and particles must be added to the wave function representation. As we have discussed previously, Albert proposes that the Hamiltonian is the one that is understood as providing the connection between configuration space and three-dimensional space as a practical matter. A similar way of thinking is the one of Rimini about GRW, one of the proponents of the theory, who provides a similar explanation (Rimini and Nicrosini 2003). It is because the Hamiltonian has the form that it has that the world appears at us as if it were three-dimensional, even if it is not.

Let us now consider the following question: What are the reasons for which the Hamiltonian is the way we write it? It seems rather straightforward to me that the reason we use a certain Hamiltonian H and not another, call it $\tilde{H}$, is that we already assume that physical space is three-dimensional space and H, the actual Hamiltonian of the theory, works better in explaining the behavior of matter in three-dimensional space than $\tilde{H}$, the one that we did not choose. Therefore, it seems that the explanation structure in Albert's view is upside down: is it the structure of the Hamiltonian that explain the appearance of the three-dimensional world or the existence of such a world that explains the Hamiltonian?

In addition, arguments against the possibility for the Hamiltonian to provide the correct rule of correspondence are given by Bradley Monton (Monton 2002). On the one hand, Albert stresses that the Hamiltonian determines that the world appears three-dimensional to its inhabitants, even though such appearances are nonveridical. But Monton responds that

> [...] the naturalness of the correspondence does not get us anywhere. It isnt the case that we can select the natural correspondence and forget about the rest; since there is no three-dimensional space, each correspondence is equally real, or (if you prefer) equally unreal. To say that one correspondence is natural is to make an

> epistemic claim about how we judge correspondences. There is no ontological import to that claim (Monton 2002).

As already noted, this is connected with the problem of the preferred basis: since the wave function can be written in many basis, what determines the preferred basis? What makes that particular basis the "right" choice? Some have replied (see (Barrett 2003) for a review) with some sort of evolutionary argument: evolution has selected us in such a way that it is advantageous for us to have them in that basis. But it is not at all obvious that there is some evolutionary advantage to this.

Others have proposed that decoherence should play a role in this (see (Wallace 2003)) but, as already noticed, this is very much a work in progress.

5.2 Macroscopic World as Emergent

In addition to a map that should be used to explain why we believe we are in a world that is three-dimensional, we also need some account of the properties of the objects we see all around us. This is connected to the so called problem of the tails discussed in (Albert and Loewer 1996) and summarized previously. In the case of GRW∅, as we saw, Albert and Loewer suggest that the so called eigenvalue-eigenstate link cannot account for the properties that we observe in the framework of GRW theory, since the wave function after the random collapse would not be an eigenstate of any operator. Their proposal therefore is that we should use a different rule of correspondence, which has been called the fuzzy link, according to which if the square of the wave function as a function of $x \in \mathbb{R}^3$ integrated in a certain region S is "sufficiently" close to 1, then we are entitled to say that there is a "particle" localized in region S, where "sufficiently" is characterized by a certain parameter p, such that $1 - p \sim 1$. In this way, they say, it is possible to recover what we usually mean when we talk about localizable objects on the macroscopic level and the appearances of those objects to be localized while they are not.

Peter Lewis discusses what he calls the problem of interpretation of spontaneous collapse theories (P. Lewis 2006). He starts from the problem that, if the wave function constitutes physical objects and if it randomly and spontaneously collapses, we need to account of why we perceive physical objects having certain properties. This is the problem of the tails:

> In its most general form, the worry is that the GRW theory cannot, after all, ensure that objects have the determinate properties that we observe them to have, since determinate properties require that the wavefunction is an eigenstate of the relevant operator, and the GRW theory does not in general yield such eigenstates. But this is just the measurement problem all over again; the tails problem calls into question the GRW theory's claim to solve the measurement problem in the first place.

In the case of GRW, three different links, he says, have been proposed to solve this problem: the fuzzy link, the mass density link, and the flashy link. They provide a rule of correspondence between the descriptions provided in terms of the language of the wave function and our everyday description. In other words, they are translation rules. Lewis notices that that one can take these links to be ontologies rather than rules of translation. The mass density and the flash link are alternative links to the fuzzy link.

The crucial thing to notice is that all these links are not additional ontologies in any way: they do not represent anything in the physical world. Rather, they are just practical rules that should be added for epistemic purposes. Therefore, in this framework, since the links provide just different translations, GRWm, GRWf and fuzzy GRW are not really different theories. In addition, it does not matter that these rules are not precise: it is just a matter of understanding how appearances of the world arise from a fundamental physical theory, how we can match the language that we use every day to describe the physical objects with the description provided by the theory.

This approach is completely different to the approach based on the notion of primitive ontology: in GRWf and GRWm, an ontological rather than practical rule has been introduced. It is not just a rule to recover the three-dimensional space out of the configuration space but it is much more that that: what is defined by this new rule is what the theory is about, it is what tables and chairs are made of. Specifying that "rule" amounts to specifying the primitive ontology of the theory.

The fuzzy link has never been considered as an ontology, so Lewis focuses only on the mass density link and on the flashy link. He provides two arguments, and none of them is very convincing in my opinion, that we should regard these links as practical and not ontological. The first argument he provides against these translation rules being ontologies is that the mass density link and flashy link supervene on the wave function. Because of this supevenience, there is no real need to add them over and above the wave function. I think that this argument is unconvincing: first of all, the fact that we have (unspecified) supervenience of the mass density and the flashes on the wave function does not have any bearing on them being additional ontology, as we have already discussed in the previous chapter. We will come back to it also later in this chapter, in the section about reductionism. One might say that we would not be having the problem of the tails if we would drop the idea that the wave function constitutes physical objects. Lewis' second argument is that there seem to be no reason to do so: after all, it is part of the dynamical laws. Also, I hope that the previous chapter would have been sufficient to show how this is false.

Lewis, and Albert and Loewer have discussed the problem of the tails in the case of GRW∅, but, as we saw, also cBM and MW∅ have the problem of the tails and therefore need a "link". A link should be proposed also for MW∅, otherwise it is not clear what properties of macroscopic objects correspond to. From what we have seen before, we have different possible links for MW: the fuzzy link corresponds to MW∅, the mass density link corresponds to Sm, and theflash link to Sf. Again, if we take them to be rules of translations, they are not different theories. Also in BMc we should provide links. The fuzzy link seems to be appropriate. What about the possible descriptions in terms of the other possible links, the mass density and the flashes? In any case, they are just different descriptions of the same theory, BMc.

David Albert [1] argues that it is possible for these links to emerge from the description in terms of the wave function, but this is unclear how it is supposed to be accomplished.

5.3 Many Worlds and the Stability of Macroscopic Properties

Supporters of MW∅ have emphasized that, in order for the theory to be sensible, it needs to account for the fact that the properties that we attribute to the macroscopic

[1] Private communication

objects stays the same in time. That is, we need to guarantee the stability of the classical world. That would amount to guarantee, in MW∅, that, since objects are patterns defined by the wave function, the different branches of the wave function do not interfere anymore after a while. This is supposed to be accomplished through the effect of decoherence: after a short time, the interference terms go to zero so that we can consider the two branches as constituting different worlds.

But if objects are defined as above, then all the bare theories described here (GRW∅, MW∅, and arguably BMc) are many worlds theories. As emphasized by Alberto Cordero in his *Are GRW Tails as Bad as They Say?* (Cordero 1999), in theories like GRW∅, when the wave function collapses due to the stochastic evolution, the small piece corresponding to the collapsed piece of the superposition still remains, and a small piece is still a piece. So why is that the small term in the wave function does not represent physical objects, if it possesses all the same features as the big one? In all these theories we need decoherence to separate the branches and guarantee the stability of the classical world.

In cBM, since we deny the particles trajectory a primitive status, one can arrive to the conclusion that the trajectories have no influence on the outcomes of experiments and are thus superfluous. In addition, since the wave function does not collapse, one would then have to regard, as one is supposed to in the many worlds view, all outcomes of a quantum measurement as being realized. So also in cBM we need decoherence. An argument that the trajectories are not really necessary has been made in the already mentioned paper by Brown and Wallace (Brown and Wallace 2005). They take up an argument by David Deutch in which he claimed that, since the way in which Bohmian mechanics solves the measurement problem does not involve the corpuscles in any way and therefore, the particles are not really needed and they are redundant. As a consequence, we can think of

> pilot-wave theories are parallel-universes theories in a state of chronic denial (Deutch 1996).

Wallace and Brown continue noticing that Everett, since it has a simpler structure, is a better solution of the measurement problem when compared with Bohmian mechanics.

In detail, here is the argument:

1. In the case of one particle, the wave function determines the actual measurement result (what Brown and Wallace call "Bohm's result assumption"),
2. If [1] is true for one particle, then it is true also for N particles.
3. Therefore, the particle does not serve any purpose.

Obviously, if one instead insists that in Bohmian mechanics the wave function is not a physical object, the argument falls apart. Peter Lewis in (P. Lewis 2007) takes on this challenge and proposes to hold that the wave function is physical but that "empty" branches are different from "full" ones. But if we identify macroscopic objects with patterns of behavior, as Wallace suggests (Wallace 2003) following Daniel Dennett (Dennett 1991), then empty and full waves seem to instantiate the same pattern. If that is true, then what is the difference introduced by the particle? A possibility is to stipulate that empty waves and waves with particles do not instantiate the same pattern and therefore represent different macroscopic objects. They would represent the same object if we identify patterns with what they do, but we can stipulate differently what patterns are identified by.

5.4 Configuration Space: Wild, Wild Metaphysics

It is crucial to notice that the picture of the world that this view gives us is very radical: for example, if GRW∅ is true as complete description of the world, then the world is composed by the wave function, which lives in this very highly dimensional space. The same is the case for MW∅. And if cBM is true, then there are a particle and the wave function, both in configuration space. And all the rest is there: all the complexity, all the variety, all the identity, all the multiplicity of things is in this value: $\psi(q)$, where $q \in \mathbb{R}^{3N}$ is the configuration of the universe. Tables and chairs, apples and planets, reptiles and cats, humans and Martians, ... : they are not made of particles, they are not made of fields in $\mathbb{R}^3$, rather they are all there, together, described by the wave function in $\mathbb{R}^{3N}$. You, me, our friends and families, serial killers, Mother Theresa, George Bush, ... : we are all there. As also Monton (Monton 2002) points out, this view is even more radical than the "brain in the vat" scenario: at least in that case brains are in space-time, while in this view there are basically no brains as we think of. There are no individuals at all! I believe I am in a three-dimensional space, but I am mistaken; I believe that there are objects separated from me, but I am mistaken; I believe that when I say "there is a table over there" I am saying something (at least in a reasonable sense) true about the world, but I am mistaken; I believe I have an identity as an individual, but I am mistaken; I believe that I am having individual experiences, but I am mistaken; I believe that I am using the word "I" coherently, but I am mistaken! That is, I am (and you are) mistaken about the whole entirety of my beliefs, including the one that Descartes was telling us that I cannot be mistaken about, namely that there is somebody who thinks! And this is one of the least believable thing of all, since it seems clear, starting from Descartes, that I can put into doubt almost anything but the fact that I have experiences!

Now, as a matter of methodology, as we did in the case of Wigner's solution to the measurement problem, I think that before accepting some radical view we should establish whether or not there are strong reasons to really reject more "natural", less revisionary perspectives. Or, if we can gain some further understanding adopting the more radical view. Albert's theory seems far too radical than is needed: it is possible that the world is like he claims it is but but there seems to be no reason to believe it is the case. In fact, it seems we can perfectly do without it: what is so wrong with the alternative view that the world *is* actually three-dimensional to justify the acceptance of such a revisionary view of the world? I do not see any justification. What further understanding is gained adopting this new view? I cannot see anything either. Rather, just the opposite: if we adopt this radical view we need to supplement what we have so far with further explanations.

More work needs to be done, and the amount would be enormous. In fact, as we saw in the previous section, Albert's approach with the use of the Hamiltonian is primarily concerned with how to explain the (false) appearances of the world around us as three-dimensional given that the world is actually $\mathbb{R}^{3N}$. But before explaining why my perceptions mislead me in thinking I live in $\mathbb{R}^3$, in this theory I should explain why my perceptions mislead me in thinking I am myself and I have perceptions at all. In fact, as we have pointed out above, there are no "individuals" of any kind in the theory, just the big, holistic, highly dimensional wave function. In contrast, according to the alternative based on a primitive ontology in space-time, the world *is* three-dimensional, we are three-dimensional objects, and our individual perceptions about them are not put into question. In fact the idea behind these theories is that starting from a microscopic ontology in $\mathbb{R}^3$ as the physical space, we would end up recovering all macroscopic physical properties (temperature of a gas, ductility of a metal, elasticity of material, transparency of glass, and so on). If we adopt this approach we do not have to worry about explaining misleading perceptions. We are still left with the gap between the physical and the mental, but whether one considers this gap in principle unclosable by physics or not, it has no implication in physics as we know it today, since physics does not directly talk about perceptions. This is the main reason for which it seems more sensible to choose right from the beginning a primitive ontology in three-dimensional space to explain the behavior of macroscopic in three-dimensional physical objects instead of having a primitive ontology in some abstract space and then be obliged to derive my own (mistaken) perceptions of the world. As a consequence, a necessary condition for an adequate primitive ontology is that it is three-dimensional space.

To put it differently, there are two problems: first, we want to explain the behavior of macroscopic objects in three-dimensional space in term of the motion of microscopic objects in three-dimensional space. Then the second problem is to try to explain why we have the perceptions that we have, given that the physics is what it is. I think physics should deal with the first problem, the second problem being the mind-body problem. If a fundamental physical theory is intended to be about a primitive ontology in three-dimensional space, the mind-body problem is left to a (future) theory of consciousness or a more complex physics. Once we have left perceptions out, we can dedicate physics to the description and the explanation of the motion of bodies in three-dimensional space. On the wave function approach, in contrast, we have a completely different view of what physics is supposed to be: physics is required to explain right away the origin of perceptions in order to start explaining everything else. Physics as we know it and the theory of consciousness are completely merged in this approach and therefore everything becomes more difficult at any level, since no one has a theory of consciousness. Notice that what it is claimed here is not that we should not aim at such a complete theory. But given that the physical world is or seems to be causally closed (given that we have been able to do physics until now), we can do physics without having to have a theory of consciousness. What is gained, then, not to do that and entertain the wave function view? To put it differently, I agree with Chalmers when he writes:

> [...] heat is naturally construed as the cause of heat sensations. Does this mean that we have to explain heat sensations before we can explain heat? Of course we have no good account of heat sensations (or of experience generally), so what happens is practice is that that part of the phenomenon is left unexplained. [...] To be sure, no explanation of heat will be complete until we have an account of how that causal connection works, but the incomplete account is good enough for most purposes. It is somewhat paradoxical that we end up explaining almost everything about a phenomenon except for the details of how it affects our phenomenology, but it is no problem in practice. It would not be an happy state of affairs if we had to put the rest of science on hold until we had a theory of consciousness (Chalmers 1996).

As also emphasized by Maudlin (Maudlin 2006), we do not need to mention the mind-body problem in classical mechanics because the evidence is stated in the language of local physical fact and not in the language of experience. All you need to do is to explain how experiences, say, *as of* a rock are experiences of a rock and then physics can take care of that. If we don't have a primitive ontology in space-time, the only connection can be made at the level of experiences. So you cannot avoid to discuss about how conscious experiences come about. Indeed, if we don't have a primitive ontology in three-dimensional space, physics would be very different than what it is right now.

One should also notice that, if one considers the gap between the mental and the physical as unclosable in physics (as, for example, a particle dualist would think), then the approach of the quantum theories about the wave function will *never* be successful: one needs to be a physicalist in order to believe in cBM, GRW∅ and MW∅. If one really wishes to insist that consciousness has a role in physics, a theory like Wigner's or the many minds theory seem more adequate. In contrast, a non-physicalist that wishes to keep consciousness out of physics would for sure prefer a quantum theory without observer based on a primitive ontology in three-dimensional space as described in the previous chapter.

5.4.1 Is Configuration Space really $3N$ Dimensional?

Incidentally, I would like to note that Peter Lewis (P. Lewis 2004) provides a reply to the objection that the wave function ontology is vastly revisionary and that has to recover three-dimensional space since the real space is $\mathbb{R}^{3N}$. His strategy, as I take it, is to deny that, for all practical purposes, physical space is configuration space. Rather, also the wave function can be suitably identified as living in $\mathbb{R}^3$, even if, in some respect, it can be thought as living in $\mathbb{R}^{3N}$. That is, there is a sense in which the wave function lives in three-dimensional space and another one according to which it lives in configuration space. I find this argument very unconvincing, but for completeness, here it is:

1. The Hamiltonian H has to be invariant under choice of coordinate system, so its sufficient to specify the origin and the three axes,

2. H has to be invariant under arbitrary origin shifting (so that we can pick up any location to be the origin): this would represent an N-particle system where all the particles have some position in $\mathbb{R}^3$,

3. In order for H to be invariant, the only possible transformations are the one that perform a transformation on the triplets in Alberts grouping,

4. Therefore the wave function lives in $\mathbb{R}^3$ if we define dimensionality in terms of possible coordinate transformation and not in terms of number of parameters.

As Monton in his *Quantum Mechanics and 3N-Dimensional Space* (Monton 2006) replies to Lewis, in the argument above [4] does not follow from [3] since the definition of dimensionality provided by Lewis does not capture our standard concept of

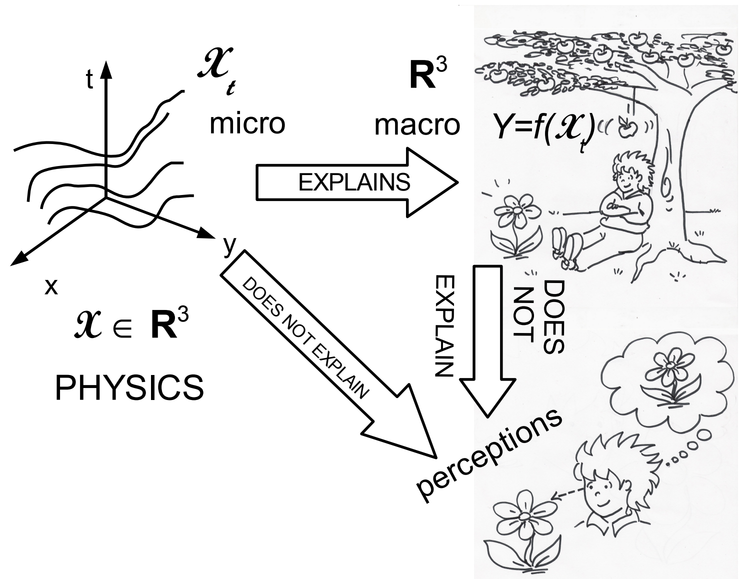

Figure 5.1: Explanation in quantum theories about a primitive ontology in three-dimensional space.

dimensionality. According to Monton, the correct notion of dimensionality is constructive: we first start from the the point, which is the 0-dimensional primitive object, and then we construct on it the higher dimensions. Dimensionality is a metaphysical matter while Lewis's argument makes it "scientific". My take is that Lewis's approach to dimensionality is just bait and switch: we all know what "dimension" means and what Lewis is doing here is just using the same term with a different meaning. Lewis's strategy solves a real, metaphysical issue with a terminological turn and this is cheating. It is like responding to the problem of evil argument that suffering is not real: who do we think we are convincing? The argument is far fetched and really hard to believe, but this is what needs to be done if we wish to insist that the wave function is a physical field.

5.4.2 Configuration Space and the Space of String Theory

Here is another parenthetical remark. One might think that, given what we concluded above, that is, that three-dimensional space is special, we should also reject string theory. The reasoning might be the following: in string theory physical space is supposed

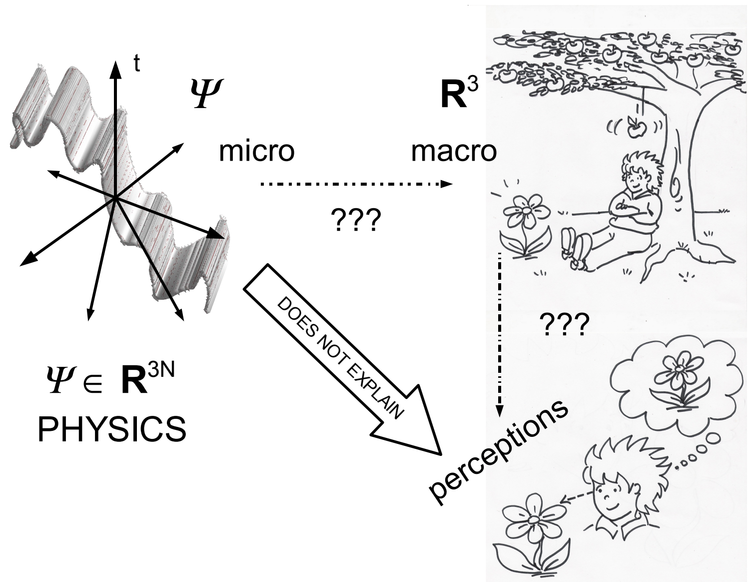

Figure 5.2: Explanation in quantum theories about the wave function.

to have 10 dimensions; in configuration space we have M dimensions, with M some non-prime natural number; 10 is a non-prime natural; therefore, the case of string theory should suffer from the same kind of objections we raised for configuration space.

I do not think this reasoning is correct. In fact in string theory the starting point is that the number of dimensions of physical space is greater than three. At the same time, though, it is assumed that all dimensions except three are compactified and scientists still looks for a mechanism that would explain why that is the case. Indeed, string theorists hope to find a unique way of compactify the extra dimensions, that is, a unique string theory. Then in this case, assuming extra dimensions and then compactifying them, string theory could explain features of our world (such as the vacuum state) that have been left unexplained by the previous theories.

This should make clear that the approach of string theory is completely different from the one of configuration space. In string theory the extra dimensions are added but they are promptly compactified, in order to keep the world *objectively* always like $\mathbb{R}^3$. There is no need to talk about perceptions in order to explain the world as three-dimensional.

In this respect, let me note that one could raise a very similar objection as the one that Cordero had to bare GRW∅ also to GRWm arguing that GRWm is a many worlds theory without the objection being so devastating: the small portion of the collapsed wave function defines a small mass density field that instantiates the same pattern (that is, that describes the same macroscopic object) as the mass density field defined by the big, peaked portion of the wave function. But this is just a counterintuitive consequence of GRWm, not a knock-down critique: as in string theory all of the dimensions except three are "compactified" such that we do not see them, in GRWm there are small copies of objects that always come together with the one we experience.

5.4.3 Configuration Space and Space-Time

I would also like to stress the following. Consider the theory of relativity: one could think that this theory brings about the lesson that space and time are not fundamentally distinct because they appear into the laws in the same manner. But what is the explanation of the fact that we perceive them differently? Einstein warned us that we will never be able to explain, in physics, our sensations, included the one concerning the passing of time. In any case, on the one hand relativity suggests that space and time are of the same kind, on the other it does not explain why we do not perceive them equally. A analogy can be drawn again with quantum mechanics: on one hand it suggests that physical space is $\mathbb{R}^{3N}$, on the other hand it does not explain why we perceive it as $\mathbb{R}^3$. If one accepts the position that in relativity physical space is $\mathbb{R}^4$ and not $\mathbb{R}^3$, then why not accept that in quantum mechanics physical space is $\mathbb{R}^{3N}$?

A possible position is the one of Tim Maudlin in his *Remarks on the Passing of Time* (Maudlin 2002), that rejects both positions: space is three-dimensional and it is fundamentally different from time. A position that might seems a little more difficult to justify is the one I would like to entertain, that is, we can accept that space is fundamentally not distinct from time while we do not accept that physical space is highly dimensional. Indeed, it seems sufficient to recall that in relativity it is always possible to separate space from time and to recognize the objects around us without invoking some theory to explain our perception of time (something that it is not possible to do in the case of physical space being $\mathbb{R}^{3N}$) to stress the difference between the two cases in favor of my position. In other words, given an observer, the metric (which should be included in the primitive ontology of the theory of relativity) gives us the natural separation of $\mathbb{R}^4$ into space and time: for a given observer, there is always a way to tell how things are for a given world line. Therefore, the situation in relativity is, in my opinion, far less extreme and therefore far more acceptable, that the one required by bare theories like GRW$\varnothing$, that is indeed not acceptable.

5.5 Different Kinds of Reductionism

One might asks the following question: What kind of explanation is the one provided by this bare approach? If the world consists only in a field in some high-dimensional space and in its evolution in time, do we consider such a theory a good scientific theory? Do we think it really explains the world around us? As we just saw, taking the wave function as primitive ontology seriously involves a very radical reductionist position: we have to explain our perception of three-dimensional space, the appearance of the existence of tables and chairs, apples and cats as individual and localized objects in that space, from the behavior of the wave function, that lives in a highly dimensional space.

In the previous sections we discussed emergence. When we say that a given macroscopic property Y emerges from the microscopic description provided by X we mean that Y can be recovered from or (suitably) reduced to X. The proponents of bare quantum theories without observer have claimed that the perception of three-dimensional space and the macroscopic world in general emerge from the description provided by the wave function.

While the reductionism of the macroscopic properties seems to be acceptable, the one of the perception of three-dimensional space does not. One might notice that, something similar, it seems, has been accomplished in the case of color: it is not necessary to postulate that red or redness exists, once we have described something red in terms of electromagnetic waves interacting with our retina (even if with that we did not explain why we perceive it as "red"). Calling an object "red" means that the wave length that

	QTWO$\varnothing$	QTWO$\mathscr{X}$
3d world	emergent (H, deco)	fundamental
Macro World	emergent (links)	function of $\mathscr{X}$
Stability of Macro	deco	deco

Table 5.1: Comparison between bare quantum theories without observer (QTWO$\varnothing$) and quantum theories with a three-dimensional primitive ontology (QTWO$\mathscr{X}$).

it reflects belongs to a given interval. The convention is purely pragmatic because that interval is arbitrary: excluding some wavelengths on the boundaries does not make much difference. Why cannot we play a similar game in the case of our perception of three-dimensional space? It might seem that there is no use of postulating that $\mathbb{R}^3$ is the physical space because, as Albert would claim, we can explain why it appears to me like that because of the shape of the Hamiltonian.

While it seems reasonable to postulate that the color can be reduced, the case of the perception of three-dimensional space seems a different kind of reductionism. A color, like "red", is just a macroscopic property, we could have used "round" or "solid". The claim that we can explain them away seems acceptable. In fact, notice that I said "color", not "perception of color". In contrast, the case of the reduction of the perception of color, is of the same degree of magnitude as the reductionism of the perception of three-dimensional space: we have the problem of the explanation of perceptions, the mind-body problem, that suggests that they are not reducible to microphysical facts.

5.5.1 Reductionism of the Macroscopic World

We have seen in Section 4.9 that in the framework of quantum theories with a primitive ontology in three-dimensional space the properties of macroscopic objects can be completely described in terms of the primitive ontology, since there is logical supervenience. Also in the framework of the quantum theories about the wave function in principle we could have a similar logical supervenience and therefore a similar reductionism of the appearances to the wave function, the links discussed above providing the supervenience function. But these links do not express logical supevenience. To understand why it is the case, let is analyze an example of failure of reductive explanation.

5.5.2 The Impossibility of the Reductionism of Perceptions

Let us analyze, as an example, the case of the relation between the mental and the physical. As Chalmers has emphasized in more than one occasion (Chalmers 1996), (Chalmers 2002), there are two problems, on the line of those discussed above: the "easy" problem and the "hard" problem. The easy one amounts to explaining certain cognitive functions, and it seems more like solving a problem rather than a mystery. The hard problem is the problem of experience: Why is the performance of these functions accompanied by experiences? How does the physical give rise to the mental? Philosophers have tried to provide a reductive explanation of consciousness: explaining consciousness in terms of natural principles. We have physicalist solutions, according to which consciousness is reducible to the physical, and we have non-physicalist solutions, according to which consciousness is not physical. Chalmers, among others, argues that we cannot have any reductive explanation of consciousness. Let us consider the "explanatory argument", one of the standard arguments against materialism, that is against the possibility of reductively explain consciousness in terms of physics:

1. Physics accounts only for functions and structure,

2. Functions and structure are not enough to explain consciousness,

3. Therefore the physics cannot explain consciousness and materialism is false.

As Chalmers emphasized (Chalmers 2002), this argument tries to establish an epistemic gap between the physical P and the phenomenal Q: It denies an epistemic entailment from physical truths to phenomenal truths. And once one has established the epistemic gap, one then infers that there is in addition an ontological gap between P and Q:

1. There is an epistemic gap between P and Q,

2. If there is an epistemic gap, there is an ontological gap,

3. Therefore materialism is false.

To say that there is an epistemic gap but not an ontological gap amounts to saying that reductionism is possible, at least *in principle* even if not in practice, while to say that there is also an ontological gap is to deny the possibility of any kind of reductionism. There are standard objections to this argument, that correspond to different materialistic positions: they are classified by Chalmers as Type A, Type B and Type C materialism. According to "Type A" materialism consciousness *is* functional. If Type A materialism is true, then we have entailment from the physical to the phenomenal and therefore we have reductive explanation of the mental in terms of the physical. Another popular answer to the arguments above is to accept that there is an epistemic gap but to deny that there is an ontological gap. This is what is called by Chalmers "Type B" materialism. According to this position, consciousness is not ontologically distinct from the physics. Conscious states are identical to physical or functional states, they are identities like "water = H_2O". They are not derived by analysis but they are discovered empirically, we have *a posteriori* identification. So there is an epistemic gap but not an ontological gap.

Notice that, even if a type B materialist would deny it, if type B materialism is true, we would have failure of reductive explanation of the mental in terms of the physical:

> Let us say that the property dualist is right about consciousness, and that consciousness is connected to the physical only by contingent laws. These laws might be inferred, in principle, from psychophysical regularities in the actual world. Given the presence of these laws, we can still arguably have some sort of explanation of consciousness and its properties, in terms of physical processes and the psychophysical laws. [...] But this will not be a case of reductive explanation, precisely because of the need for principles in our explanatory base over and above what is present in P. The laws themselves are not explained: they are epistemically primitive, in that they are not implied by more basic truths.[...] And these substantive, epistemically primitive principles play a central role in the explanation of the phenomena. So there is no transparent explanation of the phenomena in physical terms alone, and reductive explanation fails (Chalmers and Jackson 2001).

In other words, if type B materialism is true, P implies Q but it is not *a priori.* There are some identities that are not explained, they are epistemically primitive. Ontologically, this view is very similar to type A; while epistemically it is more similar to property dualism. In both cases, we do not have a clear explanation of consciousness.

Ontologically, these primitives could be different from laws, but epistemically they are the same. Calling them "bridge principles" or identities, one might continue to call himself a materialist but the explanatory structure is so far away from the one the materialist would hope for. One could insist that identities do not require explanation but this is a strange position to have: most of the time identities can and indeed are explained, like "water is H_2O". We come to know identities by deducing them, so that:

> If the identities in question were epistemically primitive, then explanations of the macroscopic phenomena in terms of microscopic phenomena would have a primitive "vertical" element, and science would have established a far weaker explanatory connection between the microscopic and the macroscopic than it actually has (Chalmers and Jackson 2001).

For completeness, let me mention the other strategy against the arguments abov, which is to claim that the epistemic gap is closable in principle. This is the position of "Type C" materialism, for example of Thomas Nagel and Colin McGinn. In this view, the unclosability of the gap is apparent, due to our limitations as humans. According to Nagel (Nagel 1974) we need conceptual revolution; according to McGinn (McGinn 1989) we will never be able to close this gap because humans do not, intrinsically, have the right concepts to grasp consciousness. So reductive explanation is possible in principle, but not in practice by us.

This discussion about the philosophy of mind is important for our purposes because if one wishes to insist on the wave function being the primitive ontology of quantum theories without observer, then one would have to solve the hard problem (the problem of explaining experiences) and the easy problem (the problem of explaining the behavior of physical objects in three-dimensional space) at once. Arguments with the same structure as those seen above can be written in the case of the wave function ontology to show that there cannot be reductive explanation of the macroscopic truths in terms of the truths expressed in the language of the wave function. For example, one could write an argument similar to the explanatory argument as follows:

1. The wave function accounts only for the world in configuration space,
2. Our experiences are of three-dimensional space so the wave function alone in configuration space is not enough to explain our experiences,

3. Therefore the wave function cannot explain the world, so wave function monism is false.

This argument shows that we need a rule to connect $\mathbb{R}^{3N}$ to $\mathbb{R}^3$. And, as for the case between physical and the phenomenal, between the wave function picture and the picture that comes to our senses, we have an epistemic gap from which we can infer an ontological gap:

1. There is an epistemic gap between P (the picture of the world provided by the wave function) and Q (the picture of the world provided by our experiences),

2. If there is an epistemic gap, there is an ontological gap,

3. Therefore, wave function monism is false.

If we recognize that there is an ontological gap, it would amount to considering the "links" discussed above as ontologies, not just pragmatic rules. As in the case of materialism, one could deny there is an epistemic gap (type A), one could accept there is this gap but deny that there is an ontological gap (type B or C). That is, people that hold the wave function ontology view and deny that we need anything more than that might be compared with the Type A materialist. A problem with this view is that it denies the obvious: as in the case of consciousness, there is something to be explained!

The view of those people like Albert, Loewer, Rimini, and Lewis discussed above seems to be close to type B materialism, since they recognize that there is something to explain (an epistemic gap) and this is because we need to add additional information to the one provided by the wave function, without giving to these rules of correspondence an ontological status. There is, as there is for type B materialism, one big worry for this view. Type B materialists claim that the perception of three-dimensional space is in the wave function in the same way that water is H_2O. Since the latter identity is *a posteriori*, so it is the former. But this analogy does not seem to be a good one: for example, there are not any epistemic arguments for water but there are for the wave function, as the argument above shows. In fact, there is no epistemic gap between water and H_20, given the complete physical description and the truth about water, while there is in the case of the wave function. The identity "water=H_20" is empirical but not primitive, since it can be deduced from the compete physical description. Type B materialists instead have to hold that the gap is primitive otherwise type A materialism would be true instead. The very same thing can be said also in the case of the wave function ontology: the rules of correspondence between wave function description and macroscopic description are *a posteriori* but not primitive. But what kind of identity is one in which there is an epistemic gap? This what is usually found in the laws of nature: to label these links identities and not laws might allow to maintain the label of materialism but this sacrifices most of the spirit of materialism or reductionism. One could insist that identities do not need explanation but in some cases they can be explained (like "water=H_2O") and so this suggests that all of them could and should be explained.

5.6 The Wave Function and Symmetry Properties

If the wave function is what quantum theories without observer are about, then they turn out not to have any symmetry property. Consider the case of Galilean invariance. There seems to be no good reason to justify any other transformation for the wave function under Galilean transformations other than that of a scalar field, given that, mathematically, it seems to be such an object. In other words, the transformation that one finds in physics books according to which the wave function transforms by multiplication of some strange exponential factor is simply unjustified. And if this is the case, that is, if the wave function transforms as a scalar field, then the Schrödinger's evolution is not Galilean invariant.

Here is the argument: Consider an N particles system evolving freely. The non relativistic Schrödinger equation is the law of the time evolution of a $N+1$-component scalar field ψ in three-dimensional space. It seems part of thinking of ψ as a concrete classical physical object to suppose that the way it transforms under a pure Galilean transformation of magnitude v is

$$\psi(q_1, ..., q_N, t) \rightarrow \psi(q_1 + vt, ..., q_N + vt, t), \tag{5.1}$$

and not

$$\psi(q_1, ..., q_N, t) \to \exp\left[\frac{i}{\hbar}\sum_{k=1}^{N} m_k \left(q_k \cdot v - \frac{1}{2}v^2 t\right)\right]\psi(q_1, ..., q_N, t), \tag{5.2}$$

as usually assumed. Note that if $\psi(q,t)$ is a solution to the free nonrelativistic Schrödinger equation, then $\psi(q+vt,t)$ is not. And so the corresponding Schrödinger equation (construed as a theory of the motions of a classical scalar field) is not invariant under Galilean transformations.

My first remark is that, the wave function in quantum mechanics is a ray or direction in Hilbert space. This means that the wave functions ψ and $c\psi$, where c is a non zero complex number, represent the same physical state. While if one believes that the wave function is part of the primitive ontology of the theory, then ψ is not defined up to a phase and it is indeed more like a scalar field. Let us suppose the transformation of a scalar field under Galilean transformation defined by equation (5.1) is the correct one. In the framework of a quantum theory of a primitive ontology in three-dimensional space, the wave function, as we saw in Chapter 4, is naturally a ray. The natural transformation of the ray under a Galilean transformation, since it is a projective object, can be determined by analyzing the projective representation of the Galilei group (see (Lévy-Leblond 1971)). There are five unitary and projective representations, among which only one is chosen to be the "natural" transformation, the one that leaves the physics the same. This transformation is exactly (5.2), the one that one finds in physics books. As a consequence the theory is Galilean invariant.

Since the wave function is a matter scalar field, the bare theories are not Galilean invariant. As a consequence, they provide the wrong results in the classical limit. Consider the example of a free particle on a line. Usually in ordinary quantum mechanics it is described by a wave packet. To simplify calculations, consider the case of a Gaussian wave packet. In taking the classical limit one should obtain results that are compatible with the Galilean invariance of classical mechanics. The point is that the transformation (5.1) gives the wrong results for the classical velocity, assuming that Newtonian mechanics is Galilean invariant. In order to obtain the correct result one has to adopt a different transformation, namely the transformation we find in physics books. Consider a (one dimensional) Gaussian wave packet centered in $x_0 = 0$ at time $t = 0$

$$\psi(q,0) = \frac{1}{(2\pi\sigma_0)^{1/4}} \exp\left[-\frac{q^2}{4\sigma_0^2} + i\frac{mu}{\hbar}q\right] \tag{5.3}$$

(Note that in the previous discussion we always had $\hbar = 1$ for convenience. Now, given that we will take the classical limit, we will have to reintroduce the parameter.) In the expression, σ_0 is the initial spreading of the packet and u is the velocity. At time t the packet has evolved to the form

$$\psi(q,t) = \frac{1}{\left[\sqrt{2\pi}\sigma_0\left(1+\frac{i\hbar t}{2m\sigma_0^2}\right)\right]^{1/2}} \exp\left[-\frac{(q-ut)^2}{4\sigma_0^2\left(1+\frac{i\hbar t}{2m\sigma_0^2}\right)} + i\frac{mu}{\hbar}\left(q-\frac{u}{2}t\right)\right] \tag{5.4}$$

We can write it as

$$\psi(q,t) = R(q,t)\mathrm{e}^{\frac{i}{\hbar}S(q,t)}, \tag{5.5}$$

where S is given by

$$S(q,t) = muq - \frac{1}{2}mu^2 t + \frac{1}{2}\left(\frac{\hbar t}{2m\sigma_0^2}\right)^2 \frac{(q-ut)^2\,\frac{m}{t}}{1+\left(\frac{\hbar t}{2m\sigma_0^2}\right)^2} \tag{5.6}$$

and R by

$$R(q,t) = \frac{1}{\left[\sqrt{2\pi}\sigma_0\left(1+\frac{i\hbar t}{2m\sigma_0^2}\right)\right]^{1/2}} \exp\left\{-\frac{(q-ut)^2}{4\sigma_0^2\left(1+\left[\frac{\hbar t}{2m\sigma_0^2}\right]^2\right)}\right\}. \tag{5.7}$$

According to the general formulation of the classical limit in the framework of orthodox quantum mechanics (Maslov 1981), the classical velocity associated to the particle is given by

$$V = \frac{\nabla S^0}{m}, \tag{5.8}$$

where S^0 is obtained by S in the limit "$\hbar \to 0$"[2]. In the case of a free Gaussian wave

[2]This limit should be intended as the mathematical implementation of considering a situation in which the relevant action of the system is big compared to $\hbar$.

packet, we have

$$S^0(q,t) = muq - \frac{1}{2}mu^2t + O(\hbar^2), \tag{5.9}$$

and therefore

$$V = u. \tag{5.10}$$

If this is the velocity of the particle associated to the packet in the classical limit, we should expect that it would transform under a pure Galilean transformation $g = (0,0,v,1)$ (in one spatial dimension) of velocity v corresponding to $V \to V + v$, since classical mechanics is Galielan invariant. This is not the case if the wave function transforms according to (5.1). In fact suppose that under the transformation $x \to q + vt$ the wave function transforms as $\psi(q,t) \to \psi(q+vt)$. Then, the action S will transform as

$$S(q,t) \to S(q+vt,t) \tag{5.11}$$

where $S(q+vt)$ is

$$S(q+vt,t) = muq - \frac{1}{2}mu^2t + \frac{1}{2}\left(\frac{\hbar t}{2m\sigma_0^2}\right)^2 \frac{[q-(u-v)t]^2\,\frac{m}{t}}{1+\left(\frac{\hbar t}{2m\sigma_0^2}\right)^2}. \tag{5.12}$$

In the classical limit it becomes (up to a factor $O(\hbar^2)$)

$$S^0(q+vt,t) = muq - \frac{1}{2}mu^2 \tag{5.13}$$

that is , V is

$$V = u, \tag{5.14}$$

that is not the expected result $u + v$.

In contrast, suppose the wave function transforms under a pure one dimensional Galilean transformation according to

$$\psi(q,t) \to \exp\left[\frac{i}{\hbar}\left(mvq + \frac{1}{2}mv^2t\right)\right]\psi(q,t). \tag{5.15}$$

We can see that with that transformation the correct classical limit is now obtained.

In fact the action S transforms as

$$S(q,t) \to m(u+v)q - \frac{m}{2}\left(u^2 - v^2\right)t + \frac{1}{2}\left(\frac{\hbar t}{2m\sigma_0^2}\right)^2 \frac{(q-ut)^2\,\frac{m}{t}}{1+\left(\frac{\hbar t}{2m\sigma_0^2}\right)^2}. \tag{5.16}$$

that gives in the classical limit (up to a factor $O(\hbar^2)$)

$$S^0(q,t) = m(u+v)q - \frac{m}{2}\left(u^2 + v^2\right)t \tag{5.17}$$

and therefore

$$V = u + v \tag{5.18}$$

as it is expected.

Chapter 6

From Local Beables to Primitive Ontology: The Case of Classical Electrodynamics

In the framework of quantum theories without observer, one of the options was to consider the theories to be about the wave function. We have seen this view is problematical, mostly because of the fact that the wave function lives in the (highly dimensional) configuration space. Therefore I argued that the alternative, the view that the wave function is not part of the primitive ontology but rather it is part of the law that generates the motions of the primitive ontology, has to be preferred. That is, an adequate primitive ontology is one in three-dimensional space.

Let us now consider a different example. In classical electrodynamics we have, as mathematical variables, both particles and electromagnetic fields. What is the primitive ontology of the theory? One possibility would be to say that they are both part of the primitive ontology. Such a view would not have the same problems as the corresponding view in the framework of quantum theories without observer regarding the wave function as a primitive ontology, since in this case the electromagnetic fields are in $\mathbb{R}^3$. I wish to argue in this chapter that we should regard nonetheless the electromagnetic fields not as part of the primitive ontology, even if they live in a morefamiliar space than configuration space and they are local beables in space-time. This is where the notions of local beables and primitive ontology come apart. One might still be puzzled: What is the difference between the particles and the fields in classical electrodynamics? After all, since they both live in $\mathbb{R}^3$ they both seem to be possible candidates to be part of the primitive ontology, they both seem to physically exist. Wouldn't differentiating among them amount to saying, similarly to what we have discussed in the framework of quantum theories without observer, that there are different degrees of reality? How exactly is the notion of primitive ontology different from that of local beable? What is the crucial feature of primitive ontology if it isn't the fact that it is localizable in a certain space-time region? The notion of local beable focuses on the localizability in a bounded region of space-time, while for the notion of primitive ontology, even if it also takes three-dimensional space as fundamental, this is not sufficient. What is also needed to be an appropriate primitive ontology is also to have a suitable role in the structure of the fundamental physical theory. Let us see this in the framework of classical electrodynamics.

6.1 Time Reversal in Classical Electrodynamics

There seems to be a connection between primitive ontology and symmetry properties of the theory. We have seen in Chapter 5 that if we take the wave function as part of the primitive ontology of a quantum theory without observer, it turns out to be not Galilean invariant. Now, if one takes seriously the idea of the electromagnetic fields being part of the primitive ontology of classical electrodynamics, we will see that the theory is not symmetric under time reversal transformation. In contrast, if one takes the particles but not the fields as the primitive ontology, one can make sense very naturally of the fact that the electromagnetic fields transform, under a given symmetry transformation, in a way that does not seem to be that straightforward, if one just looks to what kind of mathematical objects they are.

Consider the recent debate about classical electrodynamics. In his book *Time and Chance* (Albert 2001), Albert has suggested that, in contrast to what commonly believed, classical electrodynamics is not a time reversal invariant theory. The transformation T of time reversal is such that T acts on a solution of the theory x_t such that $T(x_t) = x_{-t}$. A theory is invariant under time reversal if the time-reversed solution $T(x_t)$ is also a solution. That is, it describes is a possible state of affairs. For classical electrodynamics to be time reversal invariant would require that the magnetic field B flip sign. Albert questions the justification of such a transformation for B, and concludes that classical electrodynamics is not time reversal invariant. Contrary to this point of view, David Malament in his *On the Time Reversal Invariance of Classical*

Electromagnetic Theory (Malament 2004) claims to have found a justification for the transformation of the magnetic field under the transformation of time reversal. In fact he can provide an intrinsic definition for the electromagnetic fields that would account for such a transformation. Therefore, he considers it perfectly justifiable to regard classical electrodynamics as a time reversal invariant theory. Similar arguments are provided by Frank Arntzenius in his *Time Reversal Operation, Representation of the Lorentz Group and the Direction of Time* (Arntzenius 2004). Let s analyze these positions in some detail.

6.1.1 Primitive Ontology and State

In his book Albert says that:

> According to Newtonian mechanics, or at any rate according to the particularly clean and simple version of it that I want to start off here, the physical furniture of the universe consists entirely of point particles. The only *dynamical variables* of such particles - the only physical attributes of such particles that can *change with time* - are (only in this theory) their positions; [...] (Albert 2001).

At page 9 he goes on saying:

> Let's start thinking what it means to give a complete description of the physical situation of the world at an instant. There would seem to be two things you want from a description like that:
>
> [**a**] that it be genuinely *instantaneous* (which is to say that descriptions of the world at different times have the appropriate sort of logical or conceptual or metaphysical *independence* of one another, that a perfectly explicit and intelligible *sense* can be attached to *a temporal sequence* [. . .] as a *story of the physical world*); and
>
> [**b**] that it be complete (which is to say, that *all* the physical facts about the world can be read off from the full temporal set of its descriptions).
>
> [. . .] let us call whatever satisfies [a] and [b] an instantaneous physical *state* of the world (Albert 2001).

In the following he claims that what satisfies the two requisites in the case of Newtonian mechanics is only the particles' positions. He then contrasts this fact with what in the physical literature is usually called the instantaneous state, that is the couple of positions and velocities. According to him, this is not a genuine instantaneous state because actually the positions and velocities of all particles at one time are

> *not* conceptually *independent* of specifications of the positions and velocities of all the particles in the world at all other times (Albert 2001).

while the objects in the instantaneous state (IS) should be, according to his previous definition. At page 17 he adds that

> a *specification* of those positions and velocities at some particular instant is *not* a specification of the world at an instant *alone*— it is not a description of the world at that instant *as opposed to all others*, at all! (Albert 2001)

He continues saying that what is referred into the literature as instantaneous state is actually what could be called the *dynamical condition* (D) at an instant. And this is characterized by providing

> all the information about the instant in question - which or all the information which can in one way or the other be uniquely *attached* to the instant in question - which is required in order to bring the full predictive resources of the *dynamical laws of physics* to bear. (Albert 2001)

The trouble with that is that dynamical conditions at different times, he says, are not logically independent. Therefore they cannot be identified with the instantaneous states of the world. Using the words of Frank Arntzenius:

> [velocity] is a feature of the development of states, it is a property of a finite or infinitesimal history of states, and is not determined by the instantaneous state of the particle (Arntzenius 2004).

According to Albert, in classical mechanics the dynamical condition is given by (q, v), while the instantaneous state is given by q alone. In the terminology I used so far, the instantaneous state IS is the primitive ontology $\mathscr{X}$ while the dynamical condition D includes both the primitive ontology and the nomological variables, that is, it is what I previously called the state of the system. In quantum theories without observer, we have $D = (\mathscr{X}, \Psi)$. In classical mechanics, the instantaneous state (the primitive ontology) is the positions $q = (q_1, ..., q_N)$ of the particles while the dynamical condition (the state) is given by the positions together with the velocities, that is by the couple (q, v).

Albert claims that what is in the instantaneous state should not change under time reversal transformations. That is, the time reversal operator T is supposed to be an operator that leaves the instantaneous state untouched. In the case of classical mechanics this means $T(q(t)) = q(t)$. The velocities, given that they are defined in terms of positions as $v = \frac{dq}{dt}$, transform under time reversal transformations according to their definition, that is

$$T(v) = T\left(\frac{dq(t)}{dt}\right) = -\frac{dq(t)}{dt} = -v. \tag{6.1}$$

Therefore, while the instantaneous state (the primitive ontology) is left unchanged under time reversal transformation T, the dynamical condition (the state) transforms as $T(q, v) = (q, -v)$.

Let us now consider classical electrodynamics. The dynamical condition is given by (q, v), where now in v there are the electromagneic fields. What is the instantaneous state in this case? Albert believes that, in order to have a complete and faithful view of the world, we need to add to the instantaneous state of the worlds also the

> the magnitudes and directions of the electric and magnetic fields at every point in space (Albert 2001).

That is, in the case of classical electrodynamics the instantaneous state is given by the specification of the triplet (q, E, B). This is, in our terminology, the primitive ontology of classical electrodynamics. He then says that

> the magnetic field is not —either logically or conceptually— the *rate of change* of anything (Albert 2001).

Therefore there is no reason whatsoever, according to Albert, that it flips sign under time reversal transformation, as we are taught it does. Time reversal transformation is defined by Albert as follows: if the sequence of states $IS_1, IS_2..., IS_N$ is possible, then also the sequence of states $IS_N, IS_{N-1}..., IS_1$ is also a possible one. In our terminology, if a history of a primitive ontology forward in time $\mathscr{X}_t$ is a possible history, then also the history of the primitive ontology backwards $\mathscr{X}_{-t}$ is a possible history. If the instantaneous state (the primitive ontology) is given by (q, E, B), then classical electrodynamics is not time reversal invariant. It can be, according to Albert, only if we allow for a transformation of the fields that allows for the backward sequence of dynamical conditions, and not for the backward sequence of instantaneous states, to be in accordance with the laws of classical electrodynamics. That is $T(E) = E$ and $T(B) = -B$. According to Albert, instead, we should look to the instantaneous state not to the dynamical conditions and therefore we should drop this transformation for the fields and let the state remain untouched at all. In this way, electrodynamics is not time reversal invariant. In fact there are histories of the primitive ontology, sequences of instantaneous states $IS_1, ..., IS_N$ such that their backward history does not happen.

6.1.2 Intrinsic Geometrical Definition of the Electromagnetic Fields

As I understand it, the reasons why Albert thinks that the electromagnetic fileds must belong to the primitive ontology is connected to the fact that according to Albert, the fields are, unlike velocities, logically independent from positions and therefore have to be added to the instantaneous state in order to complete the picture of the world at one time. Notice though that to say that one needs to add an entity does not mean that we have to admit that it is physically existing, as we saw in the case of quantum theories without observer for the wave function: in BMp, for example, the wave function is needed as much as the particles in order to complete the description but nonetheless it is not part of the primitive ontology.

Be that as it may, a reason to justify the transformation of B into $-B$ under T can be provided, as David Malament has emphasized (Malament 2004). It can be shown in fact that the B field happens to have an intrinsic definition with well defined symmetry properties. Under time reversal transformations, the B field actually flips sign. Arntzenius (Arntzenius 2004) and Malament (Malament 2004) have in fact shown that it is possible to give reasons why magnetic fields change sign under time reversal operations. In fact in their work they have shown that we can understand the symmetry properties of the fields if we consider its intrinsic geometrical definition.

Let us look in particular at the results of Malament. According to him, the elctromagnetic fields are defined as follows. Let us consider a smooth, connected 4-dimensional manifold M and a pseudo-Reinmaninan metric g_{ab} with signature (1,3) on M, i.e. a relativistic space-time. The world line of any (massive) point particle can be described therefore as a smooth curve on M. The electromagnetic force can be represented just by a map from the tangent line to the curve to force vectors, regardless any temporal orientation, in any point:

$$(L, q) \to F(L, q). \tag{6.2}$$

To choose a temporal direction, we take a direction of the 4-velocity v^a. So the force is represented by the map

$$v^a \to F^a(v^a, q). \tag{6.3}$$

In requiring that this map has the desired properties (that is, it is linear in q, the force is orthogonal to the velocity, ...) we get that it has to be represented by an antisymmetric tensor:

$$F(v^a, q) = qF^a_b v^b. \tag{6.4}$$

From F_{ab} and the currents we can recover classical electrodynamics (Lorentz force and Maxwell equations).

The time reversal operation is naturally understood as taking one temporal orientation and comparing it to what happens if we take the opposite choice of the orientation. Given that we have $F(v^a, q) = qF^a_b v^b$ it is straightforward that $T(F_{ab}) = -F_{ab}$ because the choice of an orientation is a choice of the direction of v^a. From which follows (see Malament (Malament 2004)) that Maxwell's equations are time reversal invariant.

Suppose we want to know how fields transform. We can recover Maxwell equations in terms not of F_{ab} but in terms of E and B as soon as we specify additional structure (again, see Malament (Malament 2004) for details). We need a spatial structure to specify B, in addition to a temporal orientation. In this way we see that, under the properly defined time reversal invariance, we get the correct transformations connected with the fact that E turns out to be not a scalar or vector field but a polar vector, and B an axial vector.

6.2 Different Metaphysics for Classical Electrodynamics

Malament does not commit himself in saying what is the primitive ontology of classical electrodynamics. He seems to be saying that there is a way of understanding the transformations of the electromagnetic field according to its intrinsic geometrical definition given above. In particular, Malament's results show us that it is wrong to believe that it is impossible to find a definition of B that accounts for the transformation $(E, B) \to (E, -B)$ under T transformations. Therefore, if we regard B as a part of the primitive ontology, that is, if $\mathscr{X} = IS = (q, E, B)$, we can make sense of its changing sign under time reversal transformation T and classical electrodynamics, as a theory of particles and fields, is time reversal invariant. But under the transformation $\mathscr{X}$ changes, because B gets transformed. This seems to be the position of Arntzenius (Arntzenius 2004). He holds that the state should change under time reversal transformation. Indeed

> the transformations of the fundamental quantities under time reversal (and parity) are not arbitrary but are determined by the type of irreducible representation that they correspond to. These, in turn, determine the transformational properties of all non-fundamental quantities (Arntzenius 2004).

Another possibility is to continue to keep the fields inside the primitive ontology, i.e. $\mathscr{X} = (q, E, B)$, but to reject the definition of B *á la* Malament. In this way, we can leave the primitive ontology unchanged under the transformation, but the theory turns out to be not invariant anymore. This is the position taken by Albert in his book. Notice that it is possible only for a primitive ontology that excludes the fields to be left unchanged under the symmetry transformation, if at the same time we want the theory to be invariant. In other words, if $\mathscr{X} = q$, then the state is unchanged by the transformation, $T(q) = q$, and the theory is invariant. This is the view that I will defend.

To sum up, there are three alternatives for the metaphysics of classical electrodynamics. Depending on what we choose to be in the primitive ontology and how it should behave, we have different symmetry properties. Here are the different possibilities:

[**a**] Classical electrodynamics is a theory of particles and fields. That is, the primitive

ontology (the instantaneous state) of the theory is $\mathscr{X} = (q, E, B)$, when E and B are scalar fields. The primitive ontology is untouched by the transformation so that $T(\mathscr{X}) = T(q, E, B) = (q, E, B) = \mathscr{X}$. The theory is therefore not invariant. While there is a possible history $\mathscr{X}_t$, the time reversed one, $T(\mathscr{X}_t)$, is not possible.

[**b**] Classical electrodynamics is a theory of particles and fields. That is, the primitive ontology (the instantaneous state) of the theory is $\mathscr{X} = (q, E, B)$. B is intrinsically defined as such that the primitive ontology naturally transforms under T as $T(\mathscr{X}) = T(q, E, B) = (q, E, -B) = \mathscr{X}' \neq \mathscr{X}$. That is, the primitive ontology changes under a symmetry and the theory is invariant: $\mathscr{X}_t$ and the time-reversed $T(\mathscr{X})$ are both possible histories.

[**c**] Classical electrodynamics is a theory of particles. That is, the primitive ontology (the instantaneous state) of the theory is $\mathscr{X} = q$. In order to complete the description, we need to specify also the fields, but they are not part of the primitive ontology. They are just part of the state. The primitive ontology is untouched under the symmetry so that $T(\mathscr{X}) = T(q) = q = \mathscr{X}$, and the theory is invariant: both $\mathscr{X}_t$ and its time-reversed $T(\mathscr{X}_t) = \mathscr{X}_{-t}$ are both possible histories.

Note that position [b] and [c] are similar in a certain respect: at least they both accept Malament's results in considering the fields as having an intrinsic geometrical definition. Nonetheless, there are reasons to somehow prefer position [c] to position [b]: I think we should recognize that what Malament's analysis really shows us is that the fields do not have the same role in the theory as the particles. The intrinsic definition of B dictates the way in which it transforms. But B is defined as such *because of the role it has in the theory*: it generates the trajectories of the particles, that we want to remain the same under the symmetry transformation. It is because of this that we define B *á la* Malament. Position [b] does not recognize that the reason for the electromagnetic fields intrinsic definition is that they are *not* primitive objects, while the latter does. If we understand the velocities as generated by the fields, then, given that we regard in classical mechanics the velocities as part of the dynamical condition but not of the instantaneous state, there is no reason to regard the field as part of the instantaneous state, the primitive ontology of the world. Notice that in position [b] the primitive ontology changes under the transformation but the theory is invariant, disconnecting completely the notion of symmetries to the notion of primitive ontology. So what are symmetries in this view? In the case of position [c], instead, symmetries and primitive ontology are connected: symmetries provide the possible histories of the primitive ontology, if starting from possible histories. For this reason, position [c] seems in some sense more natural than position [b]: we regard the primitive ontology as given by the position and the fields are a part of the state.

Let us now compare position [a] and position [c]. They differ totally in spirit: in the latter the primitive ontology is given by positions only and the fields are considered not as fundamental objects. In the former, the basic idea is to regard the fields as something that characterize the primitive ontology just as position [b] does. As we already saw, the consequences of choosing one primitive ontology or another will influence the symmetry property that the theory happens to have: Both in [a] and in [c] the primitive ontology is connected with symmetries but, while in [a] the theory is not invariant, in [c] it is. As we have seen, in the first case the classical electrodynamics is time reversal invariant, in the second case it is not. It might worthwhile noting that theory [c] is not physically equivalent to [a] because they have different histories of the primitive ontology.

According to Albert in classical electrodynamics as physicists usually think about, we invent a rule (i.e. $T(B) = -B$) in order to get back time reversal invariance. And, according to him, this is highly artificial. It is only a move to preserve this symmetry property. But if we think as he does we do not allow for this transformation, we totally disentangle the fields from the particles. The role of positions is different from the one of fields in classical electrodynamics as the role of position is different from the role of the wave function in quantum theories without observer. And if we define symmetries in terms of the primitive ontology, the way in which the non primitive variable transforms is far from being unnatural. Rather, their transformation properties are determined by what function they are of the primitive entities. One might object that position [c] is unacceptable because the primitive ontology in this way is not complete. In Albert's words, we should understand "complete" as

> [...] that *all* the physical facts about the world can be read off from the full temporal set of its descriptions (Albert 2001).

But the primitive ontology $\mathscr{X}$ never exhausts all the information that one needs to provide: the primitive ontology is not the state. As we have seen before, also in quantum theories without observer what provides the complete information is the couple $(\mathscr{X}, \Psi)$. Here the situation is the same: the complete information is given by the state, not by the primitive ontology.

6.3 Epistemology, Metaphysics, and Architecture

We can see a parallel between classical electrodynamics and quantum theories without observer as theories about a primitive ontology in space-time: on the one hand we have the primitive ontology, and then we have the wave function that implements the law of evolution for the primitive ontology. Note, though, the following difference between quantum theories without observer and classical electrodynamics. Suppose we take quantum theories without observer with the wave function in the primitive ontology and electrodynamics with the electromagnetic fields in the primitive ontology (in addition to the particles). In quantum theories without observer, since the wave function lives in configuration space, it is somehow easier to accept that it does not constitute physical objects, while in classical electrodynamics the electromagnetic fields are in $\mathbb{R}^3$. But as noted above, what is important is the role they have into the theory: both the wave function in quantum theories without observer and the fields in classical electrodynamics are part of the law to generate the motion of the primitive ontology. In this view, electromagnetic fields do not constitute physical objects.

The discussion above seems to suggest that electromagnetic fields in classical electrodynamics and the wave function in quantum theories without observer play the very same role in the theory. In addition, it suggests that any fundamental physical theory has a certain structure: it is constructed starting from the simplest choice of primitive ontology. In a fundamental physical theory we choose some variables (the primitive ones) that we believe represent what is out there and we try writing an equation of motion for them. In doing so, we inevitably introduce new variables, to account for the correct motion of the primitive variables. Suppose the world is made of particles only. If it happens that we cannot explain the phenomena only with particles then we will have to introduce other variables. This is actually the way in which historically E and B have been introduced. The reason why electromagnetic fields were introduced in the first place was that, in the case of charged particles, it was not sufficient to specify their positions in order to describe the world faithfully in the framework of classical mechanics: we need to specify also the fields in order to recover the correct trajectories of the particles. This suggests that the fields have been introduced parasitically on the particles, underlining the fact that they are not what the theory is about. Therefore, the fields are not a part of the primitive ontology of classical electrodynamics. Fundamental physical theories provide a tool with which we discover what there is and we explain what is its behavior. Therefore, we structure them on a primitive ontology, which describes what tables and chairs are made of, and we use additional variables to explain how they evolve in time. In the framework of classical electrodynamics, particles are the primitive ontology, while the fields are these nomological variables. Another example of that is the wave function in quantum theories without observer and the electromagnetic fields in classical electrodynamics. In this view, therefore, the theory is not flat, the variables are not all on the same level. A fundamental physical theory therefore has an architecture, a structure, in which the different variables are collocated: there are the "basic" variables, the primitive ontology $\mathscr{X}$, and then we have other variables whose purpose is to explain the behavior of what we have chosen to be the primitive entities. The emphasis is on the role of the objects inside the architecture of the fundamental physical theory and this is where the primitive/nonprimitive distinction comes from.

This is connected with what a fundamental physical theory is supposed to be: we have seen that only a theory with a clear primitive ontology is able to explain the macroscopic world in satisfactory way. Specifying the ontology is fundamental but not enough, the explanation works in terms of the primitive ontology in three-dimensional space. The non primitive variables have been historically introduced in order to implement a dynamics for the primitive ones. As we have seen, if we do not postulate a primitive ontology in three-dimensional space, the explanation becomes much more complicated. If quantum theories without observer are thought of as theories about the wave function we do not even have the tools to start the explanation, since we do not even have what we might consider one of the most primitive of our concepts, namely three-dimensional space; in classical electrodynamics the situation is not that bad, since the electromagnetic fields are fields in $\mathbb{R}^3$, but if we allow the fields to be in the primitive ontology, the structure of the theory falls apart and also the possibility of an explanation through that theory can be in danger

This seems to be an epistemological and not a metaphysical distinction: we did not know that there are fields but now we know they are there. The idea is in fact the following:

- We needed fields to account for the motion of the particles,
- Fields obey Maxwell's equations, according to which there are free fields,
- So there are free fields and we should recognize that, *even if* they were introduced parasitically on the particles, we should consider them as independent piece of primitive ontology.

Notce that the same argument would be valid in the case of the wave function in Bohmian mechanics:

- We needed the wave function to account for the motion of the particles,
- The wave function obeys Schrödinger's equations, according to which there are free wave functions,
- So there are free wave functions and we should recognize that, *even if* it was introduced parasitically on the particles, we should consider it as independent pieces of primitive ontology.

My intuition, though, is different: while it is true that we arrived at the conclusion of the existence of fields or the wave function as a part of the ontology, we should not say that they are part of the primitive ontology. This is because with a fundamental physical theory we try to account for and to explain the behavior of the world around us and we do it in a way that, inevitably, depends on our concepts and on our way of understanding things. So on the one hand we should remember that explanation is always explanation for us, but on the other hand this does not mean that we will never be able to discover anything true about the world: the amazing fact that, its intrinsic limitations notwithstanding, physics seems to be able to do it! We discover what there is, and this is independent of us, but when we provide an explanation, and this is what is parasitic on what kind of being we are. The theory has the structure that it has and we cannot change it without any undesirable consequences. And the structure tells us the electromagnetic fields have a different role than the particles. If we assume there is not such a difference we see the world differently, and, I argue, we do not understand or explain much.

Chapter 7

The General Structure of Fundamental Physical Theories: Mathematical Structures Grounded on an Adequate Primitive Ontology

In the previous chapters, we discussed the importance of a primitive ontology in three-dimensional space for a quantum theory without observer and for classical electrodynamics as what the theory is about. In this chapter I would like to emphasize what I believe is the common structure of any fundamental physical theory in connection with what it is supposed to explain: The main idea is that a fundamental physical theory is a mathematical structure grounded on a primitive ontology with histories in space-time, that has the purpose to explain the time evolution of the macroscopic objects we observe around us.

7.1 The Need of a Clear Ontology

On thing that must be first understood is the necessity, for any theory, of a clear primitive ontology. That is, if one wants to be a realist about a fundamental physical theory, one must be clear about what are the entities that the theory is talking about, that is “out there” in the world according to the theory. If we do not specify the primitive ontology, the theory is only empty mathematics and we do not have any connection between the entities of the theory and the ones in the physical world.

7.2 From Macro to Micro

Quite ironically, the positivists understood the importance of basing a theory on something secure. In particular, they recognized experience to be this kind of undeniable, safe, secure thing on to which we could base our theories. That is, their theories were based on a sort of macroscopic ontology. “What is real is what is verifiable” was the popular slogan of the movement. I am aware that what they actually believed was more radical than actually saying that there might be different theories for describing the world and that it is empirically impossible to say which one is the true one. I am aware that they were not just saying that the theories are underdetermined by experience but that they actually had in mind that it makes no sense at all to look for a true microscopic theory. But let us forget about that for the moment. Suppose we think there is a fact of the matter of what the world is. If we want to be able to describe it we have to start from somewhere. It is the natural guess to start describing the world in terms of what is macroscopically around us. The very idea of choosing a macroscopic primitive ontology is problematic in itself because the notion of macroscopic is ill defined. As we saw at the beginning, the Copenhagen interpretation of quantum mechanics provides an example of a theory with a macroscopic primitive ontology. In fact, it is about the behavior of certain macroscopic objects, namely the pointer positions, whose dynamics is governed by the wave function. Even if it does not suffer from the measurement problem, this theory still has the problem of the cut: What is a macroscopic object? The most obvious way to solve this problem is to shift the ontology of the theory from the macroscopic, messy level to the microscopic, sharp-cut level.

In any case, I believe in this sense positivism has provided an important step toward realism because it stressed the importance of having a clear primitive ontology. They did not stress the importance of it being macroscopic, an aspect instead emphasized by Einstein: our scientific theories are the refinement of what our common sense is telling us there is, and in order to explain what we think there is in space and how it behaves in time we construct theories. As a matter of fact, in this process we ended up postulating microscopic, unobservable entities that are supposed to constitute the building blocks (what I have called the primitive ontology) of the physical (three-dimensional) world, in terms of which we proceed to explain the macroscopic phenomena. It has been straightforwardly so up to when quantum mechanics and the Copenhagen interpretation came into place. But instead now the quantum theories without observer outlined in

Chapter 4 ground the explanation on a well defined, microscopic primitive ontology.

7.3 The General Scheme of Mechanics: The Primitive Ontology and its Dynamics

The great challenge of physics is to provide us with a view of the world. But what is a fundamental physical theory?

A paradigmatic example of a fundamental physical theory is given by the theory of Newtonian mechanics. This theory has had an enormous success, unifying the explanation of the behavior of a large variety of physical objects, from planets to gases, under the same theoretical scheme. Physical bodies are supposed to be satisfactorily described by points in three-dimensional space. For example, certain behavior of extended bodies can be accounted for only considering the motion of their center of mass under the action of definite forces; the behavior of gases and fluids are accounted for considering them as composed by identical particles that collide one with the other; and so on. Newtons theory is therefore a theory about particles, described by their position in the ordinary three-dimensional space. These particles are, therefore, what the theory is about, the primitive ontology.

Note that the theory is not complete: once the primitive ontology is specified, we need additional information. What else is needed to completely characterize a fundamental physical theory? It is not sufficient to specify a primitive ontology as what constitutes matter, we also need to specify how this primitive ontology "behaves". In order to do that, there is the need of a law of motion to express the evolution of the primitive ontology in time. In the particular case of classical mechanics the law of evolution for the primitive ontology is given by Newtons law of motion:

$$m\frac{d^2q_t}{dt^2} = -\nabla V(q), \tag{7.1}$$

where V is a generic potential. Depending on the potential, we have different theories.

Therefore, the general scheme of a mechanical explanation of the world can be summarized as follows, in the words of Einstein:

- [**a**] The concept of material object: a massive object that, as far as the position and the state of motion are concerned, can be described with sufficient exactness like a point of coordinates X_1, X_2, and X_3. This state of motion (with respect to space) can be described giving X_1, X_2, e X_3 as a function of time.
- [**b**] The inertial laws: it is the annulment of the components of the acceleration of a material point that is sufficiently far away from the other points.
- [**c**] The law of motion (for the material point): force = mass × acceleration.
- [**d**] Laws of the force (actions and reactions between material points (Einstein 1969).

In this scheme, point [b] is just a particular case of point [c]. A complete theory is given only when the laws of the force are given. Classical mechanics is just a empty scheme: it becomes a theory only when we specify the laws of the force [d], like those given by Newton for celestial mechanics.

Now, let us rewrite classical mechanics in a different framework, that will help to see the general structure of all fundamental physical theories: in classical mechanics the state of the system is given by the couple (q, p) of the position and momentum of the particles. With this we mean what gives us the complete dynamical description of the world at any instant of time. The momentum p is what allows the equations to be closed and therefore that allows for a dynamical description of the evolution of the position q in time. The evolution of the positions is therefore given by

$$\frac{dq}{dt} = \frac{1}{m}p = u(q, p), \tag{7.2}$$

where m is the mass of the particle. And the evolution of the momentum itself is given by

$$\frac{dp}{dt} = -\nabla V(q) = f(q, p), \tag{7.3}$$

where f is the force. The two first-order equations can be written in a second order equation plugging in the second equation into the derivative with respect to time of the first (assuming that the mass remains constant in time). In this way we get back the usual law

$$f = m\frac{d^2q}{dt^2}. \tag{7.4}$$

In the case of Newtonian mechanics, therefore, the complete description of a physical body is given by the couple (q, p) whose components satisfy equations (7.2) and (7.3).

$$u(q,p) = \frac{dq}{dt} = \frac{1}{m}p, \tag{7.5}$$

$$f(q,p) = \frac{dp}{dt} = -\nabla V(q). \tag{7.6}$$

There is a free variable: Such a scheme becomes Newtons theory as soon as we provide an additional recipe to specify the form of potential V (or the force f) acting on the particles. It can be the gravitational force between two particle of mass m and M at a distance r, derivable from the gravitational potential $V_g = -G\frac{mM}{r}$ (where G is the gravitational constant, given by $G = 6.61\ 10^{-11}$ $\mathrm{Nm}^2/\mathrm{kg}^2$). Or it can be the electrical force between two charges e_1 and e_2 at a distance r, derivable from the potential $V_e = -\frac{1}{4\pi\epsilon_0}\frac{e_1 e_2}{r}$ (where ϵ_0 is the vacuum dielectric constant, given by $\epsilon_0 = 8.85$ 10^{-12} $\mathrm{C}^2/\mathrm{Nm}^2$).

As noted by Duhem and later by Einstein (Duhem 1906), (Einstein 1969), such recipes for V, as well as the theory altogether, cannot be derived from any well-defined logical scheme; they are simple, "intuitive" laws, created by the fantasy of the scientist and they are preferred to others only on the basis of their empirical success. They comply with the observed data and allow making predictions that are verifiable experimentally.

This scheme is not a peculiarity of classical mechanics, but arguably a general trait of a fundamental physical theory, as we will see now. In fact it can be extended to quantum theories without observer in a very straightforward way and also to classical electrodynamics.

The state of the system is given by the couple $(\mathscr{X}, \eta)$, where $\mathscr{X}$ is the variable representing the primitive ontology, what the theory is about, and η represents the variables that can allow for the closure of the equation of motion for $\mathscr{X}$, the nomological variables.

Different mathematical choices for $\mathscr{X}$ are possible: It can be for example a point in $\mathbb{R}^3$, a function from $\mathbb{R}^3$ to $\mathbb{R}^3$, a functions from $\mathbb{R}^{3N}$ to $\mathbb{R}^3$, and so on. One should pick one. Mathematics always need a physical interpretation if what we want to do is physics through mathematics so that if $\mathscr{X}$ is a point in three-dimensional space, its the natural physical interpretation is that it describes particles' positions; if it is a function $f(x)$ it will be a scalar field, and so on.

The theory is determined once the histories of the primitive ontology $\mathscr{X}$ are given, so that different mathematical choices of $\mathscr{X}$ correspond surely to different theories. Once the primitive ontology $\mathscr{X}$ is fixed, different mathematical choices of η, the variables to implement the law of motion of $\mathscr{X}$, can be made, since the same trajectories can be generated by different η.

In case of quantum theories without observers with a deterministic evolution for the primitive ontology, the general scheme of mechanics is generalized rather straightforwardly. The evolution of $\mathscr{X}$ and η in time can be described by two functions, u and f, such that one can write an equation for the evolution of $\mathscr{X}$ in terms of an appropriate function $u(\mathscr{X},\eta)$ and one can write an equation for η in terms of another function $f(\mathscr{X},\eta)$. In classical mechanics, given equations (7.5) and (7.6), we have that $\mathscr{X}=q$, $\eta=p$, $u=p/m$ and $f=-\nabla V(q)$. In Bohmian mechanics the world is composed by particles, described by points in three-dimensional space, like in Newtonian mechanics. The couple (Q,Ψ) provides the complete description of the world. To stress the formal analogies with Newtonian mechanics, let us use the same notation we used before. We have: $\mathscr{X}=Q$, $\eta=\Psi$. The functions u and f are defined as follows:

$$u(q,\psi)=\frac{dq}{dt}=\frac{\hbar}{m}\mathrm{Im}\left[\frac{\psi^*\nabla\psi}{\psi^*\psi}\right](q), \tag{7.7}$$

$$f(q,\psi)=\frac{\delta\psi}{\delta t}=\frac{1}{i\hbar}\left[\frac{\hbar^2}{2m}\nabla^2+V\right](q). \tag{7.8}$$

In classical electrodynamics, we have $\mathscr{X}=x$ and $\eta=(E,B)$. The u function is the same as classical mechanics but in the f function the potential V includes the electromagnetic fields.

In quantum theories without observers in which the primitive ontology evolve according to a stochastic law (like GRWf), the generalization is less evident but it is still there. The law u for the primitive ontology $\mathscr{X}$ amounts to the specification of possible probability distributions, for example, by specifying the generator or the transition probability, of a Markov process. For example, in GRWm the primitive ontology $\mathscr{X}$ is the mass density field $m(x,t)$ and the u function is given by equation (3.3). The η variable is the wave function and f is the stochastic GRW-evolution. In GRWf, $\mathscr{X}$ is the set of flashes F_k, and even if there is no u since the theory is presented directly in a space-time framework, the possible histories of the primitive ontology are determined by equation (2.14). As in GRWm, η is the wave function, which evolves to the stochastic GRW-evolution.

The common structure of Bohmian mechanics and GRWf and GRWm has been emphasized in (Allori et al. 2007). More on the similarity between deterministic and stochastic theories will be discussed in Chapter 8.

7.4 Theories as Choices

As mentioned early, any choice of $\mathscr{X}$, η, u and f is just a conjecture: it will never be logically justified by experiments. Theories are in this sense not deducible from experimental data. In this way, we can say that theories are creations composed by axioms (postulates) logically disconnected from experience. Theories are our best tools to describe, understand, and explain reality. We use theories with these features because we need hypotheses that, obligatorily, transcend what they want to explain. Therefore, the choices we make can only be corroborated *a posteriori* by the fact that the theory developed upon this assumption describes and predicts the behavior of bodies in the world.

To summarize, we have different possible choices (that of course must be limited by the comparison with the experimental data):

- Freedom in choosing $\mathscr{X}$: changing $\mathscr{X}$ we change the theory. This is to say that we change the mathematical objects with which we describe the world. It is exactly what happens for example in theories like string theory, in which the basic objects are one-dimensional, rather than dimension-less, as in classical mechanics or like field theories.

- Freedom in choosing η: changing η does not matter if the evolution of the primitive ontology stays the same. This is what happens, for example, in classical electrodynamics using different gauges for the potentials, or in GRWf or in its linear reformulation presented in Section 4.2.1. If we change η such that the evolution for $\mathscr{X}$ changes, then we have a different theory. This is what happens between BMp, GRWp or classical mechanics and electrodynamics.

- Freedom in choosing u: changing u amounts to change the law of evolution for the primitive ontology, so the theory changes. An example of a change of u keeping fixed the primitive ontology is given by the passage from classical mechanics to Bohmian mechanics or stochastic mechanics or classic electrodynamics.

- Freedom in choosing f: changing f we change the law of evolution for the η and two things can happen. Either this change also affects the evolution of the primitive ontology or it does not. In the former case the theory changes, as we can see if we compare classical mechanics with classical electrodynamics, in which the change is f amounts to the change in the potential. In the latter case the theory is the same as before. The same u for a given $\mathscr{X}$ means that the theories are physically equivalent, even if we have a different f. An example of this is GRWf and its linear version.

Since, as we have seen in Chapter 4, the defining feature of a fundamental physical theory is the histories of the primitive ontology $\mathscr{X}_t$, we can conclude, as Goldstein has stressed [1], that there are two crucial ingredients for the theory, as shown in Figure 7.1, namely: the space-time and the decoration of the space-time, the histories of the primitive ontology. These decorations are entities (world-lines of material points, tubes-lines of strings, local fields, ect.) that, together with the pure geometrical fields that defines the space-time (metric tensor and connection), allow for a complete representation of physical reality.

[1]Private communication.

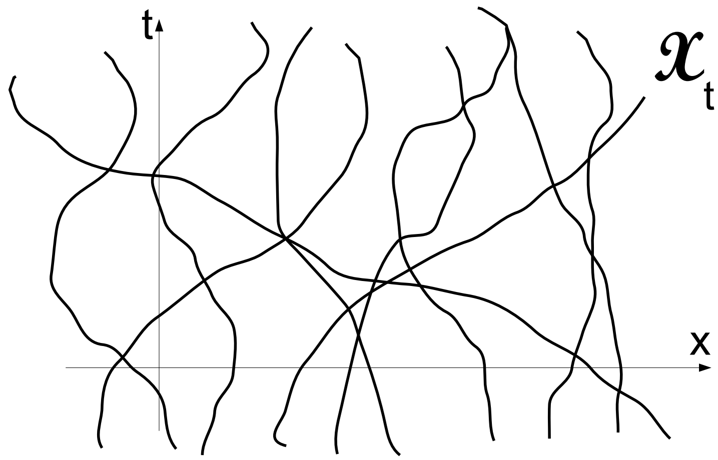

Figure 7.1: The fundamental ingredients of a fundamental physical theory: the space-time and the decorations of space-time.

7.5 Semicolon and the Nature of Reality

In a fundamental physical theory there are different kind of variables, a rough list of which can be summarized as follows:

$$(\mathscr{X}, n\mathscr{X}).$$

$\mathscr{X}$ are the primitive variables, directly describing the what physical objects are made of. As we have seen, in classical mechanics, classical electrodynamics, Bohmian mechanics, stochastic mechanics, BMW and GRWp they are the positions of particles, in GRWf and Sf they are the flashes and in GRWm and Sm they are the mass density. All the other variables are non-primitive variables, $n\mathscr{X}$. In particular, there are the nomological variables η that are necessary to specify the dynamics of the primitive ontology. Examples of nomological variables are the momentum in classical mechanics, the electromagnetic fields in classical electrodynamics, the wave function in quantum theories without observer. The state $(\mathscr{X}, \eta)$, that contains the specification of the functions u and f (that define the law of evolution for the primitive ontology $\mathscr{X}$ and the nomological variable η), provides the complete specification of a fundamental physical theory.

The functions u and f contain some parameters π, like masses, charges, constants of nature: they should be intended as part of the definition for the law of the couple $(\mathscr{X}, \eta)$. They are similar in nature as the variables η but they do not evolve in time and therefore do not belong to the state.

We also have the M variables that are non-physical variables, such as operators as observables in quantum mechanics, that are useful to express the statistics of the measurement results.

Notice that all the other variables appearing in the fundamental physical theory do not really appear in its formulation and therefore express what we would call properties of microscopic bodies. They are "derivative" variables D, functions of $\mathscr{X}$ or of the state: $D = F(\mathscr{X}, \eta)$. An example of such a variable is given, for example, by kinetic energy $E = \frac{p^2}{2m}$.

Suppose the complete description of the world, the state, is provided by the following variables:

$$(a, b, c, d).$$

This list is complete in the informational sense, this list *per se* does not tell me anything about the primitive ontology of the theory. For this reason, I will use the symbol of the semicolon ";" to divide in the list the primitive ontology from the other variables, the primitive ontology being the variable on the left of the semicolon [2]. For example, if we write

$$(a; b, c, d)$$

what we mean is that the theory is completely specified by the variables a, b, c, d but the theory is about the entity represented by the variable a, which is therefore the primitive ontology $\mathscr{X}$ of the theory. In contrast the variables b, c, d are the non-primitive variables $n\mathscr{X}$. So, $\mathscr{X} = a$, $n\mathscr{X} = b, c, d$. Assume $\eta = b$, so that the state is given

[2] This idea is of Shelly Goldstein, private communication.

by $(\mathscr{X}, \eta) = (a, b)$. Since the theory is defined by the primitive ontology, depending in where we place the semicolon, we have different theories:

- $T_0 = (; \mathscr{X}, \eta)$
- $T_1 = (\mathscr{X}; \eta)$
- $T_2 = (\mathscr{X}, \eta;)$

In the case of quantum theories without observer, we would have, for example, $T_0 = (; \mathscr{X}, \Psi)$, while in the case of classical electrodynamics we would have $T_0 = (; q, E, B)$. In both cases, T_0 is a theory in which there is no primitive ontology at all: this theory is not a physical theory but just empty mathematics. In the case of quantum theories without observer $T_1 = (\mathscr{X}; \Psi)$ corresponds to BMp, SM, BMW, GRWp, GRWf, Sf, GRWm, and Sm, in which the wave function is not part of the primitive ontology, while cBM, GRW∅ and MW∅ correspond to $T_2 = (\mathscr{X}, \Psi;)$, in which also the wave function belongs to the primitive ontology. In classical electrodynamics $T_1 = (q; E, B)$ corresponds to the view in which the fields do not constitute physical objects, which are constituted just by particles, while $T_2 = (q, E, B;)$ to the view of Albert in which the fields are part part of the primitive ontology.

T_1 and T_2 are two different theories and they have different symmetries properties. In the case of quantum theories without observer, T_1 is Galieli invariant while T_2 is not; in classical electrodynamics T_1 is time reversal invariant while T_2 is not. The reason for the symmetry in T_1 theories comes from their connection with the primitive ontology.

7.6 Metaphysical Neutrality

As a consequence of the role of primitive ontology as opposed to the role of the wave function in defining a theory, one can redefine the notion of primitive ontology as follows: It is the *only* set of objects one has to postulate to exist in the physical world in order to build a fundamental physical theory. We have to assume primitive ontology as physically real, otherwise the theory is empty; we need to stipulate that the M variables do not physically exist, otherwise we commit redundancy and risk of inconsistency, as Bell would have said (Bell 1987).

What about η? The nomological variables belong to the state, contrarily to the π variables that are constant: $(\mathscr{X}, \eta)$. But the theory is actually *neutral* regarding the metaphysical status of the η variables. In fact, as said before, since a theory is specified once we have the evolution of the primitive ontology, what is crucial are the primitive ontology $\mathscr{X}$ and its temporal evolution u, while η and f are not. The nature of the entities mathematically described by η will depend on what our view about laws is. If we have a Humean view of laws, that is if we regard laws as the simplest and most informative summary, then we would probably tend to regard η just as mathematical fiction. Rather, if we tend to regard laws in a Platonic view, we would consider the η variables as real, even if not physically real.

Note that since the status of the other variables that appears into the theory is irrelevant to define a fundamental physical theory, their choice and the choice of their evolution equation should be made according to reasons of convenience, like simplicity or compactness of expression. So, for example, BMp is physically equivalent to its collapsed version. Nonetheless, when we present the theory we use the version in which the wave function evolves linearly, *just because* it seems mathematically nicer and not for other, metaphysical reasons.

Notice that this is exactly what has happened in Bohmian mechanics: the theory is defined using reasons of simplicity, as Dürr, Goldstein and Zanghí point out:

> Suppose that the wave function ψ does not provide a complete description of the system, that the most basic ingredient of the description of the state at a given time t is provided by the positions [...] of its particles at that time, and that the wave function governs the evolution of (the positions of) these particles. Insofar as first derivatives are simpler than higher derivatives, the simplest possibility would appear to be that the wave function determine the velocities $v_1^\psi, v_2^\psi, ..., v_N^\psi$ of all the particles (DGZ 1992).

They also point out that, in the guide equation, the fact that one has to divide for the wave function derives from the fact that the wave function is, mathematically, a ray (rather than a vector) in Hilbert space. In fact, given that the wave function is a ray, $c\psi$ and ψ provide the same physical description, for any constant $c \in \mathbb{C}, c \neq 0$. So the velocity field should not change under the transformation from ψ to $c\psi$: $v^{c\psi} = v^{\psi}$. [3] The form of the velocity field is determined by reasons of symmetry. The requirement for the theory to have some symmetries puts constraints on the form of the guide equation. In particular, Galilei invariance of particle trajectories requires the presence of the factor $\frac{\hbar}{m}$, time reversal invariance of particle trajectories gives rise to the imaginary part, and their invariance under rotations gives us the gradient. If we have a magnetic field, we should take ∇_k as the covariant derivative. To take spin into account, one should consider the wave function as a spinor and the velocity field should be rewritten as

$$v_k^{\Psi} = \frac{\hbar}{m_k} \mathrm{Im} \left[\frac{\Psi^* \nabla_k \Psi}{\Psi^* \Psi} \right], \tag{7.9}$$

when now in the numerator and denominator appears the scalar product in the spinor space.

7.7 The Macroscopic World, Properties and Reductionism

Now the question is: How can we explain what macroscopic objects are and what are their properties in the language of a fundamental physical theory? Macroscopic objects and their properties are not metaphysically primitive entities in a fundamental physical theory. In fact, they are completely defined in terms of the primitive ontology and its dynamics, so that there is no need to add any new metaphysical entity. Objects and their properties Y are functions of the primitive ontology of the theory $\mathscr{X}$ such that $Y = \rho(\mathscr{X})$, for an appropriate function ρ: they logically supervene on the primitive ontology. As such, a reductive explanation of them can be provided.

Notice that, of course, the function ρ changes depending on the theory. It is this function ρ that has not any ontological component into it, but rather, it is just of practical interest. As already noticed, proponents of bare quantum theories without observers as for example GRW∅ claim that everything can be reduced to the description provided by the wave function Ψ so that we would have $Y = \rho(\Psi)$. As already noticed,

[3] This is to say, the velocity is homogeneous of degree zero as a function of ψ. A function $f(x)$ is homogeneous of degree δ with respect to x when $f(cx) = c^{\delta} f(x)$.

the ρ functions can be practical rules, *except* those that define the primitive ontology $\mathscr{X}$ of the theory, which should not to be as translation rules at all.

One could say that macroscopic properties are all secondary qualities, since they are functions Y of the primitive ontology. Primary qualities are the properties of the primitive ontology. So the next question can be: What are the primary qualities? This depends on what the primitive ontology is. If the primitive ontology is the one of particles, their only primary property is the one of position. What about those properties such as, for example, mass or charge? One might think it is some "intrinsic"feature, as the primary qualities of the microscopic primitive ontology. They are not functions of the primitive ontology but rather they do appear in the formulation of the theory. As I anticipated earlier, they should be considered as being parameters needed to specify the law for the primitive ontology. Newton's equation, for example, involves a parameter that we want to call "the mass" of the particle that is interpreted as the inertia coefficient that rules how the particle will accelerate if subject to a force. It is therefore a part of the law of motion, it is present in the u function, given that $u = \frac{1}{m}p$. This is the inertial mass, but also the gravitational mass appears in the f function if there is a gravitational potential $V = -G\frac{mM}{r}$. Also electric charge can be understood in this way, as a parameter in the f function if there is an electrical potential.

Therefore, mass and charge have a different status with respect to the other properties that can be defined as functions of the primitive ontology of the theory. The latter can be reduced to the primitive ontology so they are real in the same sense as the primitive ontology is real; while the former are not reducible to the primitive ontology but they are part of the law, and therefore they are real in the same sense in which the law of motion is real. The same can be said in the case of spin in the case of quantum theories without observer. While masses and charges are in the Hamiltonian, the spin is in the wave function, as it is evident for example in Bohmian mechanics in which the wave function is a spinor. But since the wave function is part of the law, there is no contradiction.

Moreover, observe that since the nomological variables η can be considered as flexible as far as they generate the "right" law for the primitive ontology, and since the masses and charges appear into u through η, also they should be considered with flexibility. As a consequence, we do not need to attach to objects the same properties if we describe the world with a different theory. We consider classical properties like mass more familiar and therefore we try to find analogs of them in the other theories. But actually it is the shape of the functions u and f that appear in the theory that specifies the parameters that are the properties of the objects. For example, given that m and $\hbar$ appear in the Schrödinger and the guide equations in the same way, we can rewrite them in the following way:

$$\frac{dq}{dt} = \frac{1}{\mu}\mathrm{Im}\frac{\psi^*\nabla\psi}{\psi^*\psi}, \tag{7.10}$$

$$i\frac{\delta\psi}{\delta t} = \left[-\frac{1}{\mu}\nabla^2 + \hat{V}\right]\psi(q), \tag{7.11}$$

where $\hat{V} = V/\hbar$ and $\mu = m/\hbar$. This new parameter μ can be regarded as a sort of "natural mass", even if it has the dimensions of $[time]/[length]^2$. It appears in the theory in a much more natural way than how m or $\hbar$ do. Moreover, if we also make use of some natural charges defined as $\epsilon = e/\hbar$, $\hbar$ completely disappears from the equations of motion: it only remains into the equations $\mu = m/\hbar$ and $\epsilon = e/\hbar$ that link the masses and the charges in microscopic natural units ($[\mu] = ([charge]/[length]^2)$,$[\epsilon] = ([charge]/[length])([time]/[mass])1/2)$ with the ones in macroscopic units proper of Newtonian mechanics ($[m] = [mass]$, $[e] = [charge]$). These parameters μ and ϵ do not have any *a priori meaning*, as m and e did not, but they acquire it only because of the role they play into the equations of motion.

7.8 Symmetry Properties and Primitive Ontology

Can the understanding of fundamental physical theories as mathematical structures grounded on primitive ontology be of some help for scientific progress? We have, for example, nonrelativistic quantum theories, but we know it is not the end of the story: we need relativistically invariant quantum theories. One of the problems here is that it is not clear what objects in the theory should be invariant under the symmetry transformation. We have already seen that there is a connection between symmetries and the primitive ontology and here we will discuss more in detail what it is. We will see that it is the histories of the primitive ontology that have to be invariant. Since there is a connection between the primitive ontology and symmetry properties of the theory, choosing one primitive ontology instead of another might lead to different symmetry properties of the theory. Therefore, the point is just to choose the right one in order to build a relativistically invariant theory.

Let us therefore proceed to investigate what does it mean for a theory to be Lorentz invariant. When we talk about the symmetry of a theory, what does the invariance refer to? To say that a theory has a given symmetry is to say that

> *The possible histories of the primitive ontology $\mathscr{X}_t$, those that are allowed by the theory, when transformed according to the symmetry, will again be possible histories for the theory, and the possible probability distributions on the histories, those that are allowed by the theory, when transformed according to the symmetry, will again be possible probability distributions for the theory.*

Let me explain this statement, using the example of Galilean invariance.

- "*The possible histories of the primitive ontology, those that are allowed by the theory...*" In classical mechanics the primitive ontology is that of particles, described by their positions in physical space, a history of this primitive ontology corresponds to a collection of particle trajectories (the trajectories $Q_k(t)$, $k = 1, \ldots, N$, in a universe of N particles) and a history is allowed if the particles obey Newton's law, i.e., if $m_k \ddot{Q}_k(t) = F_k(Q_1(t), ..., Q_N(t))$, where F_k is the Newtonian force acting on the kth particle. The theory is defined once the form of F_k is specified (for example, that the force is the Newtonian gravitational force). Consider now Bohmian mechanics: also here the primitive ontology is that of particles and a possible history of the primitive ontology (one that is allowed by BM) is a history described by the particle trajectories $Q_k(t)$, $k = 1, \ldots N$, which satisfy the guide equation *for some wave function* ψ satisfying the Schrödinger equation. The theory is defined once the Hamiltonian H in the Schrödinger equation is specified.
- "*... when transformed according to the symmetry...*" Since the primitive ontology

is represented by a geometrical entity in physical space (a decoration of space-time, as we have said earlier), space-time symmetries naturally act on it, for example transforming trajectories $Q_k(t)$ to trajectories $\tilde{Q}_k(t)$. For example, under a Galilean boost by a relative velocity v, in classical mechanics as well as in BM, the trajectories $Q_k(t)$ transform into the boosted trajectories $\tilde{Q}_k(t) = Q_k(t) + vt$.

- "*. . . will again be possible histories for the theory. . .*" Notice that $Q_k(t)$ and $\tilde{Q}_k(t)$ may arise in BMp from different wave functions. In other words, the wave function must also be transformed when transforming the history of the primitive ontology. However, while there is a natural transformation of the history of the primitive ontology, there is not necessarily a corresponding natural change of the wave function. The latter is allowed to change in any way, solely determined by its relationship to the primitive ontology. For example, consider again a Galilean boost (by a relative velocity v) in BMp: the boosted trajectories $\tilde{Q}_k(t) = Q_k(t)+vt$ form again a solution of the guide equation and the Schrödinger equation with ψ replaced by the transformed wave function [4].

$$\tilde{\psi}_t(q_1, \ldots, q_N) = \exp\Big[\tfrac{i}{\hbar}\sum_{k=1}^{N} m_k(q_k \cdot v - \tfrac{1}{2}v^2 t)\Big]\, \psi_t(q_1 - vt, \ldots, q_N - vt). \quad (7.12)$$

Since the trajectories of the primitive ontology transformed according to the symmetry are still solutions, BMp is symmetric under Galilean transformation, even though the corresponding wave function has to undergo more than a simple change of variables in order to make this possible.

- "*. . . and the possible probability distributions on the histories, those that are allowed by the theory. . .*" In a deterministic theory, a probability distribution on the histories arises from a probability distribution on the initial conditions. In BMp, a probability distribution on histories is possible if there exists a wave function ψ such that the given distribution is the one induced on solutions to the guide

[4] Under this transformation $V = V(q_1, \ldots, q_N)$ in the Schrödinger equation must be replaced by $\tilde{V} = V(q_1 - vt, \ldots, q_N - vt)$. For V arising from the standard two-body interactions, we have that $V = \tilde{V}$, and hence the theory is invariant.

equation by the probability distribution $|\psi(q_1, \ldots, q_N)|^2$ at some initial time.

More interesting is the case of nondeterministic theories. For these theories, i.e., for theories involving stochasticity at the fundamental level, the law for the primitive ontology amounts to a specification of possible probability distributions, for example by specifying the generator, or transition probabilities, of a Markov process. For example, in GRWm the history of the primitive ontology is the mass density field, and a probability distribution on the histories of this primitive ontology is possible if it is the distribution induced on the mass density field, according to equation (3.3), by some wave function ψ with probability law given, say, by (2.14) (and (2.12)). The case of GRWf is analogous: a probability distribution for the flashes $F = \{(X_k, T_k) : k \in \mathbb{N}\}$ is possible if induced by (2.14) for some wave function ψ.

- "*. . . when transformed according to the symmetry, will again be possible probability distributions for the theory.*" The probability distribution on the histories, when transformed according to the symmetry, is the distribution of the transformed histories. In other words, the action of a transformation on every history determines the transformation of a probability distribution on the space of histories. As in the deterministic case, the wave function is allowed to change in any way compatible with its relationship to the primitive ontology. For example, consider the Galilean invariance of GRWf: Let ψ and $\tilde{\psi}$ be two initial wave functions related as in (7.12), that is, by the usual formula for Galilean transformations in quantum mechanics. Let G_t denote the transformation operator in (7.12) at time t, such that $\tilde{\psi}_t = G_t \psi_t$. A simple calculation shows that

$$\Lambda_i(x + vt)^{1/2} G_t = G_t \Lambda_i(x)^{1/2}. \tag{7.13}$$

As a consequence, the distribution of the (spatial location of the) first flash arising from $\tilde{\psi}_{T_1}$ is that arising from ψ_{T_1} shifted by vT_1, and the post-collapse wave functions are still related by the appropriate G_t operator, i.e.,

$$\tilde{\psi}_{T_1+} = G_{T_1}\psi_{T_1+}. \tag{7.14}$$

Thus, the joint distribution of flashes arising from $\tilde{\psi}$ is just the one arising from ψ shifted by vt for every t.

Note that under a space-time symmetry the primitive ontology must be transformed in accord with its intrinsic geometrical nature, while wave functions (and other elements of the non-primitive ontology, if any) should be transformed in a manner dictated by their relationship to the primitive ontology.

7.9 Toward Relativistic Quantum Theories Without Observer

The flashy primitive ontology was invented by Bell as a step toward a relativistic GRW theory. About GRW, he wrote:

> I am particularly struck by the fact that the model is as Lorentz invariant as it could be in the nonrelativistic version. It takes away the ground of my fear that any exact formulation of quantum mechanics must conflict with fundamental Lorentz invariance (Bell 1987).

What Bell refers to in the above quotation is the following. An analogue of the relativity of simultaneity, i.e. of the invariance of the dynamics under boosts, in the framework of a nonrelativistic theory is the invariance under relative time translations for two very distant systems. Bell (Bell 1987) verified by direct calculation that GRWf has this symmetry. Going back to the work of Bell mentioned in the beginning of this section (Bell 1987), what Bell had to do for GRWf, and what he did, was to confirm the invariance under relative time translations of the stochastic law for $F = \{(X_k, T_k) : k \in \mathbb{N}\}$, the galaxy of flashes. And more generally the invariance of GRWf directly concerns the stochastic law for the primitive ontology; it concerns the invariance of the law for the wave function only indirectly, contrary to what is often, erroneously, believed.

Recently Roderich Tumulka (Tumulka 2006) has shown that GRWf can be modified so as to become a relativistic quantum theory without observer. In that paper the stochastic law for the galaxy of the flashes in space-time, the primitive ontology of GRWf, with suitably modified, Lorentz invariant equations, has been shown explicitly to be relativistically invariant. Hence, GRWf is Lorentz invariant, while GRWm is not. Thus, one should not ask whether GRW as such is Lorentz invariant, since the answer to this question depends on the choice of primitive ontology for GRW. For details see (Maudlin 2007).

Similar results to those of (Tumulka 2006) have been obtained also by Dowker and Henson (Dowker and Henson 2002) for a relativistic collapse theory on the lattice with a primitive ontology of lattice locations (see also (Dowker and Herbauts 2004), (Dowker and Herbauts 2006)).

Other proposals for a relativistic quantum theory without observer have been put forward. In a relativistic version of BMp, developed in (Muench-Berndl *et al.* 1999), there is an additional physical object that is fundamental, that is the foliation. Such a foliation divides space-time into space-like hypersurfaces, defines absolute simultaneity and temporal ordering of space-like separated points. If we consider this foliation being part of the primitive ontology of this theory then we are exactly in the same scheme as above and we can also analyze the hypothesis of the foliation actually evolving itself in time. The question is whether this choice is against the spirit of relativity, whatever that is supposed to mean.

What about general relativity? Note that in this theory the structure of space-time cannot be ignored. It does not seem correct to regard relativity just as a theory of space-time detached from the metric. It seems as if in Newtonian mechanics we have ignored the metric structure of space time because it was not affecting the dynamics and the primitive ontology was only constituted by points or world-lines. Now the situation is different: the metric is part of the primitive ontology.

Chapter 8

Explanation and Typicality: the Example of Statistical Mechanics

In which sense are the macroscopic regularities we observe describable in terms of the microscopic description? What does it mean to explain the macroscopic world in terms of a fundamental physical theory? We have answered this question in Chapter 7, in particular in Section 7.7. In this chapter we will try to analyze the same question in those situations in which the macroscopic world presents statistical regularities. This issue has a particularly straightforward answer in the derivation of thermodynamics from statistical mechanics in which the underlying fundamental physical theory is deterministic. I argue, following (Allori et al. 2005), that the same explanation can be provided also in the case of a fundamental physical theory in which the evolution equation for the primitive ontology is stochastic.

The problem of deriving thermodynamics from statistical mechanics has been addressed (even if not exactly in these terms) by many people, for example by Albert in his book *Time and Chance* (Albert 2001) and by Goldstein in (Goldstein 2001). While Albert does not mention it explicitly, Goldstein suggests that an explanation of thermodynamic behavior in terms of an underlying microscopic description in term of a fundamental physical theory must be in terms of typicality. It is important to connect this issue with the notion of primitive ontology.

8.1 Deterministic Theories

The best example of how to understand a macroscopically "probabilistic" universe governed by a deterministic fundamental physics is the one of Boltzmann's explanation of thermodynamics in terms of classical mechanics. We will recall the situation in the next section, and we will describe how a similar account can be provided for deterministic quantum theories like Bohmian mechanics.

8.1.1 Classical Mechanics

The evolution of macroscopic objects is generally very complicated. Nonetheless, their behavior is governed by simple and general physical laws. First of all, by the laws of thermodynamics. If we believe that classical mechanics is a complete fundamental physical theory, we would like, at least in principle, to be able to account for the behavior of macroscopic objects (thermodynamics) in terms of the fundamental theory that rules the behavior of the microscopic objects (classical mechanics). But a problem promptly arises: on one hand we have the fundamental dynamical laws (Newtonian laws) that are symmetrical under time reversal transformations, and on the other hand we have macroscopic phenomena, that on the contrary are irreversible: while when two atoms (roughly thought as two rigid billiard balls) collide elastically we cannot tell if the collision is forward or backward in time, a cup of coffee always cools down, never warms up. Ludwig Boltzmann (Boltzmann 1896), (Boltzmann 1897) was the one who solved this tension between the irreversibility of macroscopic phenomena and the reversibility of the underlying microscopic dynamics from which the macroscopic behavior is supposed to be derivable.

Boltzmann's starting point was the conviction that mechanics is a compete theory of the world. For a body composed by N particles, or in general for the entire universe, the state provides a complete dynamical description at a fixed time t is given by the so called microstate $X = (q_1, ..., q_N, v_1, ..., v_N)$. The set of all the microstates constitutes the phase space Ω. Any macroscopic state, given that we think that the mechanical theory provides a complete description of the world, must be determined by the microscopic one. But not *vice versa*: there actually exist many microstates from which one could derive the same macrostate.

There exists a particular macrosctate, called the equilibrium macrostate, that can be described by a few relevant macroscopical (thermodynamical) variables, like pressure, temperature and volume. Alternatively, any macrostate can be described by means of statistical (empirical) distributions, that is, given a population of individuals totally characterized by their microstate X and a particular property, the distribution function establishes how the property is distributed in percentage among the individuals. In the case of a gas, the individuals are the atoms and the only possible properties that can be associated with them are their positions and velocities. Therefore the statistical distribution $n(X) = n(r, v)$ in this case will describe the percentage of atoms with position r and velocity v. It turns out that, for example, the equilibrium macrostate of the gas is characterized by the Maxwell–Boltzmann distribution of velocities. Another distribution will describe another macrostate. In general, for any microstate X, take the set $\Gamma(X)$ of microstate with the same empirical distribution that X has. This set represents the points in phase space that are macroscopically similar the one to the other and to X. In this way we obtain a direct geometrical representation of the macrostate as a region of Ω. The totality of all possible distributions gives a partition of phase space into disjoint regions: different regions correspond to different macrostates and the macrostates altogether define a partition of phase space into macrostates.

The microstates in the equilibrium macrostate $\Gamma_{eq}(X)$ all have the same macroscopic properties. From a microscopic point of view, the thermodynamical variables are functions of the microstate X. For example, the temperature T of a gas is the mean kinetic energy of its particles, therefore it is the function:

$$T = T(X) = \frac{1}{k_B} \sum_{k=1}^{N} \frac{1}{2} m_k v_k^2, \tag{8.1}$$

where K_B is the Boltzmann constant ($K_B = 1.38 \times 10^{-23} \mathrm{m}^2 \mathrm{kg}\ \mathrm{s}^{-2}\ \mathrm{K}^{-1}$) and m_k, v_k are respectively the masses and the velocities of the k-th particle. The equilibrium macrostate $\Gamma_{eq}(X)$ has the following empirical features: all systems will tend to reach the equilibrium state and when a body is in equilibrium state at a given time it will tend to remain there later on. A simple example of this is a gas completely spread out in a room. This is its equilibrium state and the gas will tend not to change its temperature or pressure or volume. That is, macroscopic phenomena are irreversible. How do we explain this behavior if the fundamental physical theory is classical mechanics, which is reversible?

First of all, it turns out that the equilibrium state $\Gamma_{eq}(X)$ is incredibly bigger than any other macrostate. In order to understand why this is the case, consider the example of the gas. Initially, the gas is in the right side of a box divided in two regions of volume V. When the separation wall is removed, the gas starts to expand into order to occupy the whole volume of the box $2V$. This is the equilibrium state. The phase space volume increases of the same amount for *any* particle of the gas, therefore there is a transition form a region of volume V^N in phase space to a region of volume $2^N V^N$ in phase space. Therefore, the direct computation of the ratio of the volumes of the two macrostates, the equilibrium one and the initial one, $V_{eq}/V_{initial} = 2^N \simeq 2^{10^{23}} \simeq 10^{10^{23}}$, shows how huge the equilibrium state is compared with any other.

We see, then, why it is the case that (almost all) microstates seem to "converge" to equilibrium and then stay there: any microstate, in its wandering through phase space because of the dynamics, will sooner or later fall into the equilibrium state and it will stay there for a long time. This is because the equilibrium state is so huge with respect to the others macrostates that the time the microstate will spend inside it before exit will be extremely long. From what we just said, only for extremely few microstates the dynamics will be such that they will avoid entering the equilibrium state. There could be some exception to this behavior corresponding to particular initial conditions but the volume of these special conditions is really small[1]. Therefore, the typical behavior of a microstate is the correct (empirically observed) thermodynamical behavior, as illustrated in Figure 8.1.

Moreover, if we define the notion of entropy as a measure of the volume of the phase space occupied by a given microstate X as

$$S(X) = k_B \log \Gamma(X), \tag{8.2}$$

it can be shown that it is equivalent to the thermodynamical entropy. Therefore, given that typically the microstate moves from a region of phase space of a smaller volume

[1] It is of the order of $10^{-10^{23}}$.

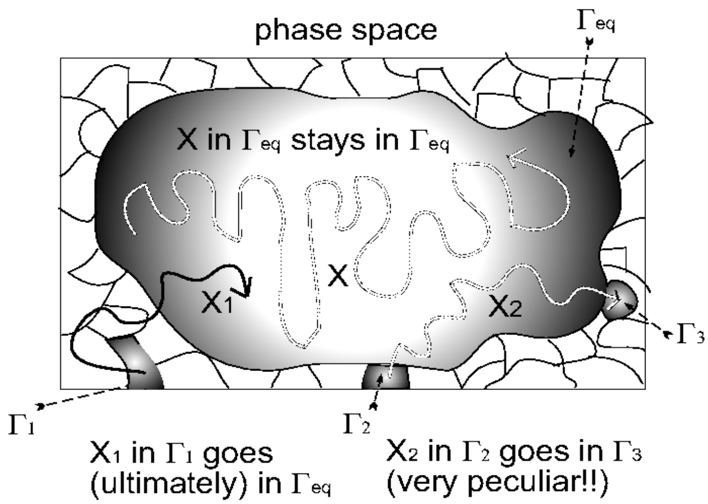

Figure 8.1: The phace space Ω.

to one of a bigger volume, the entropy will typically increase.

It is interesting to observe that, if on the one hand it is important to recognize the role of initial conditions in this explanation, it is also important to stress how the appeal it makes to them is of a kind that we are ready to accept. In fact this explanation regards the *great majority* of the initial conditions. In contrast, an unacceptable explanation in terms of initial conditions would be the one that explains a phenomenon appealing to one very particular initial condition. Note that this would not be an explanation at all: anything could be "explained" in this way. For example, in this framework, we could "explain" why a monkey, randomly pressing the keys of a typewriter, will end up writing the *Divine Comedy*: because it is always possible to find an initial condition, if we search carefully enough, such that it will be the case.

Note that, as observed in (Allori et al. 2005), the objections raised by Lodschmidt and Zermelo to Boltzmann show the nature of the Boltzmann's explanation itself: an explanation based not only on laws but also on initial conditions. Lodschmidt critiqued the whole project, pointing out, according to him, how it is in principle impossible to explain irreversible phenomena with reversible laws. Zermelo, instead, used Poincaré recurrence theorem (that states that the great majority of the microstates in a macrostate will return into its initial macrostate) to claim that none of the two behaviors, thermodynamical and anti-thermodynamical, can be compatible with the Newtonian laws. What Lodschmidt and Zermelo did was nothing else than note that there exist initial conditions that are anti–thermodynamical. But what they did not realized is that the fact that the volume of the equilibrium state is so big guarantees that they are really a minority. Moreover, they did not perform any calculation: to answer Zermelo, the average time needed for a microstate to return back to its initial state is much greater than the life of the Universe [2]. Therefore, there is no incompatibility between Poincaré's time and Boltzmann's explanation: there is only a total discrepancy of temporal scales. In fact, on the one hand we have the time scale of irreversible everyday macroscopic phenomena to which Boltzmann's explanation applies; on the other we have the time scale of Poncaré, in which everything comes back, that is totally irrelevant for Boltzmann's perspective being long as it is.

To sum up, then, Boltzmann's explanation can be stated as follow: since Γ_{eq} is incerdibly big, the great majority of the microstates associated with the same initial macrostate evolve according to the empirically observed thermodynamical behavior.

But how do we have to intend "great majority" of microstates? In the sense of the extension of the corresponding regions in phase space. This notion can be defined introducing a measure μ that roughly is a function that assigns to any subset A of the phase space Ω its measure $\mu(A)$. Such a measure will allow us to count the exceptions E of a given behavior. "Great majority" means therefore that the exceptions are few, that is $\mu(E) << 1$. This measure is called a measure of typicality. This terminology is used by many authors, among which there are Dürr, Goldstein and Zanghì (DGZ 1992), (Goldstein 2001), Bricmont (Bricmont 2001), Ruelle (Ruelle 1991) and (Allori et al. 2005). The precise mathematical sense of $\mu(E) << 1$ can be given in the framework of measure theory and mathematical theory of probability by means of three different theorems of probability theory with different degrees of generality (the laws of large numbers, the theory of large deviations and the theorem of the central

[2]It is again of the order of $10^{10^{23}}$.

limit). All of them guarantee, roughly, that the measure of the set of exceptions to the thermodynamical behavior gets smaller and smaller as N, the number of atoms, gets bigger and bigger.

In the Newtonian case, the natural measure is the uniform one, so called Lebesgue-Liouville measure. It is considered natural not really because it is uniform but instead because it has the crucial feature of being invariant under time evolution so that we are not really privileging any instant in time. This means that if we take a subset A of the phase space Ω and the subset A_{-t} composed of all the points that evolving according to the law of motion will end up in A at the time t, then $\mu(A) = \mu(A_{-t})$.

8.1.2 Bohmian Mechanics

What if the fundamental physical theory is quantum mechanics? If the fundamental theory of the world will turn out to be Bohmian mechanics, one might expect Boltzmann's explanation to go roughly along the same lines as in the case of classical mechanics at least because the dynamics is of the same type as the classical one and the phase space might be plausibly thought as the space of the couples (Q, ψ), instead of (Q, P). As anticipated, in Bohmian mechanics the same role of the uniform measure is played by an equivariant measure in which the form of the distribution remains constant in time: a distribution on the primitive ontology $\mathscr{X}$ is equivariant if it changes compatibly with the law of evolution of $\mathscr{X}$. That is, if at time $t = 0$ $\mathscr{X}_0$ is distributed according to the equivariant measure μ_0, then at time t $\mathscr{X}_t$ is distributed according to the measure μ_t, evolved in time according to an equation compatible with the one of $\mathscr{X}$. In the case of Bohmian mechanics, we have that $|\psi|^2$ is equivariant: if the initial configuration Q_0 is distributed according to $|\psi_0(Q_0)|^2$, then at time t the configuration Q_t will be distributed according to $|\psi_t(Q_t)|^2$. Graphically, this compatibility is illustrated nicely by Dürr, Goldstein and Zanghí in (DGZ 1992), and reported in 8.1.2, where $U_t = \mathrm{e}^{-\frac{i}{\hbar}Ht}$

and F_t^ψ is the evolution for the measure induced by ψ.

$$\begin{array}{ccc} \psi & \rightarrow & \mu^\psi \\ U_t \downarrow & & \downarrow F_t^\psi \\ \psi_t & \rightarrow & \mu^{\psi_t} \end{array}$$

As discussed in (DGZ 1992) and as we have already pointed out in Section 8.1.2, if $|\psi|^2$ is the measure of typicality, then this is sufficient to ensure that we can recovered the empirical statistical regularities that we find in the quantum world in terms of the microscopic description of Bohmian mechanics just in the same way as thermodynamics is recovered from classical mechanics.

Therefore, the idea of explanation based on typicality is that we have explained a phenomenon if we have shown that it is typical with respect to a certain measure. We have explained a phenomenon in terms of a fundamental physical theory if, having chosen the primitive ontology of the theory, most of its initial conditions lead to states of the world that we empirically observe.

The specification of "most" depend on the choice of a particular measure, the one that is naturally selected by the dynamics. In classical mechanics it was the uniform distribution, in Bohmian mechanics it is the equivariant distribution: both of them are such that they are compatible with the dynamics and therefore do not depend on the choice of a particular time. In other words, choosing the primitive ontology $\mathscr{X}$ corresponds to a choice of the relevant phase space Ω, and choosing the dynamics η correspond to select a natural measure μ on Ω. The natural measure is the one that is invariant under time evolution. If the measure is unique, then giving the dynamics is just yields the measure.

8.2 Typicality versus Probability

In the previous section we talked about statistical regularities. Given that in philosophy what is the correct interpretation of probability is an open problem, one might expect me to enter into the debate about the meaning of probability. Nonetheless, what I believe is that the debate about this issue is largely irrelevant to understanding the role of probability *in physics* I am considering right here, since the notion of typicality is sufficient. This has already been emphasized by Goldstein in (Goldstein 2001) and by Zanghí in (Allori et al. 2005). For this reason, let us forget for a moment all the philosophical literature about probability and start reflecting about the role of the measure of typicality μ in the theory. It counts the number of exceptions to the thermodynamical behavior. In other words, all the relevant facts concerning the mathematical and interpretative properties of a probability measure, insofar as the applications we are considering here are concerned, are the following:

- The measure μ is used to count the states,
- If E is the set of "exceptional" states (i.e. those that give rise to regularities that are not observed), we have that $\mu(E) << 1$.

$\mu(E) << 1$ expresses the fact that in any explanation given by statistical mechanics what is really relevant is that the volume of the set E of the "exceptional" (i.e. with the incorrect macroscopic behavior) microstates or initial conditions is small. More precisely, if we want to explain why a macroscopic system remains in a thermodynamic equilibrium described by the macroscopic state Γ, with $\mu(E) << 1$ we mean that the measure of the exceptions (that is the extension of the set of initial conditions whose temporal evolution brings the system out of the equilibrium state) is very small with respect of the measure of the equilibrium macrostate. In other words, the sense of typicality we have in mind when we write $\mu(E) << 1$ is the one characterized by the measure $\mu_{\Gamma_{eq}}(E) = \mu(E)/\mu(\Gamma_{eq}) << 1$. Instead, when the system is not in the equilibrium macrostate, but it belongs to a macrostate of volume Γ, with $\mu(E) << 1$ we mean that the measure of the exceptions to the correct thermodynamical behavior is very small with respect to the measure of the state Γ. To put it differently, the measure of typicality relevant in this case is $\mu_{\Gamma}(E) = \mu(E)/\mu(\Gamma) << 1$. Therefore, it is important to always keep in mind the typical behavior of the system given the macroscopic constraints that are present in the concrete macroscopic situation we want to explain in microscopic terms.

It should be now clear, then, that all the other aspects of the measure do not play any role in the physical explanation of a phenomenon. Note that we are not claiming that $\mu(A)$, where A is for example the set of microstates for which the entropy grows, cannot be interpreted as the degree of belief of an external "observer" making an inference on the probability that the entropy will arise. In fact μ might also have this role. What we are noting is instead that such a connotation does not play any role in the explanation of a physical fact. This plays a role in another context: in the justification of why, on the basis of all the empirical and theoretical evidence, we have a reasonable degree of belief about the growth of entropy. But this is not what we are considering right here.

Analogous considerations can be made about the connotation of probability in terms of relative frequencies (see (DGZ 1992)). About this, one should reflect on the obvious fact that there is only one universe, and this is a unavoidable difficulty to interpret μ in terms of relative frequencies. But a probabilistic description of the phenomena that happen in the limited portion of the universe in which we live does not present any problem. In a controllable experimental situation the probability that the physics is talking about is always a relative frequency, meaning with this a well defined regularity in a given class of experiments with initial macroscopic preparations substantially identical.

The relative frequencies predicted by a theory are always empirical distributions in a single history of the world. In fact, suppose that μ is the relative frequency in a set of universes. This would mean that if we repeat the same experiment (for example the spreading of a gas initially concentrated in the left corner of a box) in the different universes (whatever this would mean) we would obtain that the entropy grows in the vast majority of the universes. Nevertheless, this fact has no implication whatsoever in the justification of the reason why in this universe, if we prepare the gas in the left corner of a box, in the vast majority of cases it will spread. In other words, if we want to have some contact with the physics, all that is relevant are the empirical distributions: relative frequencies in this universe in a set of definite events in this Universe. This is because what is relevant is not to sample into a set of universes but to sample in the space and time of a single universe that corresponds to a macrostate fixed by its initial condition X. For this reason, once we realize that there is only one world relevant to us, the basic meaning of probability (at least as far as this concept is used in formulating the predictions of the theory) is in the specification of typicality: a specification according to which the empirical distributions theoretically predicted are typical.

8.3 Indeterministic Theories

Up to now we have considered the explanation based on typicality of the statistical macroscopic phenomena in the case in which the fundamental theory of the world is deterministic. It can explain all the experimental statistical regularities of classical thermodynamical systems, including the convergence to equilibrium, as the ones produced by histories of the primitive ontology of classical mechanics that are typical with respect to the uniform distribution. In addition, we have explained the experimental quantum statistical regularities as the ones produced by histories of the primitive ontology of Bohmian mechanics that are typical with respect to the (equivariant) $|\psi|^2$ distribution.

Suppose now that the fundamental physical theory is an indeterministic one, a theory in which the histories of the primitive ontology are governed by a stochastic law. Suppose, for example, that a theory like GRWm or GRWf is true. In this case the situation seems quite different. In fact, on the one hand we have seen that both GRWm, GRWf and BMp can be described in terms of the couple $(\mathscr{X}, \eta)$. On the other hand, in an indeterministic theory probabilities seem to be already built into the theory so that, arguably, the explanation of the macroscopic statistical regularities in terms of the fundamental physical theory could be easier than in the deterministic case.

We have seen that the natural measure should be (suitably) independent on time. For this reason, we need a notion of invariance under time transformation. But in an indeterministic theory it seems hard to make sense of this kind of notion. I will argue instead that this is not impossible and that the dissimilarities between deterministic and stochastic theories is only apparent and not fundamental. In order to see this, let us borrow the following toy example from (Allori et al. 2005) which is due to Ehrenfest and is the prototype of any indeterministic process. Suppose there are N numbered balls in two boxes, one on the right (R) and on the left (L). The microstate in this case is in general a list $X = (1, ..., N)$, such that the k-*th* entry is 0 if the k-th ball is in the right box and is 1 if it is in the left one. Now let us take, randomly, a ball from a box and put it in the other one. Consequently, the microstate changes. If the initial state is $X_0 = (1, 1, 1, ...)$ (that is, all the balls are initially in the L box), it can be shown that, increasing the number of balls randomly exchanged between the two boxes, the number of balls contained in both of the boxes will tend to be the same. The equilibrium macrostate is therefore the one in which the balls are half in R and half in L.

What is the dynamical law characterizing the possible histories of the world according to the Ehrenfest model? A first possibility would be to describe the law in pure probabilistic terms: if the system is in the state X_0 at a given initial time, at a later time the system will have certain probability to evolve in one of the 2^N possible states. This is way of proceeding is fine, but it has the defect of not making the reversibility of the law directly visible or possible at all. This is because such a description suggests an image of the world in which the past is completely fixed and the future completely open. In this view, because of this deep asymmetry between past and future, it seems that the notion of time reversal itself ceases to have any meaning at all.

Nevertheless, we are not obliged to take this route. It is possible to reformulate the law in a mathematically equivalent way that suggests a different image of the world. To understand this, let us start from the microstate X_0 at $t = 0$. There are N possible states X_1 at the time $t = 1$. At $t = 2$, the possible states are N^2, ... at $t = n$ they are N^n. In other words, the set of the possible histories of the world that originate from a initial microstate X_0 has the shape of a graph. The set of all the histories of the world for all the 2^N possible initial microstates is the union Ω of the graphs corresponding to the different initial microstates. We can see therefore that the "real" phase space, (the one in which the measure of typicality will be taken) for an indeterministic theory is this graph, because it is the space of all the possible histories of the world. In this way

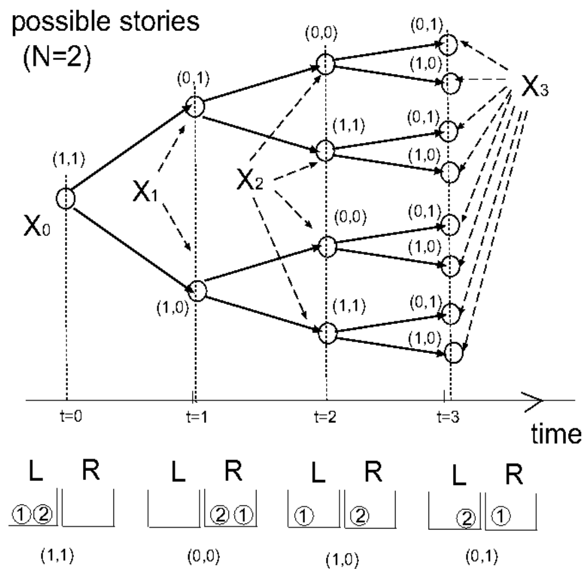

Figure 8.2: The phase space Ω of the Ehrenfest model.

it is not necessary to introduce probability in an "ontological" way as we did before: to explain all is needed is the weaker notion of typicality. But now, instead of being applied to the possible initial conditions, it is applied to the possible histories of the world. So we have the space Ω and a measure μ as before. In Figure 8.2 it is shown the phase space of the Ehrenfest model.

What is the natural measure in this case? The adequate measure of typicality in this case is given by the measure that assigns to each history the same weight. This is also the case because such measure is the one that is invariance under time evolution: if A is a set of possible histories of the primitive ontology, and A_{-t} is the set of the histories translated back in time by a time unity t, then $\mu(A) = \mu(A_{-t})$. To understand this, imagine that there is no privileged time. Therefore a possible history will extend indefinitely both in the past and the future: $X = [\ldots, X_{-1}, X_0, X_1, \ldots]$. To translate the history back in the past it means to consider another story, in which the state at time -1 coincides with the state 0 of the original sequence, the state at time 0 with X_1 and so on.

We can think of time reversal invariance in terms of the metaphor of a movie projected backwards: let T be the operator that transforms any series of photograms

$X = [\ldots, X_1, X_2, X_3, \ldots]$ in the series of photograms projected backward $T(X) = [\ldots, X_3, X_2, X_1 \ldots]$. Then if $X \in \Omega$, that is if X is a possible history of the world according to the Ehrenfest model, then also $T(X)$ will be one, that is $T(X) \in \Omega$.

The explanation of the macroscopic behavior in this case is completely analogous to the deterministic case, in the sense of $\mu(E) \ll 1$ that, without any further specification, will establish the behavior of the great majority of all possible histories, with on the average in time $N/2$ balls in the L and in the R box, and a small set E of exceptions, whose measure is negligible when N is very small. Suppose then, on the contrary, that the system at some time, let us say $t = 0$, is in a non equilibrium state, for example in which all the balls are in the R box. Then the microstate at $t = 0$ will be $X_0 = (1, 1, 1, \ldots)$. It is necessary then to consider the macrostate Γ of all the possible histories that at time $t = 0$ are macroscopically similar to X_0. In this case the entropy will increase again in the Boltzmann sense, that is to say that the great majority of the histories macroscopically similar to those originating from X_0 will evolve according to the thermodynamical behavior and the set E of the histories for which the entropy does not grow has a small measure when compared with the measure of Γ. In other words, the measure of typicality in this case is, like in the deterministic case, the measure μ_Γ obtained from μ and dividing for the measure of the macrostate Γ. This is the measure of of typicality that gives $\mu_\Gamma(E) = \mu(E)/\mu(\Gamma) \ll 1$.

8.4 Empirical Equivalence and Equivariance

I wish to make a remark concerning the agreement between the empirical predictions of Bohmian mechanics and those of standard quantum mechanics on the one hand and the disagreement between the predictions of Bohmian mechanics and the GRW theory on the other. We have seen that the difference in predictions between Bohmian mechanics and GRW is not due to the nonlinearity of the evolution of wave function, since one can reformulate, for example GRWf with a linearly evolving wave function. Because the issue of empirical equivalence of Bohmian mechanics with the predictions of orthodox quantum mechanics is ensured by the fact that the typicality measure $|\psi|^2$ is equivariant, it has been suggested (Allori et al. 2007) that the empirical disagreement between the two theories is due to the lack of equivariance of the $|\psi|^2$ distribution in the case of the GRW theory.

In fact, regardless of what the primitive ontology of a theory is, all that is required for the empirical equivalence between the theory and standard quantum mechanics is that it provide the correct $|\psi_t|^2$ probability distributions for the relevant variables Z_t. When this is so we may speak of an "effective $|\psi_t|^2$–distribution," or of *macroscopic* $|\psi|^2$ *Schrödinger equivariance*. Thus, empirical equivalence to orthodox quantum mechanics amounts to having macroscopic $|\psi|^2$ Schrödinger equivariance.

GRWf (or GRWm) predicts (approximately) the quantum mechanical distribution only under certain circumstances, including, e.g., that the experimental control over decoherence is limited, and that the universe is young on the timescale of the "universal warming" predicted by GRWf/GRWm (see (Bassi and Ghirardi 2003) for details). Moreover, we know that GRWf, roughly speaking, makes the same predictions as does the quantum formalism for short times, i.e., before too many collapses have occurred. Thus, GRWf yields an effective $|\psi|^2$–distribution for times near the initial time t_0. Now, if GRWf were "effectively $|\psi|^2$–equivariant," its predictions would be the same as those of quantum theory for all times. It is the absence of this macroscopic $|\psi|^2$ Schrödinger equivariance that renders GRWf empirically inequivalent to quantum theory and to Bohmian mechanics.

The most succinct expression of the source of the empirical disagreement between BMp and GRWf is thus the assertion that BMp is effectively $|\psi|^2$-equivariant relative to the Schrödinger evolution while GRWf is not. The macroscopic Schrödinger equivariance of BMp follows, of course, from its microscopic $|\psi|^2$ Schrödinger equivariance, while the lack of macroscopic $|\psi|^2$ Schrödinger equivariance for GRWf follows from the warming associated with the GRW evolution and the fact that GRWf is microscopically equivariant relative to that evolution. In fact, it follows from the GRW warming that there is, for GRWf, no equivariant association $\psi \mapsto \mathbb{P}^\psi$ with ψ a Schrödinger-evolving wave function.

8.5 Typicality and Primitive Ontology

To stress the point just made, let us take God's perspective. In the deterministic case God has created the histories of the primitive ontology $\mathscr{X}_t$ and a typical initial condition for the primitive ontology compatible with the initial macrostate Γ, that is, he has chosen in the phase space Ω according to the measure μ_Γ.

What about the indeterministic case? We can again say that God has created the possible histories of the world and a typical history of the world compatible with the initial condition, defined by Γ, that is he has chosen in the space of the possible histories of the world Ω according the measure μ_Γ.

The two situations are exactly analogous to each other: we have a phase space and a measure selected by the dynamics as the time invariant one. If the measure is natural in the sense just specified and it is unique, then giving the dynamics is just giving the measure μ. While fixing the primitive ontology and the variable necessary to close its dynamics (in space or in space-time) is just giving the phase space Ω.

Therefore, in both deterministic and indeterministic cases, the couple (primitive ontology, dynamics), $(\mathscr{X}, \eta)$, selects the couple (phase-space,measure), (Ω, μ), given by the space of possible histories of the primitive ontology and the measure of typicality on it, and this is what defines the theory.

This structure of the theory, the relation between physical laws and typicality, seems absolutely general. Not only deterministic theories are analogous to the indeterministic ones but also their structure is the same.

8.5.1 Typicality and Physical Laws

Therefore, in both deterministic and indeterministic cases, the couple (Ω, μ), given by the space of possible histories Ω and the measure μ, defines the laws of physics. Given any contingent fact, that is any initial condition X_0 and its corresponding macrostate $\Gamma = \Gamma(X_0)$ of all the histories macroscopically similar to those originating from X_0, the measure μ_Γ defines the correct theory, that is it provides the a clear notion of *typical* space–time histories that are compatible with the contingent empirical fact.

Two remarks: First, in this view, phase space plays an "auxiliary" role: the notion of state, of point in phase space, provides a complete dynamical description of the world at any time. In other words, the phase space point is only a convenient representation of the trajectory $Q(t)$ of the primitive ontology, given the law of motion and the initial condition. Second, the necessary feature of the measure of typicality is not of being uniform but being invariant under time translation. The reason for this is that the role of μ is to count the possible histories of the world and, in order to do this, it is sufficient for it to count the points in phase space at an arbitrary time. It will be necessary therefore to guarantee not to privilege any particular time and then to allow the measure to be invariant under time translation. Once we know μ, in order to take into account the facts of our world, we build μ_Γ, conditionalizing μ on the realization on events Γ that have happened in our world.

It is a widespread prejudice that it is necessary for the measure to be uniform. This view has been sustained by the idea that the measure, instead of being considered as a part of a law, must be considered as an additional postulate and that the properties of this postulate must reflect the state of ignorance of an external observer, somehow uniform over the possible histories of the world compatible with the information he has. According to the view we have expressed so far, that privileges the notion of typicality to the one of probability, the situation is reversed: given the law (Ω, μ) and given an empirical fact (a well defined event Γ in space-time), then it is justified to base all our statistical inferences about what happens in the past or the future of Γ, on the measure μ_Γ. In other words, the statistical inference given by μ_Γ is based on all the theoretical evidence, the physical laws (Ω, μ), and on all empirical evidence (our knowledge that the event Γ has happened).

8.5.2 Common Structure of Explanation Based on Typicality

In the last chapters of his book *Time and Chance* (Albert 2001) David Albert summarizes the ingredients of the Newtonian statistical–mechanical explanation as follows:

- The Newtonian law of motion ($F = ma$),

- The past hypothesis (according to which the world started in a particularly low entropy state)
- The statistical postulate (according to which the correct probability distribution to make inferences about the past and the future is the uniform measure on the regions of the phase space that are compatible with all other information we happen to have, both in term of laws and in terms of empirical contingent facts).

After having discussed some of the formulations of quantum mechanics, Albert arrives to the a similar characterization of the statistical explanation based on an universal theory with the dynamical scheme given by GRW. In this case, according to him the ingredients can be reduced to the following:

- The GRW law of motion (that is, the time-dependent evolution of the wave function according to GRW theory),
- The past hypothesis.

Albert underlines that in the GRW scheme, contrarily to what happens in the Newtonian case or in Bohm's theory, there is no need to add any statistical postulate. In fact, to say it plainly, given that GRW is a theory in which the evolution of the wave function is indeterministic, probabilities intervene only once, at the dynamical level, contrarily to what happens in the case of the deterministic theories, in which we need to add a statistical postulate. This fact leads Albert to conclude that the GRW theory is to be preferred to the other alternative quantum theories at least in this regard.

But if the discussion given above is correct, the two schemes, stochastic and deterministic, have basically the same structure, all is needed in both indeterministic and deterministic cases is the couple (Ω, μ), where Ω is the (generalized) phase space and μ is the measure of typicality. In these cases, then, there cannot be any real difference, not to talk about improvement, between the different explanations that the two theories, deterministic and stochastic, can give. In fact, note that the distinction that Albert makes between 1 (law of motion) and 3 (statistical postulate), that is (at least partially) crucial for his argument in favor of GRW, is actually absent in the argument I just gave in which the role of statistical postulate has been "absorbed" by the measure μ. In light of the notion of typicality as a primitive instrument on which we can base any statistical inference, the law of motion is the couple (Ω, μ). The measure of typicality μ_Γ is *not* a consequence of the statistical postulate but it is an essential part of the law, it is an essential part of the structure of all the possible histories of the world. When the law is supplemented by the past hypothesis (which is essential to guarantee that we were not produced by a fluctuation from equilibrium state), the explanation of what happens in the world is in the sense of typicality with respect to the measure μ_Γ obtained conditioning μ to the realization of the initial macrostate of the world Γ.

Both in deterministic and stochastic dynamics then, the fundamental ingredients for a mechanical explanation of the world are always the same, the couple (Ω, μ). The explanation provided by a fundamental physical theory based on a primitive ontology in three–dimensional space is always in terms of histories of primitive ontology in space–time.

8.6 Probability and Many World

It is usually argued (see for example, (Albert 1992)) that the most troublesome problem of many world is that it is not clear where the probabilistic predictions of quantum mechanics are coming from. In fact, it is argued, GRW is intrinsically written in probabilistic terms so that it explains the statistical regularities in terms of objective probabilities. In Bohmian mechanics we have particles with an intrinsic indeterminacy in their positions so that the world appear to us as if it was probabilistic. But what about many world? It is in response to this worry that people (Wallace 2003), (Deutch 1985) have proposed that in Everett probabilities should be intended in the framework of decision theory as constraints on the degrees of belief of a rational agent.

The discussion so far in terms of typicality should have made clear that:

- Probability is a much stronger constraint than typicality and typicality is the only ingredient necessary in the context of explaining empirical statistical macroscopic regularities,

- In deterministic theories like Bohmian mechanics the statistical regularities are accounted for by the typical histories of the primitive ontology,

- Therefore, in many worlds theory there is no special problem of probability, since it is a deterministic theory like Bohmian mechanics, so that whatever work in the context of Bohmian mechanics also works in the many world theory.

This last point has been emphasized by Bell in (Bell 1987) and by Everett himself:

> We wish to make quantitative statements about the relative frequencies of the different possible results [...] for a typical observer state; but to accomplish this we must have a method for selecting a typical element from a superposition of orthogonal states. The situation here is fully analogous to that of classical statistical mechanics, where one puts a measure on trajectories of systems in the phase space by placing a measure on the phase space itself, and then making assertions [...] which hold for "almost all" trajectories. This notion of "almost all" depends here also upon the choice of measure, which is in this case taken to be the Lebesgue measure on the phase space. [...] Nevertheless the choice of Lebesgue measure on the phase space can be justified by the fact that it is the only choice for which the "conservation of probability" holds, (Liouville's theorem) and hence the only choice which makes possible any reasonable statistical deductions at all (Everett 1950).

In addition, notice that stochastic theories like GRW do not necessarily imply there are objective probabilities, since they can be formulated in a way that allows an explanation of the statistical regularities again in terms of typical histories of the primitive ontology (in the relevant space and according to the relevant measure).

8.7 Typicality and Explanation

We can explain all macroscopic properties of macroscopic bodies just through the motion of the primitive ontology and its dynamics. A natural question then arises: Are all explanations, explanations of this kind? With typicality, do we explain everything? We will see that the answer is negative. Even if typicality can account for a lot, in particular it accounts for how matter gets organized, there is also a lot that remain outside.

8.7.1 Does Typicality Explain Everything?

The first problem is to understand why in the world there are such and such macroscopic properties, why matter gets organized in such and such structures (molecules, crystals, genes, viruses, cells, cats, tigers, tables and chairs). One could think that there is no explanation of why matter gets organized the way it is.

I think that the very idea of denying that there is an explanation of why the macroscopic world is the way it is in terms of the microscopic constituents and therefore considering various ontologies on different scales seems to me just to throw in the towel and give up. At least, we should try to explain how such and such structures we see in the world do arise in terms of basic laws and components. And only if for some reason we realize that we cannot do it then we should take another route. I believe that the example discussed above has taught us that we can explain some macroscopic structures in terms of primitive ontology and typicality, so we need not introduce any higher–level ontology.

What we should ask now is whether or not it is always the case. That is, can we explain all the features of the macroscopic world in terms of primitive ontology and typicality? I think we should be careful: it might well be that typicality is all we have to explain the world but what we see around us cannot be explained only in terms of fundamental dynamical laws and typicality.

Consider the following example given by Richard Feynmann (Feynman 1967)[3]. As a matter of fact, any living creature can use only a particular kind of an amino acid called alanina that is left handed. Note though that the laws of physics make it equally possible for the alanina to be left handed (L-alanina) or right handed (R-alanina) and in fact we can produce R-alanina in the laboratory. But if we provide an animal only with R-alanina, then we make him die because it cannot use it. So what would be the conclusion that we should draw from these facts? The most natural thing to think is that at the time when the first animals were developing there was much more L-alanina than R-alanina around and therefore those animals who could use the left one were

[3]He does not talk about typicality, he just provides the example to show whether physical laws are symmetric under mirror reflections.

selected. What is the explanation of the fact that there was practically only one kind of alanina instead of two? This can be explained in terms of typicality: suppose at one moment one kind alanina (say the left one) his one more in number that the right one. That is, at that moment there are 5000 L-alanina and 4999 R-alanina. Then one could say that the typical displacement from the equilibrium (meaning the two amino acids being in the same number) would engage a chain reaction that leads one to prevail in number. One can build up such a theory and then have a typicality-based explanation. Note, though, that such a theory (and any other of that kind) will only be able to explain why one kind of alanina will typically prevail. But they will say nothing about *which one*. They are not able give any reason why in the actual world it was the L-alanina and not the right to prevail. To explain that, the typicality explanation is useless. The same kind of argument would apply to the explanation of the fact that there is more matter than anti-matter in the actual world. It is not clear to me, though, what kind of explanation one would want in that case, even if I am not suggesting that there should not be one.

8.7.2 Macroscopic Language and Primitive Ontology

On the one hand, we have said that tigers and cows can be explained in terms of the primitive ontology. But on the other hand, even if tigers and cows are made of quarks and gluons, yet we do not explain their behavior (e.g., that a particular tiger in the jungle ate a particular cow that was passing by) in terms of the dynamics quarks and gluons of the cow and the tiger. Why does that explanation on the macroscopic level uses macroscopic language and not the language of the fundamental physical theory? I think that it is possible to understand why the explanation does not seem to go along with the arrow primitive ontology $\rightarrow$ macroscopic ontology. This just amounts to recognizing that high level descriptions are useful to us, which are macroscopic beings. To demand that the explanation reflects this arrow is unreasonable. What is the reason why we should think it would be the case? There is no reason at all: we should not forget that we are creatures very limited in our mathematical and computational abilities, our possibility of gathering factual information about the world is limited by our means and by intrinsic features of the world. What else could we do if not moving upward and downwards along the chain that connects the primitive ontology with the macroscopic ontology?

This does not mean that the idea of thinking of a fundamental physical theory as a mathematical structure grounded of primitive ontology is unsatisfactory. In contrast, it is the appreciation of the hierarchical structure primitive ontology $\rightarrow$ macroscopic ontology$_1$ $\rightarrow$ ... macroscopic ontology$_N$ that gives real confidence and certainty to our explanations, whether they move upward or downward. We understand in terms of a fundamental physical theory why molecules, crystals, proteins and tigers get formed. That is, in terms of physics we can understand why at certain scales certain stable structures appear and start interacting according laws which are autonomous. For this reason, we can use them for the explanation of the behavior of those objects at that given scale. And we are confident of the explanation in term of these (nonfundamental) laws because we are aware of the fact that these laws can be accounted for by the fundamental laws of physics.

Therefore, we can conclude that there is a difference between "explanation" of a state of affairs and "reductionism" of a concept to another one. We can explain what a tiger is given that it is composed by a bunch of particles, but we cannot reduce the concept that represents the tiger to the concepts of particles. We do not need to reduce or explain away the concepts. Rather what we want is the explanation of why such a macroscopic object like a tiger behaves like a tiger. And in order to do that we rely on fundamental physical theories, that uses different concepts, like atoms, fields and so on. Some have said [4] that we cannot explain why a tiger is furry in terms of physics because in physics there are atoms and they are not furry. But the point is to explain what "furry" means, what is its extension, knowing what the basic building blocks of the physical world are.

[4]Mauro Dorato, private communication.

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Publications

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