Can Bohmian Mechanics Be Made Relativistic?

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Abstract: In relativistic space-time, Bohmian theories can be formulated by introducing a privileged foliation of space-time. The introduction of such a foliation as extra absolute space-time structure would seem to imply a clear violation of Lorentz invariance and thus a conflict with fundamental relativity. Suppose however that, instead of positing it as extra structure, the required foliation could be covariantly determined by the wave function. This would seem to allow for the formulation of Bohmian theories that qualify as fundamentally Lorentz invariant. But would they also qualify as fundamentally relativistic?

Can Bohmian mechanics be made relativistic?

D. Dürr, S.G., T. Norsen, W. Struyve, and N. Zanghì

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… conventional formulations of quantum theory, and of quantum field theory in particular, are unprofessionally vague and ambiguous. Professional theoretical physicists ought to be able to do better. Bohm has shown us a way. (John Stewart Bell)

Bohmian Mechanics

$$\psi = \psi(\mathrm{q}_1, \ldots, \mathrm{q}_N)$$

Q: $\quad \mathrm{Q}_1, \ldots, \mathrm{Q}_N$

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi\,,$$

$$H = -\sum_{k=1}^{N}\frac{\hbar^2}{2m_k}\nabla_k^2 + V,$$

$$\frac{d\mathbf{Q}_k}{dt} = \frac{\hbar}{m_k}\mathrm{Im}\frac{\psi^*\nabla_k\psi}{\psi^*\psi}(\mathbf{Q}_1\ldots,\mathbf{Q}_N)$$

time evolution for ψ

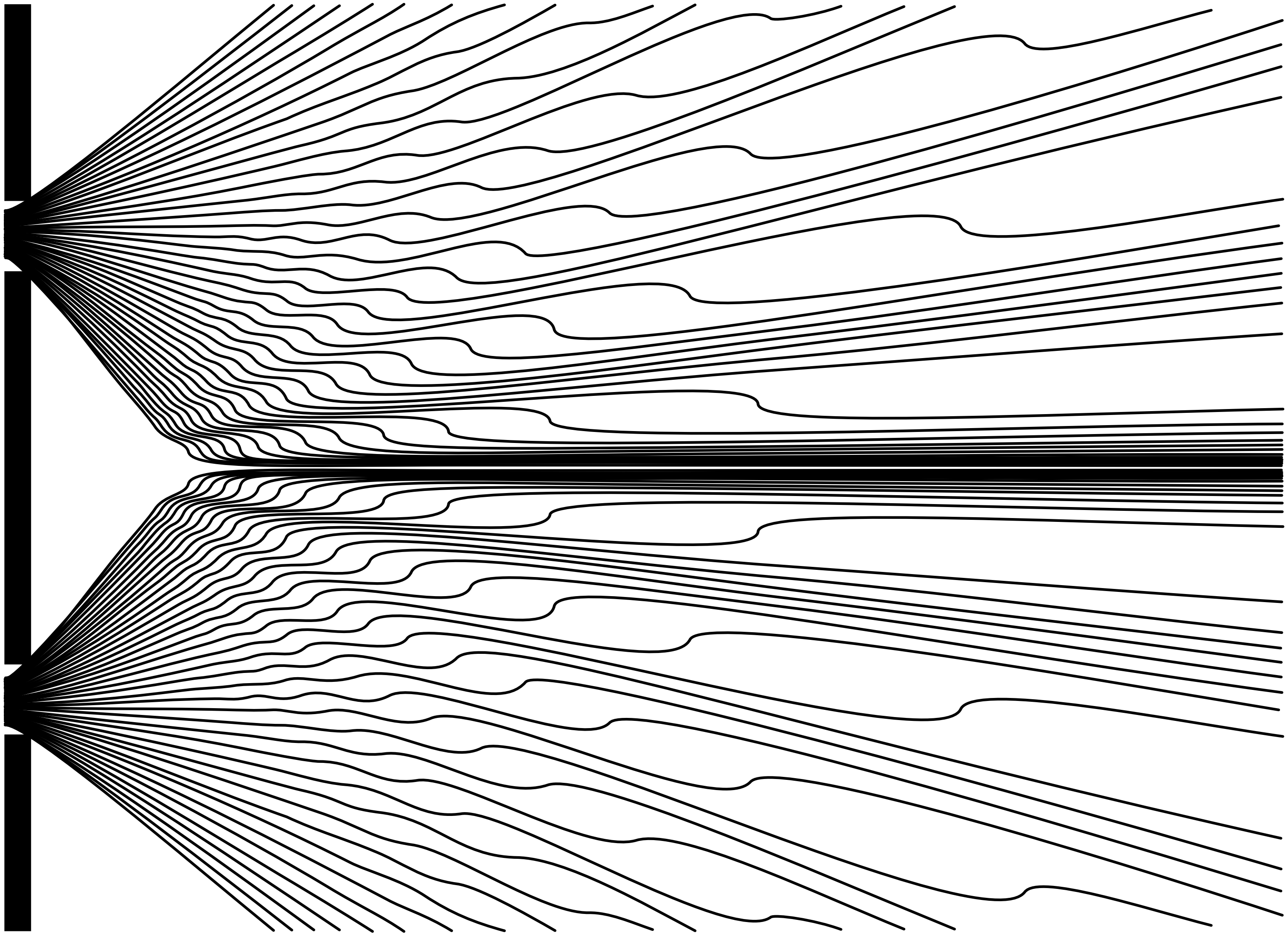
$$\nearrow$$

$$p = \hbar k$$

$$\searrow$$

time evolution for Q

$$dQ/dt = \nabla S/m$$



Is it not clear from the smallness of the scintillation on the screen that we have to do with a particle? And is it not clear, from the diffraction and interference patterns, that the motion of the particle is directed by a wave? De Broglie showed in detail how the motion of a particle, passing through just one of two holes in screen, could be influenced by waves propagating through both holes. And so influenced that the particle does not go where the waves cancel out, but is attracted to where they cooperate. This idea seems to me so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored. (John Stewart Bell, 1986)

Implications of Bohmian mechanics:

- familiar (macroscopic) reality
- quantum randomness (Dürr, G, Zanghì)
- absolute uncertainty
- operators as observables
- the wave function of a (sub)system
- collapse of the wave packet
- quantum nonlocality

The Bottom Line

Bohmian mechanics works.

Bohmian mechanics is simple.

Bohmian mechanics is obvious.

OOEOW

Bohmian Mechanics versus Bohmian Approach

• There is a clear primitive ontology (PO), and it describes matter in space and time.

• There is a state vector ψ in Hilbert space that evolves according to Schrödinger's equation.

• The state vector ψ governs the behavior of the PO by means of (possibly stochastic) laws.

• The theory provides a notion of a *typical* history of the PO (of the universe), for example by a probability distribution on the space of all possible histories; from this notion of typicality the probabilistic predictions emerge.

• The predicted probability distribution of the macroscopic configuration at time t determined by the PO agrees with that of the quantum formalism.

Objections

Bohmian mechanics makes predictions about results of experiments different from those of orthodox quantum theory so it is wrong. Bohmian mechanics makes the same predictions about results of experiments as orthodox quantum theory so it is untestable and therefore meaningless. Bohmian mechanics is mathematically equivalent to orthodox quantum theory so it is not really an alternative at all. Bohmian mechanics is more complicated than orthodox quantum theory, since it involves an extra equation. Bohmian mechanics requires the postulation of a mysterious and undetectable quantum potential. Bohmian mechanics requires the addition to quantum theory of a mysterious pilot wave. Bohmian mechanics, as von Neumann has shown, can't possibly work. Bohmian mechanics, as Kochen and Specker have shown, can't possibly work. Bohmian mechanics, as Bell has shown, can't possibly work. Bohmian mechanics is a childish regression to discredited classical modes of thought. Bohmian trajectories are crazy, since they may be curved even when no classical forces are present. Bohmian trajectories are crazy, since a Bohmian particle may be at rest in stationary quantum states. Bohmian trajectories are crazy, since a Bohmian particle may be at rest in stationary quantum states, even when these are large-energy eigenstates. Bohmian trajectories are surrealistic. Bohmian mechanics, since it is deterministic, is incompatible with quantum randomness. Bohmian mechanics is nonlocal. Bohmian mechanics is unintuitive. Bohmian mechanics is the many-worlds interpretation in disguise.

Substantive objection: Incompatibility with relativity

What is the source of the difficulty?

Take 1: Bell showed that realism is the problem!

But even supposing that somehow abandoning realism in quantum theory could preserve locality, we would have to wonder about the point of making such a bargain. Physicists have been tremendously resistant to any claims of non-locality, mostly on the assumption (which is not a theorem) that non-locality is inconsistent with Relativity. The calculus seems to be that one ought to be willing to pay *any* price—even the renunciation of pretensions to accurately describe the world—in order to preserve the theory of Relativity. But the only possible view that would make sense of this obsessive attachment to Relativity is a thoroughly realistic one! These physicists seem to be so certain that Relativity is the last word in space-time structure that they are willing even to forego any coherent account of the entities that inhabit space-time. (*Tim Maudlin*)

Take 2: Quantum Field Theory!

It does not seem possible to extend Bohm's version of quantum mechanics to theories in which particles can be created and destroyed, which includes all known relativistic quantum theories. (Steven Weinberg, 1996)

Take 3: Configuration space!

All positions at the same time:

Conflict with Lorentz invariance!

Possible response: why should we care? We will have observational Lorentz invariance.

(And do we have more with conventional relativistic quantum theory, which seems to take seriously only observations?)

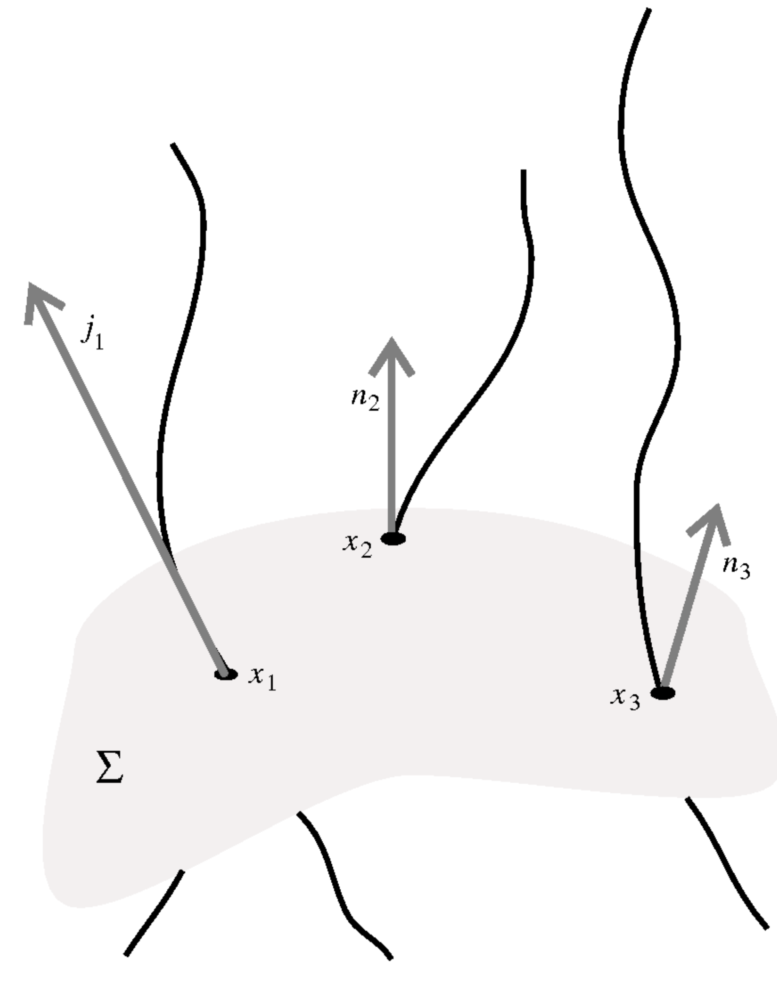
Let's accept that we should—or see what's possible even if we don't!

Lorentz frame $\rightsquigarrow$

(foliation into) $\{t = \text{const}\}$ surfaces

(simultaneity surfaces)

More generally, we need a foliation $\mathscr{F}$ into space-like hypersurfaces


j_1
n_2
x_2
n_3
x_1
x_3
Σ

$$\dot{X}_k^{\Sigma} \propto v_k^{\mathscr{F},\Psi}(X_1^{\Sigma}, \ldots, X_N^{\Sigma})$$

$$\Psi \to \mathscr{F} = \mathscr{F}(\Psi)$$

Lorentz invariance

$$\left(X', \Psi'\right) \in \mathscr{L} \iff (X, \Psi) \in \mathscr{L}.$$

$$X' \in \mathscr{L}^{\Psi'} \iff X \in \mathscr{L}^{\Psi}.$$

$$\begin{array}{ccc} \Psi & \rightarrow & \mathscr{L}^{\Psi} \\ U_g \big\downarrow & & \big\downarrow \Lambda_g \\ \Psi' & \rightarrow & \mathscr{L}^{\Psi'} \end{array}$$

$$\mathscr{L}^{\Psi} \longleftrightarrow (\underbrace{\ldots, \Gamma^{\Psi}, \ldots}_{\text{various structures}})$$

Covariance:

$$\begin{array}{ccc} \Psi & \rightarrow & \Gamma^{\Psi} \\ {\scriptstyle U_g}\downarrow & & \downarrow{\scriptstyle \Lambda_g} \\ \Psi' & \rightarrow & \Gamma^{\Psi'} \end{array}$$

Covariant Foliation

$$
\begin{tikzcd}
\Psi \arrow[r] \arrow[d, "U_g"'] & \mathscr{F}^{\Psi} \arrow[d, "\Lambda_g"] \\
\Psi' \arrow[r] & \mathscr{F}^{\Psi'}.
\end{tikzcd}
$$

Geometrical structures on space-time in QFT

- $J^\mu(x) = \langle\Psi| : \bar\psi(x)\gamma^\mu\psi(x) : |\Psi\rangle$

- $S^{\mu\nu}(x) = \langle\Psi| : \bar\psi(x)\frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right]\psi(x) : |\Psi\rangle$

- $T^{\mu\nu}(x) = \langle\Psi| : \bar\psi(x)\frac{i}{2}\left(\overleftrightarrow{\partial^\mu}\gamma^\nu + \overleftrightarrow{\partial^\nu}\gamma^\mu\right)\psi(x) : |\Psi\rangle$

- $J^{\mu_1\dots\mu_N}(x_1,\dots,x_N) = \langle\Psi| : \frac{1}{N!}\bar\psi(x_1)\gamma^{\mu_1}\psi(x_1)\dots\bar\psi(x_N)\gamma^{\mu_N}\psi(x_N) : |\Psi\rangle$

[$\psi(x)$ is the free Dirac field.]

$$D_g^{-1}\gamma D_g = L_g\gamma$$

But is it seriously Lorentz invariant?

- additional absolute space-time structure beyond the Lorentz metric?

- Modulo Ψ, there is no additional space-time structure whatsoever!

But is it relativistic?

- absolute time?
- nonlocal
- nonlocal beables
- . . .

Bruno Galvan

Does relativistic Bohmian mechanics really need a preferred foliation?

THE END

Detlef Dürr Roderich Tumulka Nino Zanghì

Bohmian Mechanics and Quantum Field Theory

Bell-type Quantum Field Theory

Minimal Processes

Equivariance

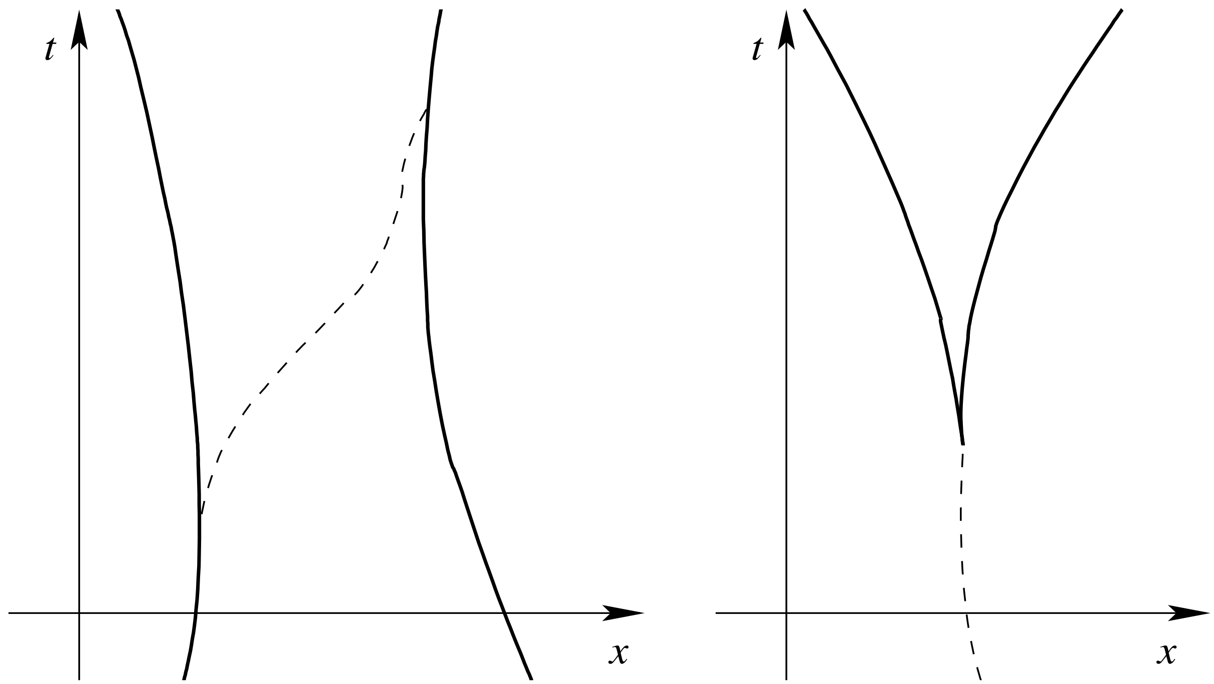
$$\left(\rho^{\Psi}\right)_t = \rho^{\Psi_t}$$

$$\rho^{\Psi}(q) = |\Psi(q)|^2$$

$\rho_{t_0}(q) = |\Psi_{t_0}(q)|^2$ *at some time* $t_0 \implies$
$\rho_t(q) = |\Psi_t(q)|^2$ *for all* t

quantum equilibrium $\rho = |\psi|^2$

thermodynamic equilibrium $\rho \sim e^{-\beta H_{\mathrm{class}}}$



(a) (b)

$$
\begin{aligned}
H &= H_F + H_B + H_I \\
&= (1/2m_F)\int d^3\boldsymbol{x}\,\nabla\psi^\dagger(\boldsymbol{x})\nabla\psi(\boldsymbol{x}) \\
&\qquad +(1/2m_B)\int d^3\boldsymbol{x}\,\nabla a^\dagger(\boldsymbol{x})\nabla a(\boldsymbol{x}) \\
&\qquad\qquad +g\int d^3\boldsymbol{x}\,\psi^\dagger(\boldsymbol{x})\phi_\varphi(\boldsymbol{x})\psi(\boldsymbol{x})
\end{aligned}
$$

$$
\phi_\varphi(\boldsymbol{x}) = \int d^3\boldsymbol{y}\Big(\varphi(\boldsymbol{x}-\boldsymbol{y})a^\dagger(\boldsymbol{y}) + \overline{\varphi}(\boldsymbol{x}-\boldsymbol{y})a(\boldsymbol{y})\Big)
$$

Data

$\mathcal{Q}$: configuration space (e.g., $\mathcal{Q} = \bigcup_{n=0}^{\infty} \mathcal{Q}^{(n)}$)

$\mathcal{H}$: Hilbert space

H: Hamiltonian

$\Psi \in \mathcal{H}$: state vector

$P(dq)$: positive-operator-valued measure (POVM) on $\mathcal{Q}$ acting on $\mathcal{H}$ so that the probability that the system in the state Ψ be localized in dq at time t is

$$\mathbb{P}_t(dq) = \langle \Psi_t | P(dq) | \Psi_t \rangle$$

Example:

$$P(dq) = |q\rangle\langle q|\, dq$$

$$\mathbb{P}(dq) = \Psi^*(q)\, \Psi(q)\, dq$$

$$P(dq)$$

$$\langle\Psi|P(dq)|\Psi\rangle$$

on $\mathcal{Q}$

$$P \times P$$

$$P(dq)P(dq')$$

on $\mathcal{Q} \times \mathcal{Q}$

$$P \times_H P$$

$$P(dq)HP(dq')$$

$$\langle\Psi|P(dq)HP(dq')|\Psi\rangle$$

on $\mathcal{Q} \times \mathcal{Q}$

$$\sigma(dq|q') = \frac{[(2/\hbar)\,\mathrm{Im}\,\langle\Psi|P(dq)HP(dq')|\Psi\rangle]^{+}}{\langle\Psi|P(dq')|\Psi\rangle}$$

$$P(dq) = |q\rangle\langle q|\,dq$$

$$\sigma(q|q') = \frac{\Big[(2/\hbar)\,\mathrm{Im}\,\Psi^*(q)\,\langle q|H|q'\rangle\,\Psi(q')\Big]^{+}}{\Psi^*(q')\,\Psi(q')}$$

Minimal Process

$$H = H_0 + H_I$$

$$L = L_0 + L_I$$

$$L_t f(q) = \frac{d}{ds}\mathbb{E}(f(Q_{t+s})|Q_t = q)|_{s=0+}$$

Example: For $dQ/dt = v(Q)$, $L = v \cdot \nabla$

$$Lf(q) \stackrel{g}{=} \mathsf{Re}\frac{\langle\Psi|P(dq)\frac{d\widehat{f}}{dt}|\Psi\rangle}{\langle\Psi|P(dq)|\Psi\rangle} = \mathsf{Re}\frac{\langle\Psi|P(dq)\frac{i}{\hbar}[H,\widehat{f}]|\Psi\rangle}{\langle\Psi|P(dq)|\Psi\rangle}$$

$$\widehat{f} = \int\limits_{q\in\mathcal{Q}} f(q)\,P(dq)$$

$$Lf(q) \stackrel{g}{=} \mathsf{Re}\frac{\Psi^*(q)\frac{d\widehat{f}}{dt}\Psi(q)}{\Psi^*(q)\Psi(q)}$$

If H is a differential operator of order at most 2, $L = v \cdot \nabla$.

(Markovian) Microscopic Processes for Quantum Field Theory?

1. baby steps

2. counterexample

3. canonical

4. challenge