TWO QUANTUM THEORIES
THAT BELL LIKED

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John Bell,
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Towards An Exact Quantum Mechanics

John Bell, 1989
What is the problem of quantum mechanics?
the measurement problem?

Erwin Schrödinger
(1887 – 1961)
Schrödinger's cat 1935
Quantum
(Empty) + (Full)
Classical
A + B
Superposition principle of QM
quantum source
power source
milk source
the deeper problem (of which the measurement problem is just a manifestation)...
Fundamental ambiguity: Nobody knows what quantum mechanics says exactly about any situation. For nobody knows where the boundary really is, between wavy quantum system and the world of particular events.

This is the problem of quantum mechanics.

It is no problem in practice — because practice is not accurate enough — and maybe never will be.
This is the problem

* Non-Relativistic Quantum Mechanics

* All the variants of Quantum Field Theory (Cut-offs, Algebraic, etc.)
Are there solutions of the problem?

Yes!

There are many, indeed!
concern

\[ \Psi = \Psi_{\text{empty}} + \Psi_{\text{full}} \]

orthodoxy
complacency

\[ \Psi \text{ is not real. what is?} \]

\[ \Psi \text{ is not all. what else?} \]

\[ \Psi \text{ is not always right. when exactly does it go wrong?} \]
forbidden words:
- system
- apparatus
- microscopic
- macroscopic
- reversible
- irreversible
- observable
- information

"for all practical purposes"

measurement

OK words:
- beable
- kinematics (possibilities)
- before
- dynamics (probabilities)
"The terminology, be-able as against observ-able, is not designed to frighten with metaphysic those dedicated to realphysic. It is chosen rather to help in making explicit some notions already implicit in, and basic to, ordinary quantum theory. For, in the words of Bohr, 'it is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms'.”

\[ A \in \mathcal{A} \quad \text{operators as observables} \]

\[ <Z> = \frac{\langle \psi, A\psi \rangle}{\langle \psi, \psi \rangle} \]

\[ \mathcal{A} \quad \text{algebra of operators on } \mathcal{H} \]

\[ \text{macroscopic variables} \]

\[ \text{classical variables} \]
“The concept of 'observable' lends itself to very precise mathematics when identified with 'self-adjoint operator'. But physically, it is a rather wooly concept. It is not easy to identify precisely which physical processes are to be given status of 'observations' and which are to be relegated to the limbo between one observation and another. So it could be hoped that some increase in precision might be possible by concentration on the beables, which can be described in 'classical terms', because they are there. The beables must include the settings of switches and knobs on experimental equipment, the currents in coils, and the readings of instruments. 'Observables' must be made, somehow, out of beables. The theory of local beables should contain, and give precise physical meaning to, the algebra of local observables.”
When von Bortkewitch collected statistics on the kicking of soldiers to death by horses, in the Prussian army, in different years, he found a Poisson distribution. Now, you don’t go out into the world looking for the Poisson distribution, you go out looking for soldiers and horses and kicks.
Ψ is not all. What else?

allowed states? jumps?
{Ψ(t, r), x(ε)...

de Broglie Bohm 1926, 1952
x's are particle pos
'pilot-wave picture'

m x(ε) = \frac{∂}{∂x} \ln \Psi(t, x(ε))

no jumps \ Ψ(0, x) = |Ψ(0, x)|^2

rational, clear, exact
agrees with experiment

Lorentz invariance?
universal wave function

\[ \Psi \]

up to the universal scale

\[ (\psi, Z) \]

down to the microscopic scale

\[ Q \]
universal wave function

up to the universal scale

\( (\psi, Z) \)

down to the microscopic scale

\( Q \)

expressing the laws which govern the behavior of the \( Q \) variables in a simple and natural way.

\( Q \) variables: what the theory is fundamentally about

for example, particles, fields, strings, et.
universal wave function \[ \Psi \]

up to the universal scale

\((\psi, Z)\)

down to the microscopic scale

\(Q\)

expressing the laws which govern the behavior of the \(Q\) variables in a simple and natural way.

\(Q = (X, Y)\)
\n\(\psi(x) = \Psi(x, Y)\)
\n\(Z = F(Q)\)

\(Q\) variables: what the theory is fundamentally about
for example, particles, fields, strings, et.

universal wave function

\(\Psi\)

up to the universal scale

\((\psi, Z)\)

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\(Q\) variables: what the theory is fundamentally about
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$\Psi$

up to the universal scale

$(\psi, Z)$

down to the microscopic scale

$Q$

$Q$ variables: what the theory is fundamentally about for example, particles, fields, strings, et.

$\psi(x) = \Psi(x, Y)$

$Z = F(Q)$

$Z \rightarrow A <Z>_\psi = \frac{\langle \psi, A\psi \rangle}{\langle \psi, \psi \rangle}$
universal wave function \( \Psi \) expressing the laws which govern the behavior of the \( Q \) variables in a simple and natural way.

up to the universal scale

\[ (\psi, Z) \]

\[ Q \]

down to the microscopic scale

\( Q \) variables: what the theory is fundamentally about

for example, particles, fields, strings, et.

\[ Q = (X, Y) \]

\[ \psi(x) = \Psi(x, Y) \]

\[ Z = F(Q) \]

\[ |\Psi|^2 \]

\[ Z \rightarrow A \quad <Z>_{\psi} = \frac{\langle \psi, A\psi \rangle}{\langle \psi, \psi \rangle} \]
plug the actual configuration $Y$ of the environment into the second slot of $\Psi(x, y)$ to obtain a function of $x$,

$$\psi(x) = \Psi(x, Y)$$

(Almost) all implications of BM follow from this formula

$$P_\Psi(X \in dx \mid Y) = |\psi(x)|^2 dx$$
GRW

$\psi$ is not right since when?

$\frac{1}{2} (\psi_{42} + \psi_{62}) \rightarrow \psi_{42}$ or $\psi_{62}$

when SC noticed? \{ 'observer'
when you noticed? \} 'mind'
when I noticed? \}

When the difference became noticeable?

stochastic? nonlinear?

Philip Pearle

Spontaneous wavefunction collapse
GRW spontaneous wf collapse. In 'one particle' system:

\[ \psi(t, r) \rightarrow \psi' \propto j(x-r) \psi(t, r) \]

with probability \[ \frac{\Delta}{\int dtdx |j(x-r)\psi|^2} \]

There are \[ \tau^{-1} = \int dx |j(x-r)|^2 \]

jumps per unit time.

In a 'many-particle' system

\[ \psi(t, r_1, r_2, \ldots, r_N) \]

independent jumping for each argument \( r \) gives \[ \frac{N}{\tau} \]

jumps per unit time. GRW:

\[ \tau \sim 10^{15} \text{ sec.} \sim 10^8 \text{ year} \]

width of \( j(x) \) \( \sim 10^{-5}\text{ cm} \).
\( \psi = \psi(q_1, \ldots, q_N), q_i \in \mathbb{R}^3, i = 1, \ldots, N \)

- for any point \( x \) in \( \mathbb{R}^3 \)
  \[ \Lambda_i(x) = \frac{1}{(2\pi \sigma^2)^{3/2}} e^{-\frac{(q_i - x)^2}{2\sigma^2}} \]
  \( \sigma \sim 10^{-7} m \)

- the evolution of \( \psi \) is the Schrödinger evolution interrupted by collapses

- When the wave function is \( \psi \) a collapse with center \( x \) and label \( i \) occurs at rate
  \[ r(x, i|\psi) = \lambda \langle \psi | \Lambda_i(x) \psi \rangle \]
  \( \lambda \sim 10^{-15} s^{-1} \)

- when this happens
  \[ \psi \rightarrow \Lambda_i(x)^{1/2} \psi / \| \Lambda_i(x)^{1/2} \psi \| \]
GRW jump spoils symmetry. Take “molecular” model of matter with “different” nuclei. For many nuclei a given value of \( \hat{\mathbf{r}} \) will be accessible only through either \( \Psi_{42} \) or \( \Psi_{62} \). Very quickly

\[
\frac{1}{\sqrt{2}} (\Psi_{42} + \Psi_{62}) \rightarrow \Psi_{42} \quad \text{or} \quad \Psi_{62}
\]

How about internal economy?

\[
\hat{\mathbf{r}} = \mathbf{\bar{r}} + ( \hat{\mathbf{r}} - \mathbf{\bar{r}} )
\]

\[
\Psi_{42,62} \propto \delta(\hat{\mathbf{r}} - \mathbf{\bar{r}} - \mathbf{\bar{a}})
\]

\[
j(\mathbf{x} - \mathbf{r}) \equiv j(\mathbf{x} - \mathbf{a} - \mathbf{\bar{r}})
\]

— approximate localization of c.o.m only — or more generally of ‘quasiclassical’ coordinates

— with internal economy little disturbed.
[...] the GRW jumps (which are part of the wave function, not something else) are well localized in ordinary space. Indeed each is centered on a particular spacetime point \((x, t)\). So we can propose these events as the basis of the ‘local beables’ of the theory. These are the mathematical counterparts in the theory to real events at definite places and times in the real world (as distinct from the many purely mathematical constructions that occur in the working out of physical theories, as distinct from things which may be real but not localized, and distinct from the ‘observables’ of other formulations of quantum mechanics, for which we have no use here). A piece of matter then is a galaxy of such events. (Bell, 1987a)
$$m(x, t) = \sum_{i=1}^{N} m_i \int d^3 x_1 \cdots d^3 x_N \delta^3(x - x_i) |\psi_t(x_1, \ldots, x_N)|^2$$

$\psi_t$ is a GRW process
Lorenz invariance

Those paradoxes are simply disposed of by the 1952 theory of Bohm, leaving as the question, the question of Lorentz invariance. So one of my missions in life is to get people to see that if they want to talk about the problems of quantum mechanics – the real problems of quantum mechanics – they must be talking about Lorentz invariance. Bell (1990)

The big question, in my opinion, is which, if either, of these two precise pictures [GRW and Bohm] can be redeveloped in a Lorentz invariant way. Bell (1990)
Lorentz invariance

\[ z' = r(z - vt) \quad c = 1 \quad t' = r(t - vzt) \]

Suppose: \( v \ll 1 \quad r \approx 1 \)

Let \( |z| \) be very large:

\[ vz \approx \pm a \]

Then Lorentz transform becomes

\[ z' = z \quad t' = t \pm a \]

i.e. for widely separated systems: L.I. \( \Rightarrow \)

relative-time invariance

--- even for nonrelativistic systems. So:

Schrödinger \( t \Rightarrow \) Dirac \( t, t', \ldots \)
What is a precise quantum theory?

(i) There is a clear primitive ontology, and it describes matter in space and time.

(ii) There is a state vector $\psi$ in Hilbert space that evolves either unitarily or, at least, for microscopic systems very probably for a long time approximately unitarily.

(iii) The state vector $\psi$ governs the behavior of the PO by means of (possibly stochastic) laws.

(iv) The theory provides a notion of a *typical* history of the PO (of the universe), for example by a probability distribution on the space of all possible histories; from this notion of typicality the probabilistic predictions emerge.

(v) The predicted probability distribution of the macroscopic configuration at time $t$ determined by the PO (usually) agrees (at least approximately) with that of the quantum formalism.
Now in what I said probably many of you think that I have been wasting not only my time but yours, and therefore I would like to end up on a more harmonious note, with some concepts of which we all approve (Fig. 21). The theories that I presented to you are certainly not beautiful. I think they are not true either; it may be that they give some hint of where truth is to be found, but in their present brutally simplistic form the truth is certainly not there. I do think however that they have a certain kind of goodness — these little spots are halos — in the sense that they are honest attempts to replace the wooly words by real mathematical equations — equations which you don’t have to talk away — equations which you simply calculate with and take the results seriously. Thank you.
TRUTH

BEAUTY

GOODNESS
quantum probability

\[ P(\hat{A} = \alpha|\hat{B} = \beta) = |< \alpha|\beta >|^2 \]

\[ |< \psi|\chi >|^2 \quad \text{probability amplitude} \]

\[ |< \psi|\chi >|^2 \quad \text{probability} \]

non- Kolmogorovian?
momentum

\[ |\langle \psi | p \rangle|^2 = \tilde{\psi}(p) \]

probability to find the value \( p \) of \( \hat{P} \) if the system is (initially) in the state \( \psi \)

Fourier transform

time of flight measurement of momentum

(Heisenberg, Bohm52, Feynman & Hibbs)

\( \psi \) wf at time 0
free evolution
measure \( \hat{X} \) at large time \( T \)

\[ \hat{X}(T) = \frac{1}{m} \hat{P}T + \hat{X} \quad \rightarrow \quad \hat{P} = \frac{m\hat{X}(T)}{T} + \frac{X}{T} \approx \frac{m\hat{X}(T)}{T} \]
Bohm

The particle has a well defined position $X$ whose evolution is guided by $\psi$

$$\dot{X}(t) = \frac{\hbar}{m} \text{Im} \left( \frac{\psi^*_t \nabla \psi_t}{\psi^*_t \psi_t} \right) (X(t))$$

$$P(X(t) = x | \psi, t = 0) = |\psi_t(x)|^2$$

$$P(\dot{X}(t) = x | \psi, t = 0) = |\psi_t(x)|^2$$

Thus the asymptotic momentum $P = \frac{mX(T)}{T} \ (T \text{ large})$ is a RANDOM VARIABLE on the space on initial conditions with probability distribution

$$P(P = p | \psi, t = 0) = |\tilde{\psi}(p)|^2$$
spin

\[
\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

\[
|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

\[
|\langle \psi | \uparrow\rangle|^2 \quad \text{probability spin up along } z
\]

\[
|\langle \psi | \downarrow\rangle|^2 \quad \text{probability spin down along } z
\]

Classically Non-Describable Two-Valuedness (Pauli)
Stern Gerlach measurement of spin

\[ H_I = \mu \vec{\sigma} \cdot \vec{B} \approx (b + az)\sigma_z \]

initial \( \Psi = \psi \otimes \Phi(z) \)
\[
\hat{Z}(T) = \hat{Z} + \frac{\hat{P}_z T}{m} + \frac{a}{2m} \sigma_z T^2
\]

Thus the RANDOM VARIABLE

\[
F_T(Z(T))_1, \quad F_T(z) = \frac{2mz}{aT^2}
\]

in the limit of \( T \) large has values

+1 with probability \(< \psi| \uparrow > |^2\)

−1 with probability \(< \psi| \downarrow > |^2\)

\( F_T(z) \) is the calibration function of the experiment

assignment of num. values to the outcome of the exp.
Morals

• Association between random variable $Z$ (numerical result of the experiment) and operators

• Operators compactly express the statistics of the experiment

\[
\bar{A} = \langle \psi | A \psi \rangle \quad \text{mean value of } Z \\
\langle \psi | (A - \bar{A})^2 \psi \rangle \quad \text{variance of } Z \\
\langle \psi | A^n \psi \rangle \quad \text{higher moments of } Z
\]
• One can completely understand what's going on in the experiments measuring momentum or spin.

• No need of invoking any putative property of the electron such as its actual z-component of spin that is supposed to be revealed in the experiment.

• There is nothing the least bit remarkable about the nonexistence of this property.

• Measurements are “active.”