TWO QUANTUM THEORIES THAT BELL LIKED

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Summer School on the Foundations of Quantum Mechanics dedicated to John Bell,
Sesto, July 2014

Towards An Exact Quantum Mechanics

John Bell, 1989

Towards An Exact Quantum Mechanics

J. S. Bell
CERN, CH-1211, Geneva 23, Switzerland

Good morning! Last night I had a dream. I dreamt that I was standing up before an audience of very distinguished people, and that I was about to give a talk, and that I started by saying that the very big difficulty in getting started in a talk like this is to find the switch on the projector. So this morning I came early and found the swtich on the projector. I don't usually dream about things like that — I think it just shows how particularly apprehensive I was about this particular talk. The work of Professor Schwinger has been among much else an extraordinary demonstration of the power and precision of quantum mechanics, and yet here I am with a title which suggests that there is something imperfect about quantum mechanics, and which might even lead you to imagine that I thought I knew how to perfect it. Well, you can only be disappointed, but I will do what I can.

The debate about quantum mechanics was and is dominated by these two men (Fig. 1). Einstein is the inspiration of those of us who think that there is a problem with quantum mechanics, and Niels Bohr is the great father figure of

What is the problem of quantum mechanics?

the measurement problem?

Erwin Schrödinger
(1887 – 1961)

Schrödinger's cat 1935

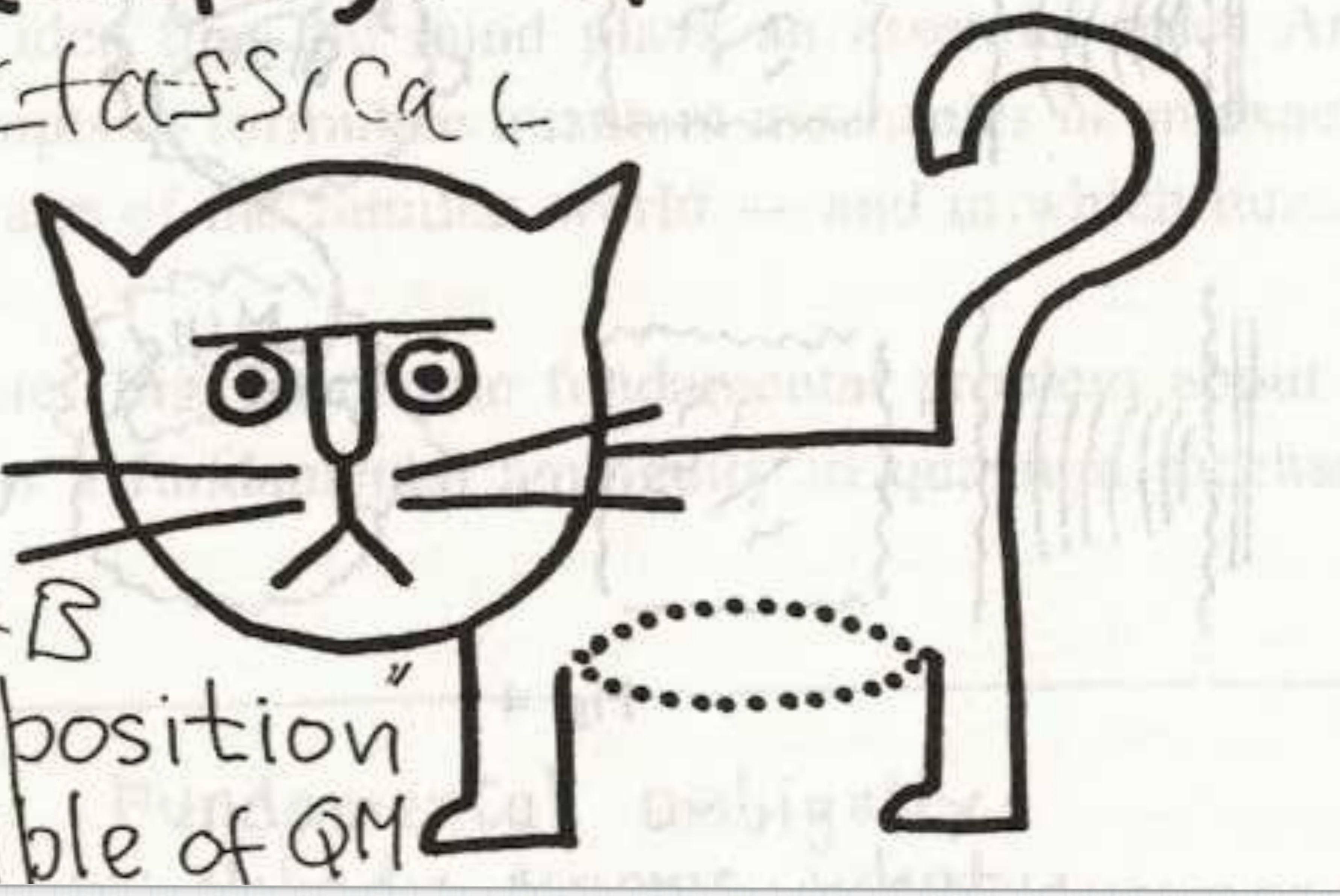
quantum

(empty)+(full)

classical

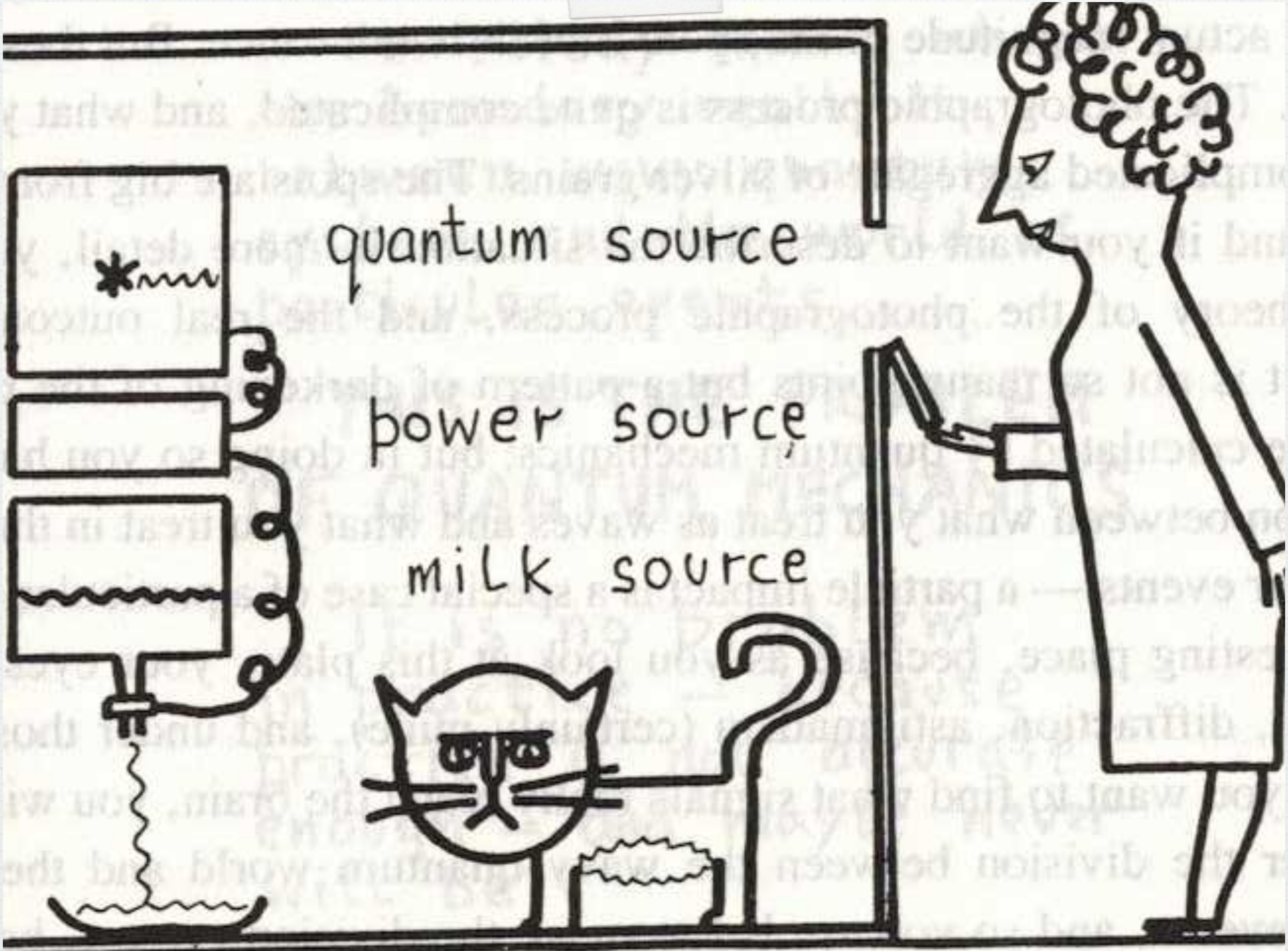
A

B



A+B

"Superposition"
principle of QM


quantum source
power source
milk source

the deeper problem (of which the measurement problem is just a manifestation)...

John Stewart Bell
(1928 – 1990)

Fundamental ambiguity:
Nobody knows what quantum mechanics says *exactly* about any situation.

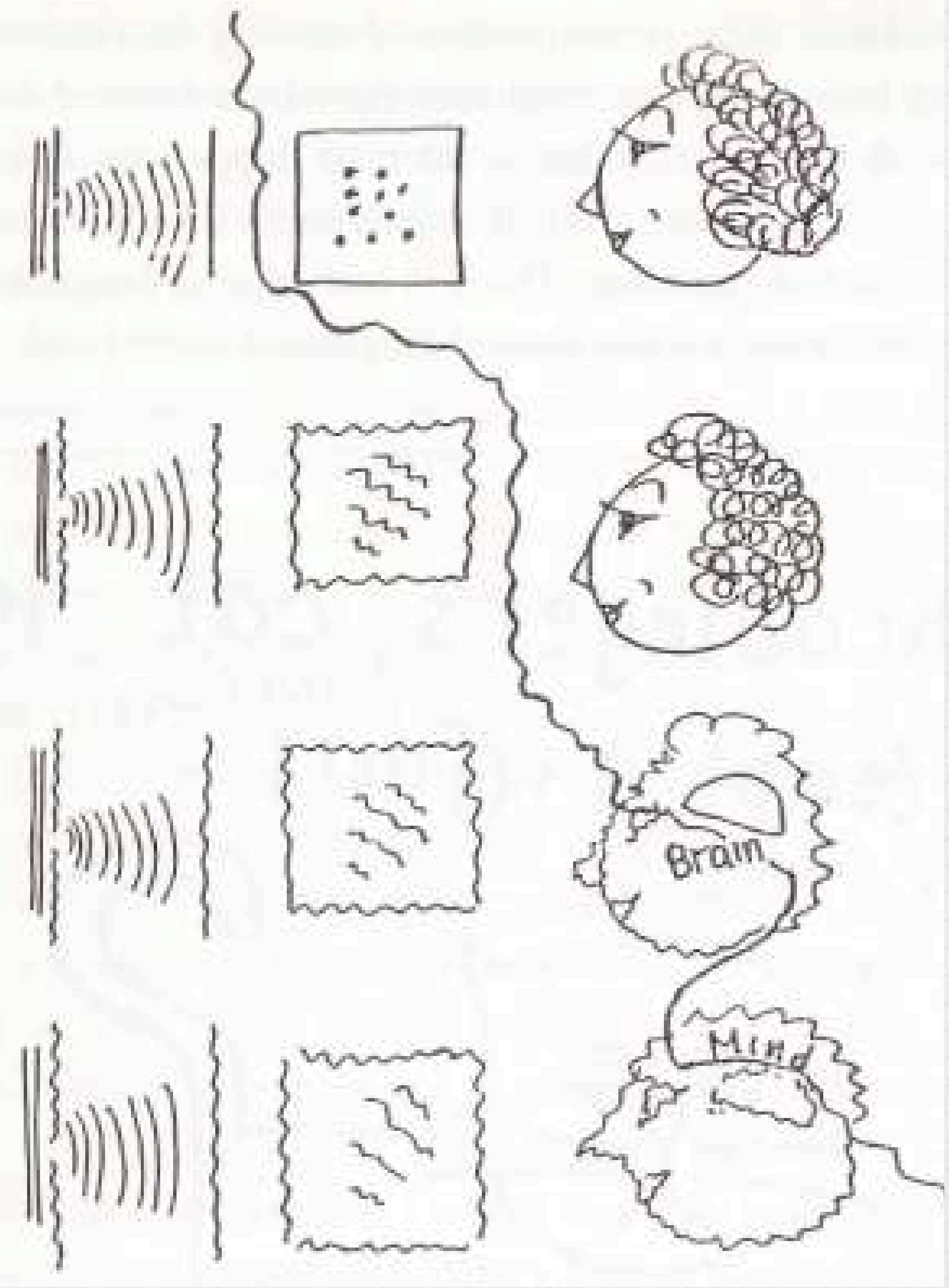
For nobody knows where the boundary really is, between wavy quantum system and the world of particular events.

THIS IS THE PROBLEM OF QUANTUM MECHANICS

It is no problem in practice – because practice is not accurate enough – and maybe never will be.

This is the problem

* Non-Relativistic Quantum Mechanics
* All the variants of Quantum Field Theory (Cut-offs, Algebraic, etc.)



Are there solutions of the the problem?
Yes!
There are many, indeed!

orthodoxy
complacency

concern

? $\Psi = \Psi_{EMPTY} + \Psi_{FULL}$

? Ψ is not real. what <u>is</u>?

? Ψ is not all. what else?

? Ψ is not always right.
when <u>exactly</u>
does it go wrong?

forbidden words:

system apparatus
microscopic macroscopic
reversible irreversible
observable
information
"for all practical purposes"
measurement

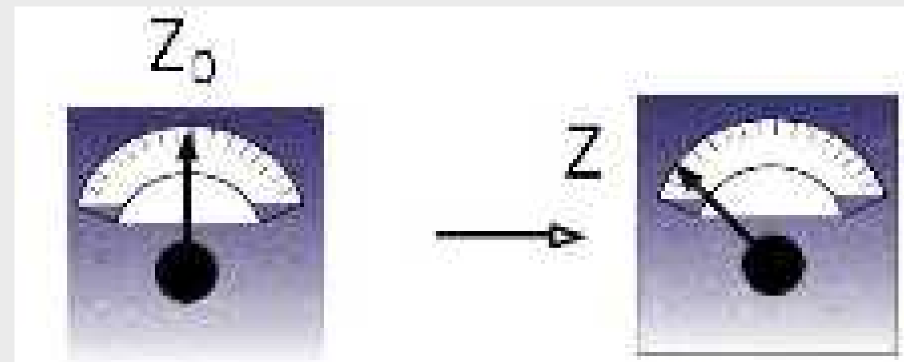
OK words:

beable

kinematics (possibilities)
before
dynamics (probabilities)

Beables

"The terminology, be-able as against observ-able, is not designed to frighten with metaphysic those dedicated to realphysic. **It is chosen rather to help in making explicit some notions already implicit in, and basic to, ordinary quantum theory**. For, in the words of Bohr, 'it is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms'."



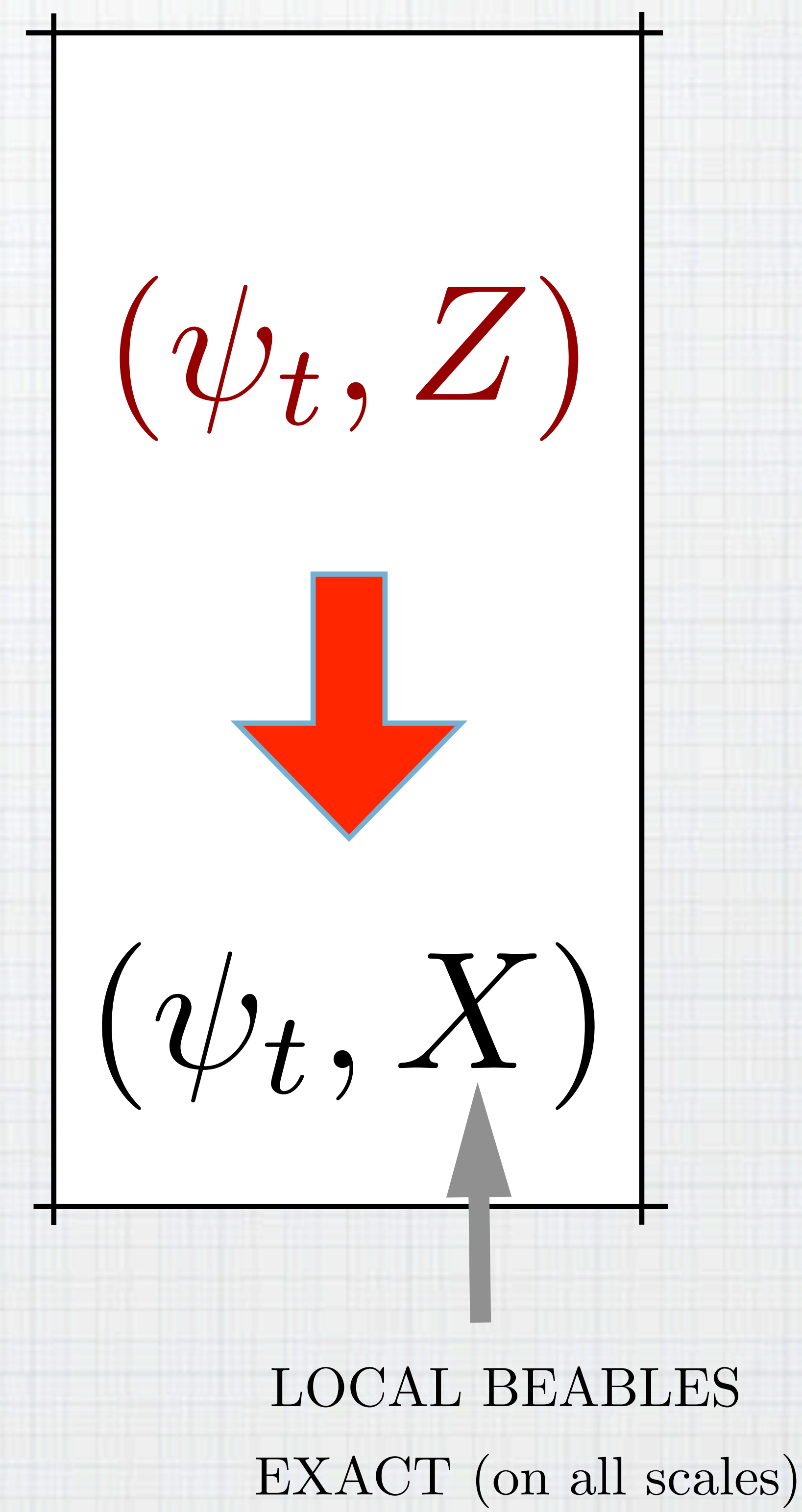
$\mathscr{A}$ algebra of operators on $\mathscr{H}$

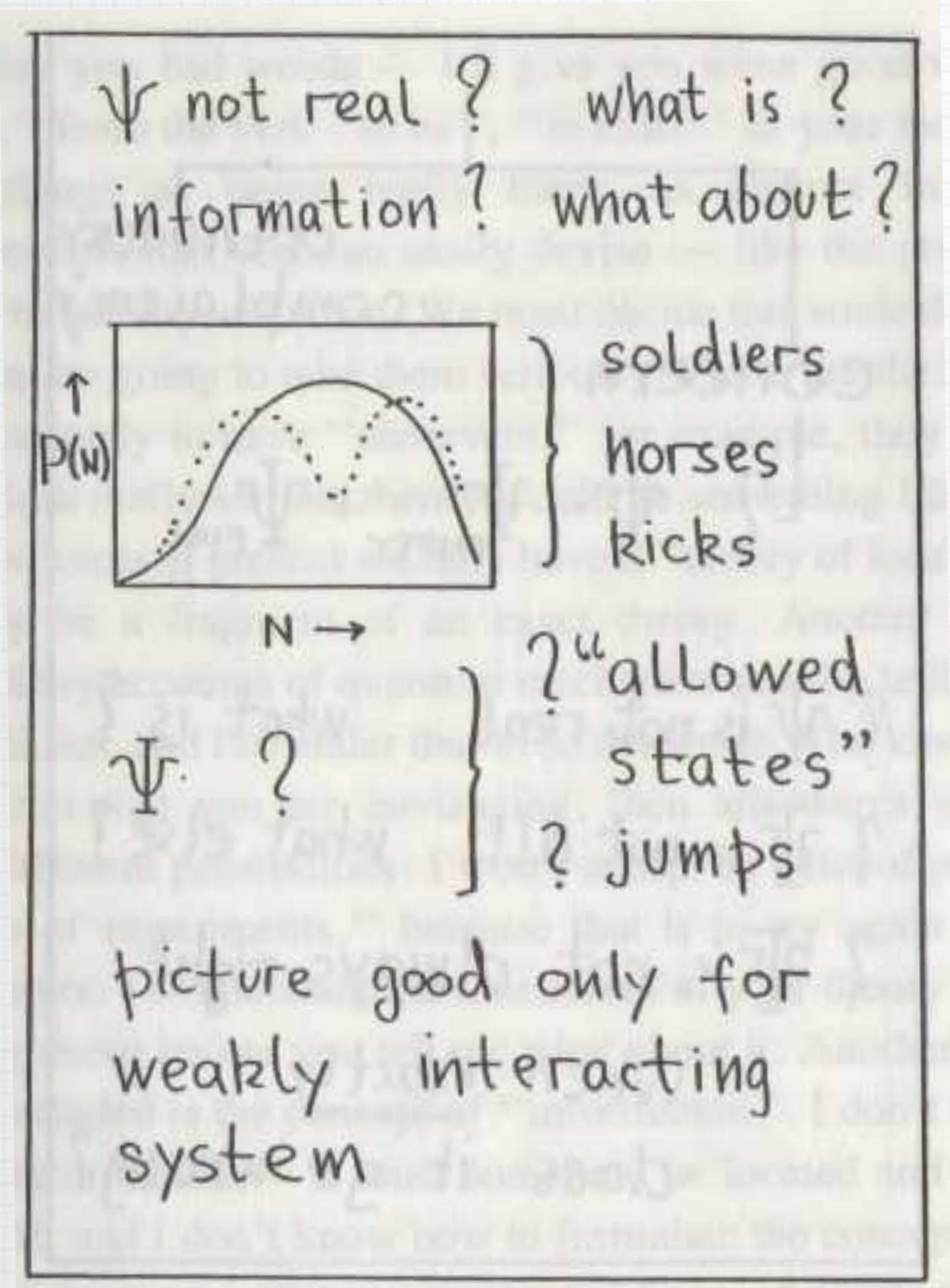
operators as observables

$A \in \mathscr{A} \longleftarrow Z$ macroscopic variables classical variables

$$< Z >_\psi = \frac{\langle \psi, A\psi \rangle}{\langle \psi, \psi \rangle}$$

“The concept of 'observable' lends itself to very precise mathematics when identified with 'self-adjoint operator'. But physically, it is a rather wooly concept. It is not easy to identify precisely which physical processes are to be given status of 'observations' and which are to be relegated to the limbo between one observation and another. So it could be hoped that some increase in precision might be possible by concentration on the beables, which can be described in 'classical terms', because they are there. *The beables must include the settings of switches and knobs on experimental equipment, the currents in coils, and the readings of instruments.* **'Observables' must be made, somehow, out of beables. The theory of local beables should contain, and give precise physical meaning to, the algebra of local observables.”**





When von Bortkewitch collected statistics on the kicking of soldiers to death by horses, in the Prussian army, in different years, he found a Poisson distribution. Now, you don't go out into the world looking for the Poisson distribution, you go out looking for soldiers and horses and kicks.

BM

ψ is not all | what else?

allowed states? jumps?

$\{\psi(t, r\ldots), x(t) \ldots\}$

de Broglie Bohm 1926, 1952

x's are particle pos

'pilot-wave picture'

$$m\dot{x}(t) = \frac{\partial}{\partial x}\, \mathrm{im} \log \psi(t, x(t)\cdot)$$

no jumps $\rho(0,x) = |\psi(0,x)|^2$

rational, clear, exact

agrees with experiment

Lorentz-invariance?

universal wave function

$$\Psi$$

up to the universal scale $\uparrow$

$$(\psi, Z)$$

down to the microscopic scale $\downarrow$

$$Q$$

universal wave function

Ψ

$\uparrow$ up to the universal scale

$$(\psi, Z) \Rightarrow (Q, \Psi)$$

$\downarrow$ down to the microscopic scale

Q

expressing the laws which govern the behavior of the Q variables in a simple and natural way.

Q variables: what the theory is fundamentally about

for example, particles, fields, strings, et.

universal wave function

$$\Psi$$

$$\uparrow$$

up to the universal scale

$$(\psi, Z) \quad \Longrightarrow \quad (Q, \Psi)$$

expressing the laws which govern the behavior of the Q variables in a simple and natural way.

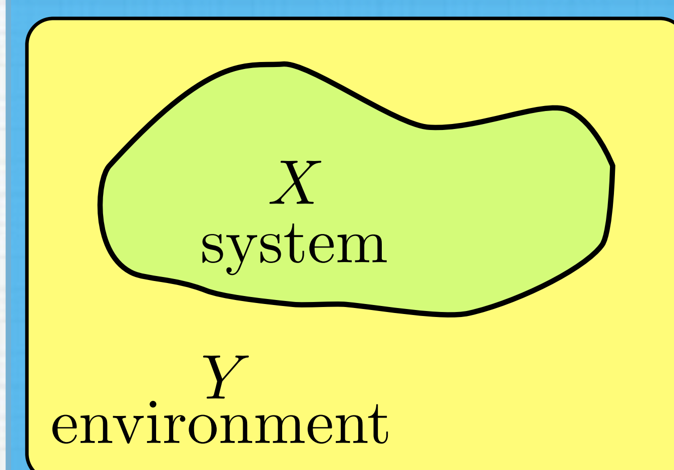
down to the microscopic scale

$$\downarrow$$

$$Q$$

Q variables: what the theory is fundamentally about

for example, particles, fields, strings, et.



$$Q = (X, Y)$$

$$\psi(x) = \Psi(x, Y)$$

$$Z = F(Q)$$

universal wave function

$$\Psi$$

$\uparrow$ up to the universal scale

$$(\psi, Z) \quad \Longrightarrow \quad (Q, \Psi)$$

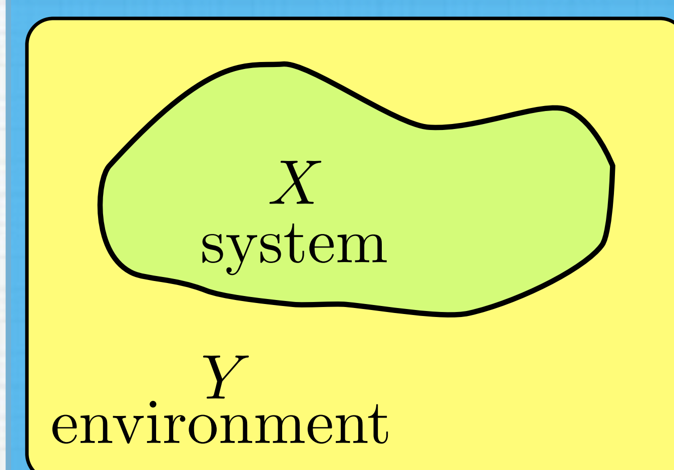
$\downarrow$ down to the microscopic scale

$$Q$$

expressing the laws which govern the behavior of the Q variables in a simple and natural way.

Q variables: what the theory is fundamentally about

for example, particles, fields, strings, et.



$$Q = (X, Y)$$

$$\psi(x) = \Psi(x, Y)$$

$$Z = F(Q)$$

$$Z \longrightarrow A \quad < Z >_\psi = \frac{\langle \psi, A\psi \rangle}{\langle \psi, \psi \rangle}$$

universal wave function

$$\Psi$$

$\uparrow$ up to the universal scale

$$(\psi, Z)$$

$\downarrow$ down to the microscopic scale

$$Q$$

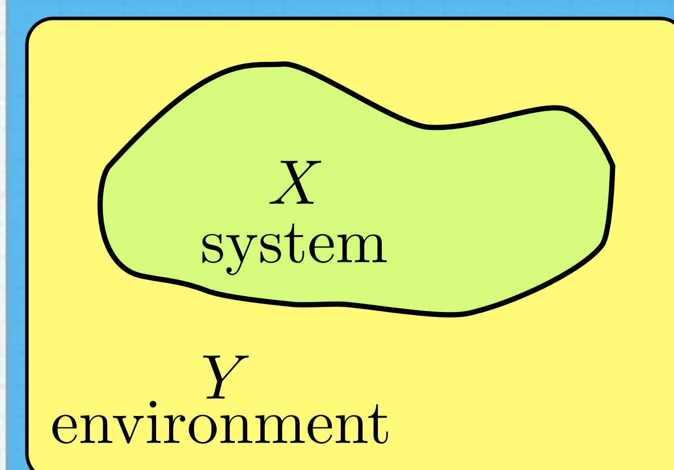
$$(Q, \Psi)$$

expressing the laws which govern the behavior of the Q variables in a simple and natural way.

Q variables: what the theory is fundamentally about

for example, particles, fields, strings, et.

$$|\Psi|^2$$



$$Q = (X, Y)$$

$$\psi(x) = \Psi(x, Y)$$

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$$Z \longrightarrow A \quad < Z >_{\psi} = \frac{\langle \psi, A\psi \rangle}{\langle \psi, \psi \rangle}$$

plug the actual configuration Y of the environment into the second slot of $\Psi(x, y)$ to obtain a function of x,

$$\psi(x) = \Psi(x, Y)$$

(Almost) all implications of BM follow from this formula

$$P_\Psi(X \in dx \,|\, Y) = |\psi(x)|^2 dx$$

GRW

ψ is not right since when?

$$\frac{1}{\sqrt{2}}(\psi_{42} + \psi_{62}) \rightarrow \psi_{42} \text{ or } \psi_{62}$$

when SC noticed ? ⎫ 'observer'
when you noticed ? ⎬ 'mind'
when I noticed ? ⎭

when the difference became noticeable?

stochastic ? nonlinear ? L+L

Philip Pearle

G.C. Ghirardi ⎫
A. Rimini ⎬ Phys Rev. D34 (1986) 470
T. Weber ⎭

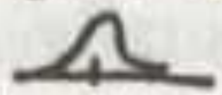
spontaneous wavefunction collapse

GRW spontaneous wf collapse.

In 'one particle' system:

$$\psi(t, r) \rightarrow \psi' \propto j(x-r)\,\psi(t, r)$$

with probability



$$dt\,dx\,|j(x-\hat{r})\psi|^2$$

There are

$$\tau^{-1} = \int dx\,|j(x-r)|^2$$

jumps per unit time.

In a 'many-particle' system

$$\psi(t, r_1, r_2, \cdots r_N)$$

independent jumping for each argument r gives

$$N / \tau$$

jumps per unit time. GRW:

$$\tau \sim 10^{15}\,\text{sec.} \sim 10^{8}\,\text{year}$$

width of $j(x) \sim 10^{-5}$ cm.

Ghirardi, Rimini, and Weber

- $\psi = \psi(q_1, ..., q_N)$, $q_i \in \mathbb{R}^3$, $i = 1, \ldots, N$
- for any point x in $\mathbb{R}^3$ $\quad \Lambda_i(x) = \frac{1}{(2\pi\sigma^2)^{3/2}} e^{-\frac{(\widehat{Q}_i - x)^2}{2\sigma^2}}$ $\quad \sigma \sim 10^{-7} m$
- the evolution of ψ is the Schrödinger evolution interrupted by collapses
- When the wave function is ψ a collapse with center x and label i occurs at rate

$$r(x, i|\psi) = \lambda \langle \psi \,|\, \Lambda_i(x)\psi \rangle \qquad \lambda \sim 10^{-15}\ s^{-1}$$

- when this happens

$$\psi \longrightarrow \Lambda_i(x)^{1/2}\psi / \|\Lambda_i(x)^{1/2}\psi\|$$

GRW jump spoils symmetry.

Take "molecular" model of matter with "different" nuclei. For <u>many</u> nuclei a given value of $\vec{r}$ will be accessible only through <u>either</u> ψ_{42} or ψ_{62}. <u>Very</u> quickly

$$\frac{1}{\sqrt{2}}\left(\psi_{42}+\psi_{62}\right) \longrightarrow \psi_{42} \ \boxed{\text{or}}\ \psi_{62}$$

How about internal economy?

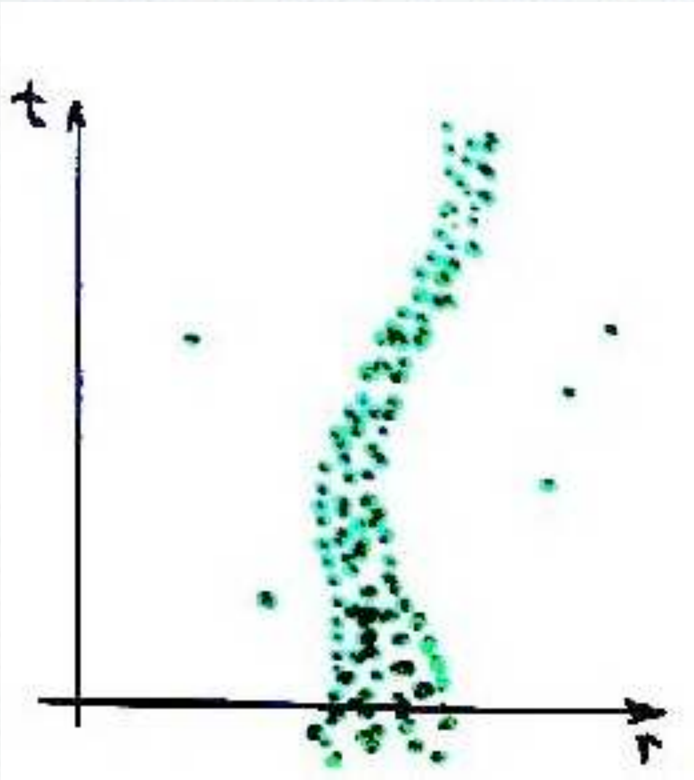
$$\vec{r} = \vec{R} + (\vec{r}-\vec{R})$$

$$\psi_{42,62} \propto \delta(\vec{r}-\vec{R}-\vec{a})$$

$$j(\vec{x}-\vec{r}) \cong j(\vec{x}-\vec{a}-\vec{R})$$

— approximate localization of c of m only — or more generally of 'quasiclassical' coordinates — with internal economy little disturbed.

GRWf



[...] the GRW jumps (which are part of the wave function, not something else) are well localized in ordinary space. Indeed each is centered on a particular spacetime point (x, t). So we can propose these events as the basis of the 'local beables' of the theory. These are the mathematical counterparts in the theory to real events at definite places and times in the real world (as distinct from the many purely mathematical constructions that occur in the working out of physical theories, as distinct from things which may be real but not localized, and distinct from the 'observables' of other formulations of quantum mechanics, for which we have no use here). A piece of matter then is a galaxy of such events. (Bell, 1987a)

GRWm

$$m(x,t) = \sum_{i=1}^{N} m_i \int d^3x_1 \cdots d^3x_N \, \delta^3(x - x_i) \left|\psi_t(x_1, \ldots, x_N)\right|^2$$

ψ_t is a GRW process

Lorenz invariance

Those paradoxes are simply disposed of by the 1952 theory of Bohm, leaving as the question, the question of Lorentz invariance. So one of my missions in life is to get people to see that if they want to talk about the problems of quantum mechanics – the real problems of quantum mechanics – they must be talking about Lorentz invariance. Bell (1990)

The big question, in my opinion, is which, if either, of these two precise pictures [GRW and Bohm] can be redeveloped in a Lorentz invariant way. Bell (1990)

Lorentz invariance

$$z' = \gamma(z - vt) \quad {}_{c=1} \quad t' = \gamma(t - vz)$$

Suppose: $v \ll 1$ $\quad \gamma \approx 1$

Let $|z|$ be very Large:

$$vz \approx \pm a$$

Then Lorentz trans. becomes

$$z' = z \qquad t' = t \pm a$$

i.e. for widely separated systems: L.I. $\Rightarrow$

relative-time invariance

— even for nonrelativistic systems. So:

Schrödinger $t \Rightarrow$ Dirac $t_1, t_2, \cdots$

What is a precise quantum theory?

(i) There is a clear primitive ontology, and it describes matter in space and time.

(ii) There is a state vector ψ in Hilbert space that evolves either unitarily or, at least, for microscopic systems very probably for a long time approximately unitarily.

(iii) The state vector ψ governs the behavior of the PO by means of (possibly stochastic) laws.

(iv) The theory provides a notion of a *typical* history of the PO (of the universe), for example by a probability distribution on the space of all possible histories; from this notion of typicality the probabilistic predictions emerge.

(v) The predicted probability distribution of the macroscopic configuration at time t determined by the PO (usually) agrees (at least approximately) with that of the quantum formalism.

Now in what I said probably many of you think that I have been wasting not only my time but yours, and therefore I would like to end up on a more harmonious note, with some concepts of which we all approve (Fig. 21). The theories that I presented to you are certainly not beautiful. I think they are not true either; it may be that they give some hint of where truth is to be found, but in their present brutally simplistic form the truth is certainly not there. I do think however that they have a certain kind of goodness — these little spots are halos — in the sense that they are honest attempts to replace the wooly words by real mathematical equations — equations which you don't have to talk away — equations which you simply calculate with and take the results seriously. Thank you.


TRUTH
BEAUTY
GOODNESS

quantum probability

$$P(\hat{A} = \alpha | \hat{B} = \beta) = | < \alpha | \beta > |^2$$

$< \psi | \chi >$ probability amplitude

$| < \psi | \chi > |^2$ probability

non- Kolmogorovian ?

momentum

$$| < \psi | p > |^2 = \widetilde{\psi}(p)$$

probability to find the value p of $\hat{P}$
if the system is (initially)
in the the state ψ

Fourier transform

time of flight measurement of momentum

(Heisenberg, Bohm52, Feynman & Hibbs)

ψ wf at time 0
free evolution
measure $\hat{X}$ at large time T

$$\hat{X}(T) = \frac{1}{m}\hat{P}T + \hat{X} \quad \rightarrow \quad \hat{P} = \frac{m\hat{X}(T)}{T} + \frac{X}{T} \approx \frac{m\hat{X}(T)}{T}$$

Bohm

The particle has a well defined position X whose evolution is guided by ψ

$$\dot{X}(t) = \frac{\hbar}{m} \mathrm{Im} \frac{\psi_t^* \nabla \psi_t}{\psi_t^* \psi_t} (X(t))$$

$$P(X(t) = x | \psi, t = 0) = |\psi_t(x)|^2$$

$$P(\hat{X}(t) = x | \psi, t = 0) = |\psi_t(x)|^2$$

Thus the asymptotic momentum $P = \frac{mX(T)}{T}$ (T large) is a RANDOM VARIABLE on the space on initial conditions with probability distribution

$$P(P = p | \psi, t = 0) = |\widetilde{\psi}(p)|^2$$

spin

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

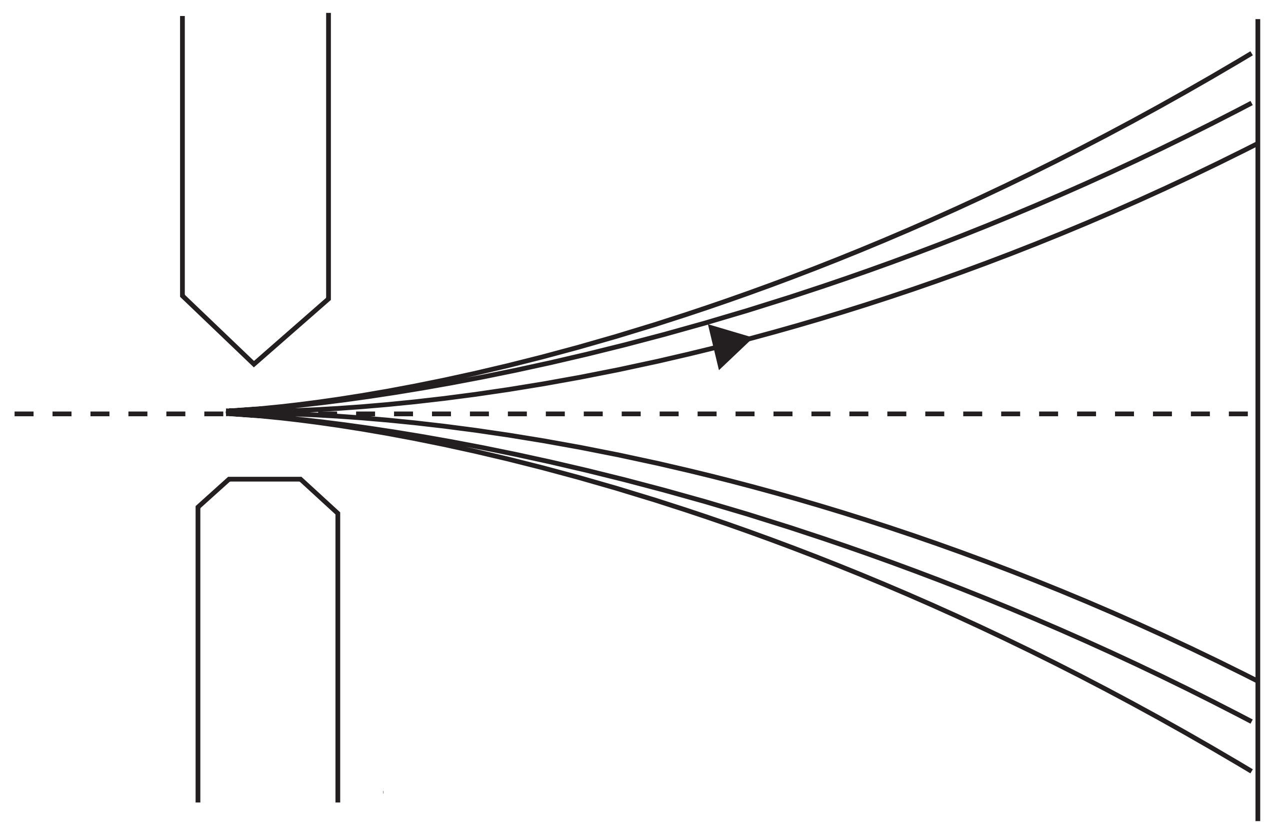
$$|\uparrow> = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad |\downarrow> = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|<\psi|\uparrow>|^2 \qquad \text{probability spin up along } z$$

$$|<\psi|\downarrow>|^2 \qquad \text{probability spin down along } z$$

Classically Non-Describable Two-Valuedness (Pauli)

Stern Gerlach measurement of spin



$$H_I = \mu \vec{\sigma} \cdot \vec{B} \approx (b + az)\sigma_z$$

$$\text{initial } \Psi = \psi \otimes \Phi(z)$$

$$\hat{Z}(T) = \hat{Z} + \frac{\hat{P}_z}{m} T + \frac{a}{2m} \sigma_z T^2$$

in the space of initial positions with prob. distribution $|\Phi(z)|^2$

Thus the RANDOM VARIABLE

$$F_T(Z(T))\,, \qquad F_T(z) = \frac{2mz}{aT^2}$$

in the limit of T large has values

$+1$ with probability $|<\psi|\uparrow>|^2$
-1 with probability $|<\psi|\downarrow>|^2$

$F_T(z)$ is the calibration function of the experiment

assignment of num. values to the outcome of the exp.

Morals

- Association between random variable Z (numerical result of the experiment) and operators
- Operators compactly express the statistics of the experiment

$$\bar{A} =< \psi|A\psi > \text{ mean value of } Z$$
$$< \psi|(A - \bar{A})^2\psi > \text{ variance of } Z$$
$$< \psi|A^n\psi > \text{ higher moments of } Z$$

- One can completely understand what's going on in the experiments measuring momentum or spin.
- No need of invoking any putative property of the electron such as its actual z-component of spin that is supposed to be revealed in the experiment.
- There is nothing the least bit remarkable about the nonexistence of this property.
- Measurements are “active.”