What is Quantum Theory about?

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The status of quantum theory

It can safely said that quantum theory is nowadays one of the fundamental columns of physics:

General Relativity	at large scales?
$\Downarrow$	
Classical Mechanics	at intermediate scales?
$\Uparrow$	
Quantum Theory	at small scales?

What is quantum theory?

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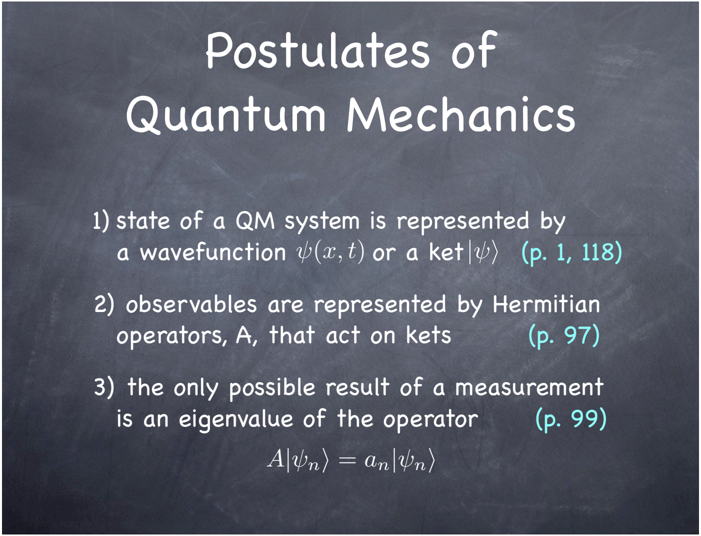

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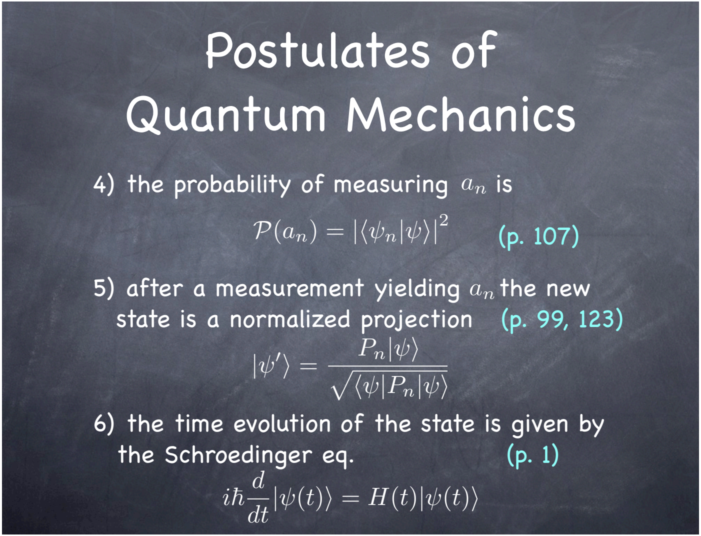


For those who are not shocked when they first come across quantum theory cannot possibly have understood it.

Niels Bohr, quoted in W. Heisenberg (1971), Physics and Beyond.

Why are we shocked? Let's revisit the six postulates again:

1. "state of a QM system is represented by a wave function $\psi(x, t)$ or a ket $|\psi\rangle$":

   A **system** *of what? Particles, or fields, or quantum stuff? And what do we mean by* **state** *of a system? Do we mean all information necessary to describe the system at a certain time?*

2. "observables are represented by Hermitian operators, A, that act on kets":

   What is an **observable***?* A thing that humans can experience? Only humans?

3. "the possible result of a measurement is an eigenvalue of the operator $A|\psi_n\rangle = a_n|\psi_n\rangle$":

   What is a **measurement***?* Looking at a pointer in a laboratory? Or is it already enough that the pointer is there without me looking at it? Is there a bijection between pointers results and Hermitian operators?

4. "the probability of measuring a_n is $\mathcal{P}(a_n) = |\langle\psi_n|\psi\rangle|^2$":

   *Does "**probability**" mean that nature is intrinsically stochastic? If so, fine, and if not, is quantum theory an effective theory of an underlying deterministic theory? Or do we live in many-worlds but we don't know exactly in which one?*

5. "after the measurement yielding a_n the new state is a normalized projection $|\psi'\rangle = \frac{P_n|\psi_n\rangle}{\sqrt{\langle\psi|P_n|\rangle}}$":

   When is **after the measurement***?* The exact time when I look at the pointer in the laboratory?

6. "the time evolution of the state is given by the Schrödinger equation $i\hbar\frac{d}{dt}|\psi\rangle = H(t)|\psi\rangle$":

   But that cannot be completely correct because it contradicts **Postulate 5***!*

Now for one century physicists, philosophers and mathematicians have tried to make sense out of the Six Postulates, and killed many many trees.

So as already J.S. Bell wrote in *Against Measurement* (1990):

"Why not look it up in a good book?"

[...] For the good books known to me are not much concerned with physical precision. This is clear already from their vocabulary.

Here are some words which, however legitimate and necessary in application, have no place in a *formulation* with any pretension to physical precision: *system, apparatus, environment, microscopic, macroscopic, reversible, irreversible, observable, information, measurement.*

The concepts 'system', 'apparatus', 'environment', immediately imply an artificial division of the world, and an intention to neglect, or take only schematic account of, the interaction across the split. The notions of 'microscopic' and 'macroscopic' defy precise definition. So also do the notions of 'reversible' and 'irreversible'. Einstein said that it is theory which decides what is 'observable'. I think he was right - 'observation' is a complicated and theory-laden business. Then that notion should not appear in the formulation of fundamental theory. Information? Whose information? Information about what?

On this list of bad words from good books, the worst of all is 'measurement'. It must have a section to itself. [...]

I think I can safely say that nobody understands quantum mechanics.

Richard Feynman (1965), The Character of Physical Law.

Nevertheless, quantum theory even in this incomplete form has led to very accurate predictions and therefore needs to be taken seriously.

Let's first understand why physicists felt urged to live with these postulates and their contradictions, and then draw a constructive conclusion.

What led to this sate of quantum theory?

In an early stage of quantum theory it was already believed that a "state" of a "quantum system" has to be described by a wave function;

in the case of N particles, a complex valued and square integrable function on configuration space $\mathbb{R}^{3N}$, whose time evolution is given by the Schrödinger equation.

This is later reflected in the **postulates 1 and 6**.

Let's see how far we can go with the state and the Schrödingier equation only.

A experiment called "measurement"

- Suppose a "system" under examination can be described by a state

$$c_1\varphi_1 + c_2\varphi_2, \qquad \text{for } |c_1|^2 + |c_2|^2 = 1,$$

  i.e. a state partly being φ_1 and partly being φ_2. One could argue that this indeterminacy at a quantum scale is not necessarily a problem.

- This "system" is coupled to a "measurement apparatus" with a pointer which is in state

$$\begin{array}{ll} \psi_0 & \text{if ready,} \\ \psi_1 & \text{if pointing to the left,} \\ \psi_2 & \text{if pointing to the right.} \end{array}$$

- If the "measurement apparatus" work properly then

$$\varphi_i \psi_0 \xrightarrow{\text{Schrödinger time evolution}} \varphi_i \psi_i.$$

- The Schrödinger equations is unitary. Hence,

$$\sum_{i=1}^{2} c_i \, \varphi_i \psi_0 \xrightarrow{\text{Schrödinger time evolution}} \sum_{i=1}^{2} c_i \, \varphi_i \psi_i.$$

- The resulting state suggests that the pointer is pointing partly to the left and partly to the right. An unacceptable result.

Furthermore, it was known from experiments that Born's rule holds. It says that the empirical distribution of the positions of the N particles of a system in state ψ is given by

$$\rho(x) = |\psi(x)|^2, \qquad \text{for } x \in \mathbb{R}^{3N}.$$

By unitarity of the Scrödinger equation

$$\int dx\ |\psi(x)|^2 = 1$$

at one time instant implies that it holds for all times. Hence, ρ can seriously be interpreted as a probability density.

Taking Born's rule seriously we compute the empirical distribution of the position of particles:

$$\begin{aligned}\rho(X,Y) = &= |c_1|^2|\varphi_1(X)\psi_1(Y)|^2\\ &+ |c_2|^2|\varphi_2(X)\psi_2(Y)|^2\\ &+ 2\mathfrak{Re}\, c_1^* c_2(\varphi_1(X)\psi_1(Y))^*\varphi_2(X)\psi_2(Y)\end{aligned}$$

Decoherence: Assuming furthermore that the apparatus consists of a large number of degrees of freedom, a thermodynamical argument gives that for all practical purposes

$$\operatorname{supp}\psi_1 \cap \operatorname{supp}\psi_2 \approx \varnothing$$

so that the pointers pointing to the left does not interfere with the one pointing to the right! Thus we may conclude for all practical purposes

$$\rho(X,Y) \approx |c_1|^2|\varphi_1(X)\psi_1(Y)|^2 + |c_2|^2|\varphi_2(X)\psi_2(Y)|^2.$$

Born's rule tells us that if we would set up an ensemble of identical experiments of this kind we would find:

$$\mathbb{P}(\text{pointer to the left}) = \int \int_{y \in \text{supp}\, \psi_1} \rho(x,y)\; dy\; dx = |c_1|^2$$

$$\mathbb{P}(\text{pointer to the right}) = \int \int_{y \in \text{supp}\, \psi_2} \rho(x,y)\; dy\; dx = |c_2|^2$$

This later gave rise to **postulate 4**, and by realizing that this game can be played not only for the position operator x but for all self-adjoint operators, also to **postulates 2 and 3**.

This is nice because we can check the accordance with the predicted empirical distributions by repeating the same experiment over and over again.

But does it resolve our initial problem?

No! The final state is still

$$\sum_{i=1}^{2} c_i \varphi_i \psi_i$$

and does not correspond to the state of the system after the experiment.

There is a difference between a shaky or out-of-focus photograph and a snapshot of clouds and fog banks.

Erwin Schrödinger,
The present situation in quantum mechanics

What's the Conclusion?

Since we only have the Schrödinger equation and its wave function ψ as the corner stones of quantum theory there are only two possibilities:

1. The Schrödinger equation is wrong!

   [e.g. Copenhagen, Dynamical Reduction Models]

2. Ψ alone does not describe the state of the system!

   [e.g. Bohmian Mechanics]

Copenhagen Interpretation

Bohr, Heisenberg and von Neumann decided for track 1: The very moment an "observer" performs a "measurement" by monitoring the experiment one has to add a new, stochastic time evolution to the Schrödinger equation which collapses the wave function ψ to either

$$\varphi_1\psi_1 \qquad \text{or} \qquad \varphi_2\psi_2$$

according to the probabilities $|c_1|^2$ and $|c_2|^2$.

So far we have accepted vagueness in words like:

"system", "state", "measurement", and "apparatus"

but now we really get into difficulties: When does the Schrödinger time evolution guide the state and when does the state jump abruptly?

It would seem that the theory [quantum mechanics] is exclusively concerned about "results of measurement", and has nothing to say about anything else. What exactly qualifies some physical systems to play the role of "measurer"? Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system ... with a Ph.D.? If the theory is to apply to anything but highly idealized laboratory operations, are we not obliged to admit that more or less "measurement-like" processes are going on more or less all the time, more or less everywhere. Do we not have jumping then all the time?

J. Bell, Against Measurement (1990).

Feynman on Obervation

"Does this mean that my observations become real only when I observe an observer observing something as it happens? This is a horrible viewpoint. Do you seriously entertain the thought that without observer there is no reality? Which observer? Any observer? Is a fly an observer? Is a star an observer? Was there no reality before 10^9 B.C. before life began? Or are you the observer? Then there is no reality to the world after you are dead? I know a number of otherwise respectable physicists who have bought life insurance. By what philosophy will the universe without man be understood?"

Richard Feynman, Lecture Notes on Gravitation

I like to think that the moon is there even if I am not looking at it.

Albert Einstein.

Ways Out of the Dilemma

Bohmian Mechanics

- *Bohmian Mechanics* like Newtonian Mechanics is a *mechanical* theory describing the motion of N particles. The state of the Bohmian universe, an N-particle system, is described by $(Q, \Psi) \in \mathbb{R}^{3N} \times \mathcal{H}$.
- The motion of Ψ is governed by the Schrödinger equation.
- The motion of the particles is governed by[1]

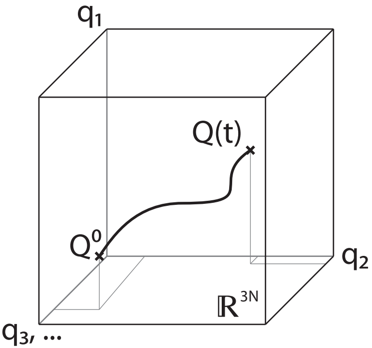
$$\frac{dQ}{dt} = v(Q,t) = \frac{\hbar}{m} \mathfrak{Im} \frac{\Psi(t)^* \nabla_Q \Psi(t)}{\Psi(t)^* \Psi(t)}(Q).$$

- The Bohmian trajectory $t \mapsto Q(t)$ in configuration space is defined as the integral curve to the velocity field $v(Q,t)$ with $Q(t)|_{t=0} = Q^0$ for some initial configuration Q^0.

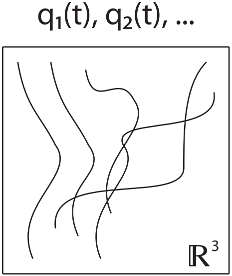
[1] Formally ∇_Q stands for $(\nabla_{\mathbf{q}_1}, \ldots, \nabla_{\mathbf{q}_N})$.

Configuration Space and Physical Space

configuration space:



physical space:



The continuity equation for the Bohmian flow on configuration space is identically fulfilled by $\rho_{eqv}(t) = |\Psi(t)|^2$:

$$\frac{\partial}{\partial t}|\Psi(X,t)|^2 + \nabla_X \cdot \Big(|\Psi(X,t)|^2 \; v(X,t) \Big) = 0.$$

That means:

If the initial configuration of particles Q^0 is distributed according to $|\Psi^0|^2$ at time $t = 0$ then the configuration $Q(t; Q^0, \Psi^0)$ will be $|\Psi(t)|^2$ distributed for any time t.

- The measure $\rho_{eqv} = |\Psi|^2$ as a local functional of Ψ is unique. So ρ_{eqv} is given by the equations of motion!
- It is timeless.
- It defines a probability $\mathbb{P}^{\Psi}(A) = \int_A dx |\Psi(x)|^2$ on configuration space.

Sub-systems

- Say $(Q = (X, Y), \Psi)$, X the sub-system, Y the rest of the Bohmian universe.
- Conditional wave function: $\psi^Y(X) := \frac{\Psi(X,Y)}{\|\Psi(\cdot,Y)\|}$. Not autonomous.
- It defines a new measure $\mathbb{P}^\Psi(A|Y) = \int_A dx |\psi^Y(x)|^2$.
- Effective wave function: ψ is the effective wave function iff

$$\Psi(X, Y) = \psi(X)\Phi(Y) + \Psi^\perp(X, Y)$$

  with Φ and $\Psi^\perp$ having disjoint Y-supports and $Y \in \operatorname{supp}\Phi$.
- If the effective wave function exists it is equal to the conditional one modulo a phase.
- Whenever the interaction $V(X, Y)\psi(X)\Phi(Y)$ is small, ψ obeys an autonomous Schrördinger equation of the sub-system.

Quantum Equilibrium

Empirical distributions of particle positions in ensembles of identical sub-systems all having an effective wave function ψ will obey the $|\psi|^2$ distribution for almost all possible initial conditions Q^0 of the Bohmian universe w.r.t to $P^\Psi(\cdot|Y)$.

This is Born's law.

Going back to our experiment we have an initial state

$$\begin{pmatrix} (X^0, Y^0) \\ \sum_{i=1,2} c_i \varphi_i(x)\psi_0(y) \end{pmatrix} \xrightarrow{\text{Bohmian time evolution}} \begin{pmatrix} (X, Y) \\ \sum_{i=1,2} c_i \varphi_i(x)\psi_i(y) \end{pmatrix}$$

What IS?

At any time the experiment has a determined configuration (X, Y), X being the one of the system and Y the one of the apparatus and the rest of the universe!

The pointer of the apparatus is unambiguously pointing somewhere at all times. The cat breathes, moves, or lies still on the ground.

After a position measurement we may find $Y \in \operatorname{supp} \psi_1$. If as before by decoherence

$$\operatorname{supp} \psi_1 \cap \operatorname{supp} \psi_2 \approx \varnothing$$

for all times after the measurement we may simplify following computations fapp by replacing $\psi = \sum_{i=1,2} c_i \varphi_i \psi_i$ with the effective wave function

$$\psi_{eff} = \varphi_1 \psi_1.$$

This is a valid approximation because in such situations Y remains in $\operatorname{supp} \psi_1$ for all times after the measurement by virtue of the Bohmian velocity law.

So what can we say about our experiment?

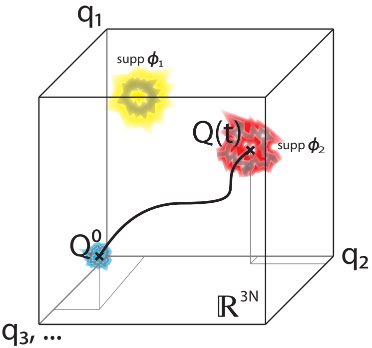
Randomness enters through our ignorance of the initial conditions! To get the probability of pointer pointing to the left or right we simply count the trajectories $Q(t) = (X(t), Y(t))$ with $Y(t) \in \operatorname{supp} \psi_i$ for $i = 1, 2$ weighted with their respective probability given by the Quantum Equilibrium:

$$\mathbb{P}(\text{pointer to the left}) = \int \int_{y \in \operatorname{supp} \psi_1} |\psi(x,y)|^2 \, dy \, dx = |c_1|^2$$

$$\mathbb{P}(\text{pointer to the right}) = \int \int_{y \in \operatorname{supp} \psi_2} |\psi(x,y)|^2 \, dy \, dx = |c_2|^2$$

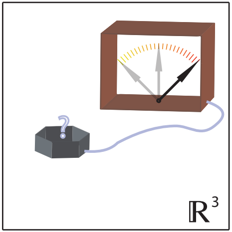
Disjoint support in configuration space

configuration space:



physical space:

q1(t), q2(t), ...



From the orthogonality of ψ_1 and ψ_2 and the unitarity of the Schrödinger time evolution we get orthogonality of φ_1 and φ_2.

This allows to introduce an extremely helpful book-keeping device for trajectory counting:

$$A := \sum_{i=1,2} \lambda_i P_{\varphi_i}$$

where $P_{\varphi_i}\varphi_j = \delta_{i,j}$ denotes the respective projectors on the sub-Hilbert space describing the quantum mechanical system and λ_i our coarse-grained measurement scale; e.g. $\lambda_1 = -1 \simeq$"pointer to the left" and $\lambda_2 = +1 \simeq$"pointer to the right".

Bohmian Mechanics delivers a well-defined mapping:

Experiments $\rightarrow$ Operators on Hilbert space.

Summary

- (Q, Ψ) describes the state of a mechanical N-particle(!) system with particle configuration Q.
- Two equations of motion, the Bohmian velocity law and Schrödinger's equation.
- All said - no talk.

Dynamical Reduction Models (DRM)

An unfairly brief run through the basic concept in DRM:

- Dynamical Reduction Models are in general theories about the mass density of an N-"particle" system which *IS*.
- The state of the system is described by the wave function Ψ on configuration space $\mathbb{R}^{3N}$.
- The mass density is given by

$$\rho(x) = \sum_{i=1}^{N} m_i \int d^{2(N-1)}(x_1 \dots \widehat{x_i} \dots x_N) \, |\Psi(x_1, \dots, x_N)|^2 \Big|_{x_i = x}$$

for masses m_i and $x \in \mathbb{R}^3$.

- The time evolution is partly stochastic and partly deterministic:
  - With a mean rate λ_i for each particle degree of freedom i the wave function Ψ undergoes spontaneous jumps.
  - In the time interval between two successive jumps the system evolves according to the usual Schrödinger equation.
  - The spontaneous jump is a localization process

$$\Psi \mapsto \frac{L_x^i \Psi}{\|L_x^i \Psi\|}$$

where the localization operator has the form

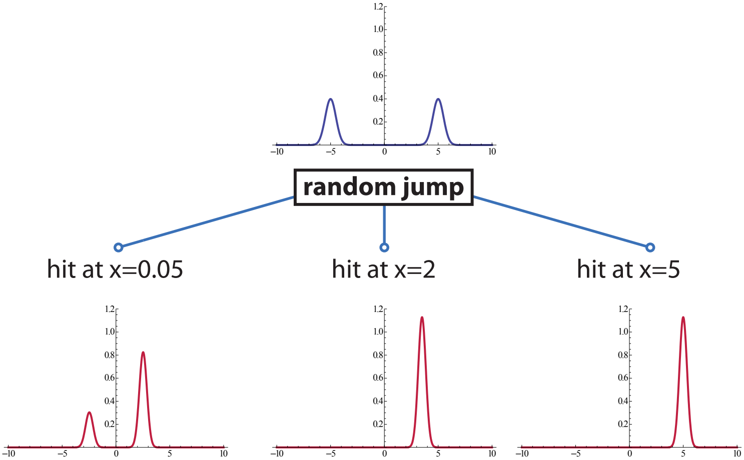
$$L_x^i := \left(\frac{\alpha}{\pi}\right)^{3/4} e^{-\frac{\alpha}{2}(\mathbf{q}_i - x)^2}$$

q_i being the usual position operator of the ith particle.

  - The probability density for an localization to occur is

$$p(x) = \|L_x^i \psi\|^2.$$

Spontaneous random jumps in DRM



Summary

- Ψ describes the state of the system. From Ψ the actual mass density can be computed at any time.
- One stochastic law of motion, the modified Schrödinger's time evolution.
- Almost all said - little talk: small deviations from quantum mechanical predictions, arbitrariness in λ_i and α.

Why shouldn't be physics about what *IS*?

Open Discussion

Come now, I will tell thee - and do thou hearken to my saying and carry it away - the only two ways of search that can be thought of. The first, namely, that **It is**, and that it is impossible for **it not to be**, is the way of belief, for truth is its companion. The other, namely, that **It is not**, and that **it must needs not be**, - that, I tell thee, is a path that none can learn of at all. For thou canst not know **what is not** - that is impossible - nor utter it; for it is the same thing that can be thought and that can be.

It needs must be that what can be spoken and thought is; for it is possible for it to be, and it is not possible for what is nothing to be. This is what I bid thee ponder. I hold thee back from this first way of inquiry, and from this other also, upon which mortals knowing naught wander two-faced; for helplessness guides the wandering thought in their breasts, so that they are borne along stupefied like men deaf and blind. Undiscerning crowds, who hold that it is and is not the same and not the same, and all things travel in opposite directions!

For this shall never be proved, that the things that are not are; and do thou restrain thy thought from this way of inquiry."

Parmenides of Elea, The Poem

"Wohlan, so will ich denn verkünden (Du aber nimm mein Wort zu Ohren), welche Wege der Forschung allein denkbar sind: der eine Weg, da [das Seiende] ist und da es unmöglich nicht sein kann, das ist der Weg der Überzeugung (denn er folgt der Wahrheit), der andere aber, daß es nicht ist und da dies Nichtsein notwendig sei, dieser Pfad ist (so künde ich Dir) gänzlich unerforschbar. Denn das Nichtseiende kannst Du weder erkennen (es ist ja unausführbar) noch aussprechen. Denn [das Seiende] denken und sein ist dasselbe.

Dies ist nötig zu sagen und zu denken, daß [nur] das Seiende existiert. Denn seine Existenz ist möglich, die des Nichtseienden dagegen nicht; das heiß' ich Dich wohl zu beherzigen. Es ist dies nämlich der erste Weg der Forschung, vor dem ich Dich warne. Sodann aber auch vor jenem, auf dem da einherschwanken nichts wissende Sterbliche, Doppelköpfe. Denn Ratlosigkeit lenkt den schwanken Sinn in ihrer Brust. So treiben sie hin stumm zugleich und blind die Ratlosen, urteilslose Haufen, denen Sein und Nichtsein für dasselbe gilt und nicht für dasselbe, für die es bei allem einen Gegenweg gibt.

Denn unmöglich kann das Vorhandensein von Nichtseiendem zwingend erwiesen werden. Vielmehr halte Du Deine Gedanken von diesem Wege der Forschung ferne."

Parmenides von Elea, Lehrgedicht

Schrödinger's cat

One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following device (which must be secured against direct interference by the cat): in a Geiger counter there is a tiny bit of radioactive substance, so small, that perhaps in the course of the hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives if meanwhile no atom has decayed. The psi-function of the entire system would express this by having in it the living and dead cat (pardon the expression) mixed or smeared out in equal parts.

It is typical of these cases that an indeterminacy originally restricted to the atomic domain becomes transformed into macroscopic indeterminacy, which can then be resolved by direct observation. That prevents us from so naively accepting as valid a "blurred model" for representing reality. In itself it would not embody anything unclear or contradictory. There is a difference between a shaky or out-of-focus photograph and a snapshot of clouds and fog banks.

Erwin Schrödinger: The present situation in quantum mechanics