Nonlocality and Relativity in Various Interpretations of Quantum Mechanics

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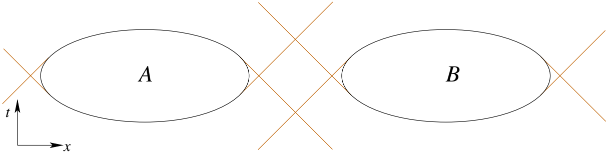
I intend to answer all of these questions in this talk.

I use this definition of locality (in single-world theories):

If the space-time regions A and B are spacelike separated then events in A cannot influence events in B.



Is Bohmian mechanics local or nonlocal?

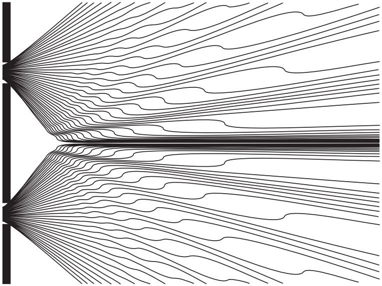
Definition of Bohmian mechanics

Bohmian mechanics (version for N nonrelativistic particles)

- N particles move along trajectories $\mathbf{X}_i(t) \in \mathbb{R}^3$ governed by

$$\frac{d\mathbf{X}_i}{dt} = \frac{\mathbf{j}_i}{|\psi|^2}(\mathbf{X}_1, \ldots, \mathbf{X}_N)$$

  with probability current $\mathbf{j}_i = \frac{\hbar}{m_i} \mathrm{Im}\, \psi^* \nabla_i \psi$.
- ψ evolves according to the usual Schrödinger equation
- At any time t, the configuration $X(t) = (\mathbf{X}_1(t), \ldots, \mathbf{X}_N(t))$ has probability distribution density $|\psi_t|^2$.



Bohmian mechanics is nonlocal

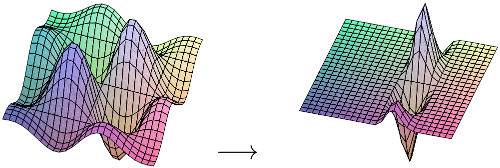
- Bell's theorem applies ($\lambda = (\psi, X)$) and shows that Bohmian mechanics is nonlocal.
- Yet, no signaling.
- It can also be seen directly that Bohmian mechanics is nonlocal: the velocity of particle 2 depends on the position of particle 1, no matter how distant and no matter whether there is an interaction term in the Hamiltonian. That is where the superluminal influence occurs.
- This influence depends on entanglement: In the absence of entanglement, the velocity of particle 2 is independent of the position of particle 1. The fact that Bohmian mechanics is local (for relativistic Hamiltonians, e.g., Dirac eq) for disentangled wave functions shows that it is necessary for proving non-locality to consider at least two particles and an entangled wave function.

Are collapse theories such as GRW local or nonlocal?

Example: GRW theories

Stochastic wave function evolution ("GRW process")
for N nonrelativistic "particles":

- ψ evolves according to the usual Schrödinger eq for a random duration T with mean $N^{-1}10^8$ years.
- At time T, ψ "collapses" according to the following precise rule:
- Nature randomly selects $i \in \{1 \dots N\}$ and location $\mathbf{X} \in \mathbb{R}^3$ with distribution $|\psi|^2 * g$ with g a Gaussian fct with width $\sigma = 10^{-7}$ m.
- $\psi(\mathbf{x}_1 \dots \mathbf{x}_N)$ gets replaced by $\psi' = \mathcal{N}\, g(\mathbf{x}_i - \mathbf{X})\, \psi(\mathbf{x}_1 \dots \mathbf{x}_N)$, ($\mathcal{N}$ = normalization factor), i.e., localized as in



- Repeat

[Ghirardi, Rimini, Weber *Phys. Rev. D* 1986]

Definition: GRW theories

GRWm

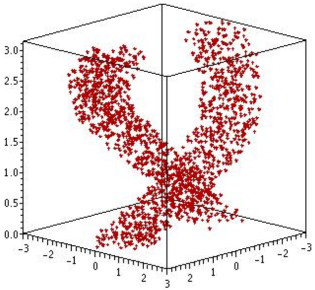
- Matter is continuously distributed in space with density

$$m(t,\mathbf{x}) = \sum_{i=1}^{N} m_i \int_{\mathbb{R}^{3N}} d\mathbf{x}_1 \cdots d\mathbf{x}_N \, \delta^3(\mathbf{x}-\mathbf{x}_i) \, |\psi_t(\mathbf{x}_1 \ldots \mathbf{x}_N)|^2 \, .$$

- ψ evolves stochastically according to the GRW process.

GRWf

- Matter consists of flashes (material points in space-time).
- Flash at $(T, \mathbf{X})$ for every collapse as ψ evolves according to the GRW process.

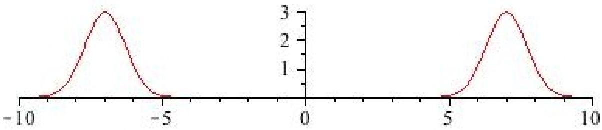


GRW theories are nonlocal

- Both GRWm and GRWf predict (with tiny deviations) the same probabilities of outcomes of EPR-Bell experiments as the quantum formalism. Thus, Bell's inequality is violated ($\lambda = \psi$), and Bell's theorem shows that GRWm and GRWf are nonlocal.
- Yet, no signaling.
- Again, nonlocality can be seen much more directly: Collapsing ψ in one variable $\mathbf{x}_1$ can instantaneously change the $|\psi|^2$ distribution for $\mathbf{x}_2$, and thus change (for GRWm) $m(t, \mathbf{x})$ in region B and (for GRWf) the probability distribution of the next flash in region B.
- Here is a third reasoning: The GRW theories are already nonlocal in a much simpler setting than Bell's, namely in Einstein's boxes.

Einstein's boxes argument

[Einstein $\sim$ 1927, unpublished], [Norsen: "Einstein's boxes" *Am. J. Phys.* 2005]



The wave function of a particle is half in a box in Paris and half in a box in Tokyo. Apply detectors to both boxes at time t (in some Lorentz frame)—at spacelike separation. One and only one detector clicks. If it is assumed that there was no fact about "where the particle actually is" before the detectors were applied, then this effect is nonlocal.

- Einstein intended this as an argument against the Copenhagen camp.
- The argument shows that any collapse theory is nonlocal.

- Both GRWf and GRWm are already nonlocal when governing a universe containing only one particle; in particular, they are nonlocal in a case (one particle) in which Bohmian mechanics is local.
  To see this, consider the wave function of Einstein's boxes example without detectors,

$$\psi = \frac{1}{\sqrt{2}}\big(|\text{here}\rangle + |\text{there}\rangle\big)$$

  Suppose that $|\text{here}\rangle$ and $|\text{there}\rangle$ are two narrow wave packets separated by a distance of 500 million light years. Spontaneous GRW collapses, likely to occur at spacelike separation, can play the role of the detectors in the argument: A collapse centered "here" precludes one "there"—a nonlocal influence.

Is the orthodox/Copenhagen interpretation local or nonlocal?

The orthodox/Copenhagen interpretation is nonlocal

- It is hard to define what the orthodox/Copenhagen interpretation (OQM) actually says.
- Nevertheless, it seems clear enough that OQM asserts that a "quantum particle" does not have a position before we do a "quantum position measurement." Thus, Einstein's boxes argument shows that OQM is nonlocal.
- Likewise, it seems clear enough that OQM denies any "hidden variables" (any further variables beyond the collapsed ψ), so Bell's theorem applies (with $\lambda = \psi$) and shows that OQM is nonlocal.

- Still, orthodox physicists usually reject nonlocality. To speculate why:
  - Historically, Einstein's boxes argument first came as an objection (at a time when locality seemed a fact) and thus had to be rejected by the OQM camp.
  - Psychologically, orthodox physicists often think in terms of hidden variables. An inclination to positivism (i.e., the idea that only operational/testable statements are meaningful/scientific) seems to keep them from appreciating the difference between whether or not the question "Paris or Tokyo?" had a well-defined answer before the detection—a non-testable answer to a non-operational question!

You cannot understand nonlocality if you do not think about *what happens out there in reality.*

Is the many-worlds theory local or nonlocal?

Definition of the many-worlds theory

S∅: Everett's (1957) many-worlds theory

There exists only the wave function ψ of the universe, and nothing else. ψ evolves according to the usual Schrödinger equation. Contributions to ψ corresponding to macroscopically different situations represent parallel, equally real worlds.

To me, this doesn't make sense as a fundamental physical theory because I think it must have a primitive ontology: variables representing matter in space-time. (E.g., flashes, $\mathbf{X}_i(t)$,...). This problem can be solved:

Sm: Schrödinger's (1926) first quantum theory

- Matter is continuously distributed in space with density

$$m(t,\mathbf{x}) = \sum_{i=1}^{N} m_i \int_{\mathbb{R}^{3N}} d\mathbf{x}_1 \cdots d\mathbf{x}_N \, \delta^3(\mathbf{x}-\mathbf{x}_i) \, |\psi_t(\mathbf{x}_1 \ldots \mathbf{x}_N)|^2 \, .$$

- ψ evolves according to the usual Schrödinger equation.

[Allori, Goldstein, Tumulka, Zanghì *Brit. J. Philos. Sci.* 2011]

Sm is nonlocal (1)

[Allori, Goldstein, Tumulka, Zanghì *Brit. J. Philos. Sci.* 2011]

You might think Sm is local because of the following fact:

$m(t, \mathbf{x})$ in B does not depend on external fields in A or on the quantum state in A (it is a function of the reduced density matrix $\rho_B = tr_A|\psi\rangle\langle\psi|$ with ψ including apparatus).

I conclude that nothing that Alice can do in A, nor any events in A, can influence $m(t, \mathbf{x})$ in B. And yet, Sm is nonlocal:

Consider Einstein's boxes at a time t after applying detectors on both sides. The possible outcomes are 01 and 10. The wave function $\psi = \psi_t$ of the universe is

$$\psi = \psi_{01} + \psi_{10} \,,$$

and correspondingly,

$$m = m_{01} + m_{10} \,.$$

Thus the world in which Alice's result is 1 is the same world as the one in which Bob's result is 0—a fact created in a nonlocal way.

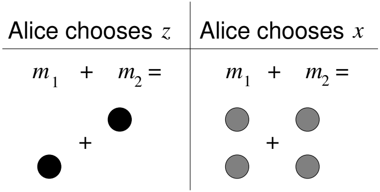
Sm is nonlocal (2)

- The m function alone, while revealing that there are two worlds in A and two worlds in B, does not encode the information conveying which world in A is the same as which world in B. That is, the pairing of worlds cannot be read off from $m(t, \cdot)$ even though it is an objective fact of Sm at time t, defined by means of the wave function ψ_t.
- Thus, the fact that Alice cannot influence m in B does not mean locality.
- One should suspect that Sm is nonlocal already when noticing that Sm involves a nonlocal object ψ and cannot (in any obvious way) be formulated without mentioning such an object.

Sm is nonlocal (3)

Moreover, even though Alice cannot influence the PO in B, she can influence other physical facts pertaining to B.

Consider a Bell experiment (with 2 electrons starting in the singlet state) in which Alice chooses either the x or the z direction for her magnet, while Bob always chooses the z direction. Suppose that at time t (in a certain Lorentz frame), Alice's detector has clicked but Bob's has not, although Bob's particle has already passed Bob's magnet. One finds that, in the region of Bob's particle,



While $m(x)$ for $x \in B$ is unaffected by Alice's choice, each $m_\ell(x)$ is affected.

The many-world theory is nonlocal

For the same reasons as for Sm, nonlocality holds for any other version of many-worlds. (What would be its relevant difference from Sm?)

Can the many-worlds theory be made relativistic?

Very easily

Let us neglect the fact that nobody can write down a Hamiltonian that is mathematically well defined, appropriately Lorentz covariant, and involves interaction. That is, let us neglect problems of ultraviolet divergence, renormalization, etc. Then it is easy to come up with a way of making Sm relativistic:

Relativistic Sm

- Consider a relativistic Hamiltonian, the Heisenberg picture with fixed state vector ψ, let $T_{\mu\nu}(t, \mathbf{x})$ be the stress-energy tensor operator for the space-time point $(t, \mathbf{x})$, and set

$$m_{\mu\nu}(t, \mathbf{x}) = \langle \psi | T_{\mu\nu}(t, \mathbf{x}) | \psi \rangle \,.$$

- Matter is continuously distributed with density given by $m_{\mu\nu}(t, \mathbf{x})$.

This theory is relativistically invariant if the underlying equations for H and $T_{\mu\nu}(t, \mathbf{x})$ are.

Can the orthodox/Copenhagen interpretation be made relativistic?

- Again, it is hard to define what OQM actually says.
- Let us say it says something like: Macroscopic quantities always have definite values, microscopic ones do not, and the interaction between the two is governed by the Born rule and the collapse rule.
- Then the mathematical laws of OQM are just the rules for making predictions.
- Let us pretend we have a relativistic Hamiltonian H.
- Then a relativistic formulation of OQM requires just the formula for the joint distribution of the outcomes $Z_1, \ldots, Z_n$ of quantum measurements of the local observables $A_1, \ldots, A_n$ at space-time points $(t_1, \mathbf{x}_1), \ldots, (t_n, \mathbf{x}_n)$:

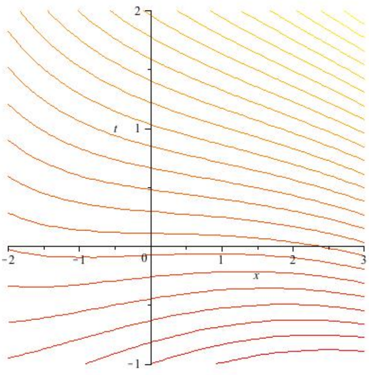
$$\mathrm{Prob}(Z_1 = z_1, \ldots, Z_n = z_n) = \langle \psi | U_1^\dagger P_1 \cdots U_n^\dagger P_n U_n \cdots P_1 U_1 | \psi \rangle$$

  numbered so that $t_1 \leq t_2 \leq \ldots \leq t_n$, with $U_k = e^{-iH(t_k - t_{k-1})}$ and P_k the projection to the eigenspace of A_k with eigenvalue z_k.
- This formula can be shown to be independent of the Lorentz frame, using that local observables at spacelike separation commute.

Can Bohmian mechanics be made relativistic?

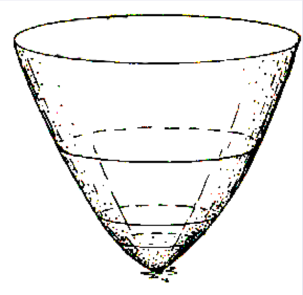
[Dürr, Goldstein, Norsen, Struyve, Zanghì *Proc. Roy. Soc. A* 2014]
[Dürr, Goldstein, Münch-Berndl, Zanghì *Phys. Rev. A* 1999]
[Tumulka *J. Phys. A* 2007]

- If a preferred foliation (= slicing) of space-time into spacelike hypersurfaces ("time foliation" $\mathcal{F}$) is permitted, then there is a simple, convincing analog of Bohmian mechanics, $\mathrm{BM}_{\mathcal{F}}$. [Dürr et al. 1999] Without a time foliation, no version of Bohmian mechanics is known that would make predictions anywhere near quantum mechanics. (And I have no hope that such a version can be found in the future.)



There is no agreed-upon definition of "relativistic theory." Anyway, the possibility seems worth considering that our universe has a time foliation.

Simplest choice of time foliation $\mathcal{F}$



Let $\mathcal{F}$ be the level sets of the function T : space-time $\to \mathbb{R}$, $T(x) =$ timelike-distance(x, big bang).

E.g., T(here-now) $=$ 13.7 billion years

Drawing: R. Penrose

Alternatively, $\mathcal{F}$ might be defined in terms of the quantum state vector ψ, $\mathcal{F} = \mathcal{F}(\psi)$ [DGNSZ 2014]

Or, $\mathcal{F}$ might be determined by an evolution law (possibly involving ψ) from an initial time leaf.

Key facts about $\mathrm{BM}_{\mathcal{F}}$

Known in the case of N non-interacting Dirac particles, expected to be true also, say, one day, in full QED with photon trajectories:

Equivariance

Suppose initial configuration is $|\psi|^2$-distributed. Then the configuration of crossing points $Q(\Sigma) = (Q_1 \cap \Sigma, \dots, Q_N \cap \Sigma)$ is $|\psi_\Sigma|^2$-distributed (in the appropriate sense) on every $\Sigma \in \mathcal{F}$.

Predictions

The detected configuration is $|\psi_\Sigma|^2$-distributed on *every* spacelike Σ.

As a consequence,

$\mathcal{F}$ is invisible, i.e., experimental results reveal no information about $\mathcal{F}$.

Can collapse theories be made relativistic?

[Diósi *Phys. Rev. A* 1990]
[Ghirardi, Grassi, Pearle *Found. Phys.* 1990]
[Tumulka *J. Statist. Phys.* 2006]
[Bedingham *Found. Phys.* 2011]
[Bedingham, Dürr, Ghirardi, Goldstein, Tumulka, Zanghì *J. Statist. Phys.* 2014]

Instantaneous collapse

Everybody's first idea:

If collapse is instantaneous (as opposed to propagating at speed c) then it must violate relativity.

That problem is easily avoided

For every spacelike hypersurface Σ there is a wave fct $\psi_\Sigma \in \mathscr{H}_\Sigma$.

E.g., $\mathscr{H}_\Sigma = \mathscr{H}_1^{\otimes N}$, $\mathscr{H}_1 = L^2\Big(\Sigma, \mathbb{C}^4, \langle\phi|\psi\rangle = \int_\Sigma d^3x\, \overline{\phi}(x) n_\mu(x) \gamma^\mu \psi(x)\Big)$.

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Relativistic collapse processes (stochastic evolution for ψ)

- [Diósi 1990, Ghirardi-Grassi-Pearle 1990]: relativistic continuous collapse processes for the state vector of a quantum field theory; however, suffers from divergences.
- [Bedingham 2011]: a modification that removes the divergences; however, not fully Lorentz invariant.
- [Tumulka 2006]: a relativistic GRW process for the state vector of N non-interacting spin-$\frac{1}{2}$ particles in an external field

- Given an initial wave function ψ_0 on Σ_0 (and possibly further data), the law for ψ specifies the joint distribution of all ψ_Σ with Σ in the future of Σ_0.
- In situations in which the unitary Schrödinger evolution would lead to a superposition $\psi_\Sigma = \sum_\alpha c_\alpha \psi^{(\alpha)}$ of macroscopically different contributions $\psi^{(\alpha)}$ (with $\|\psi^{(\alpha)}\| = 1$), the law for ψ yields $\psi_\Sigma \approx \psi^{(\alpha)}$ with probability close to $|c_\alpha|^2$.
- For any two hypersurfaces Σ, Σ' after a local measurement at a space-time point y, ψ_Σ and $\psi_{\Sigma'}$ select the same α of that measurement.

Still: given a relativistic collapse process for ψ_Σ, how do we get facts?
[Landau, Peierls 1931; I. Bloch 1967; Hellwig, Kraus 1970; Aharonov, Albert 1980]

Problem

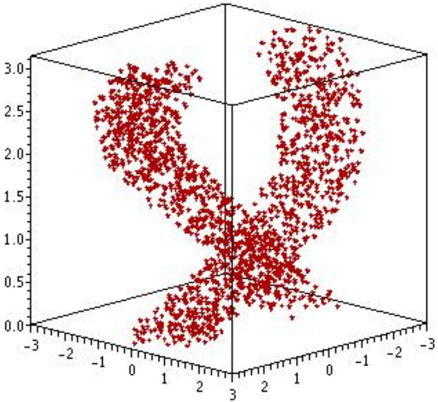
For $\Sigma = A \cup B$ with $A \cap B = \emptyset$, $\rho_A = \mathrm{tr}_B\, |\psi_\Sigma\rangle\langle\psi_\Sigma|$ depends on B: $\psi_{A\cup B'}$ may be very different from $\psi_{A\cup B}$ (collapses in-between), and $\mathrm{tr}_{B'}\, \psi_{A\cup B'}$ may be very different from $\mathrm{tr}_B\, \psi_{A\cup B}$. E.g., in an EPR experiment: If Σ lies after the exper. on particle 2 but before that on particle 1, then ρ_A will be a pure state. If Σ lies before both exper.s, ρ_A will be mixed.

Solution

Primitive ontology in space-time, such as matter density ontology or flash ontology.

BDGGTZ 2014: "... We should demand that certain local facts, such as whether a cat is dead or alive, do not depend on the choice of Σ. Fortunately, the *macroscopic* local situation is practically unambiguous... But the notion of "macroscopic" is imprecise ... The variable that defines local facts need not define a spin state for every particle. But it should define the distribution of matter in space-time and ensure that macroscopic configurations ... are unambiguous."

Flash ontology



Flashes in 2+1-dim space-time forming a binary star.

Relativistic GRWf (= rGRWf)

[Tumulka 2006]

- Involves a relativistic version of the GRW process, with a collapse occurring at a random *proper time* T after the previous collapse for the same "particle."
- A flash occurs at the center of every wave function collapse.
- There is no fact about "who influenced whom" (flashes in Paris those in Tokyo or vice versa).

Relativistic GRWm (= rGRWm)

Nonrelativistic law for m:

$$m(\mathbf{x},t) = \sum_{i=1}^{N} m_i \int_{\mathbb{R}^{3N}} d^{3N}q \; \delta^3(\mathbf{x}-\mathbf{q}_i)\, |\psi_t(q)|^2 = \langle \psi_t | \mathcal{M}(\mathbf{x}) | \psi_t \rangle$$

with mass density operator $\mathcal{M}(\mathbf{x}) = \sum_i m_i \delta^3(\mathbf{x}-\mathbf{q}_i)$.

Relativistic law for m: [Ghirardi 1999, BDGGTZ 2014]

$$m(\mathbf{x},t) = m(x) = \langle \psi_{PLC(x)} | \mathcal{M}_{PLC(x)}(x) | \psi_{PLC(x)} \rangle_{PLC(x)} \,.$$

Examples

(i) QFT, $\mathcal{M} = \mathcal{T}_{\mu\nu}$, $m = m_{\mu\nu}$; (ii) N Dirac particles,

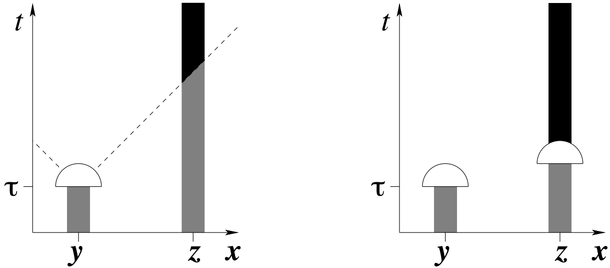
$$m_\mu(x) = \sum_{i=1}^{N} m_i \delta_\mu^{\mu_i} \int\limits_{PLC(x)^{N-1}} \Big(\prod_{j\neq i} d\sigma_j^{\mu_j}(y_j) \Big) \overline{\psi}_{PLC(x)} [\gamma_{\mu_1} \otimes \cdots \otimes \gamma_{\mu_N}] \psi_{PLC(x)}$$

with $\psi = \psi(y_1, \ldots, y_{i-1}, x, y_{i+1}, \ldots, y_N)$, measure $d\sigma^\mu(y)$ corresponding to the vector-valued 3-form $\varepsilon^{\mu}{}_{\kappa\lambda\nu}$.

Detection in Einstein's boxes example

A single particle and a detector. At time $t = 0$ in some Lorentz frame,

$$\psi_0 = \frac{1}{\sqrt{2}}\Big(|\mathbf{y}\rangle + |\mathbf{z}\rangle\Big)|\text{ready}\rangle\,.$$



Semi-circles represent detectors, dashed lines a light cone, thick lines the region where $m_1(x) = m_0$ (black) or $m_1(x) = m_0/2$ (grey).
LEFT: with one detector, RIGHT: with two detectors.

Facts about rGRWm

Given any of the known relativistic collapse processes for ψ_Σ:

- Suppose that a local measurement is made at a space-time point y. Then the m function in the future light cone of y and the ψ function on any Σ after y agree about the outcome.
- The empirical predictions of rGRWm agree approximately with those of the quantum formalism.
- Nonrelativistic limit $=$ nonrelativistic GRWm
- No signaling (except in a neighborhood of 10^{-7} m and 10^{-8} s)
- Microscopic parameter independence (except in that neighborhood)
- Nonlocal

Thank you for your attention