Introduction

1 The Title

When physics students learn about quantum mechanics, they may be intrigued by the notion that the possibility of a classical understanding of nature ended with the quantum revolution. Such a classical understanding presupposes the existence of an external real world *out there*, as well as the belief that it is the task of physics to find the basic constituents of this exterior real world and the laws that govern them. In quantum mechanics the situation seems to be quite different. One of the founders of quantum mechanics, Werner Heisenberg, writes that

> the idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them... is impossible.... [14, p. 129]

In quantum mechanics one does, in fact, talk about small particles, for example electrons in a diffraction experiment; nonetheless, one is cautioned against thinking of those particles as particles in the ordinary sense. In the textbook of Landau and Lifshitz one reads that

> It is clear that this result can in no way be reconciled with the idea that electrons move in paths.... In quantum mechanics there is no such concept as the path of a particle. [18, p. 2]

Statements of this kind abound, raising the question: What then is it that quantum physics is about? To students it must seem first of all to be about a whole lot of mathematics, for example, the abstract mathematics of Hilbert spaces and self-adjoint operators. As much as some students might hope that with the advanced mathematics insight into the genuine physical content of quantum mechanics advances as well, they soon learn that that is not so. At best they learn of the ongoing debate about the *interpretations* of quantum mechanics, about Schrödinger's cat and other paradoxes. Some students will learn that the debate is somehow related to a genuine problem in the formulation of quantum mechanics, namely the measurement problem. At the end of the day, the student will likely feel that the understanding of nature in any ordinary sense is impossible: the hallmarks of orthodox quantum theory are the denial of determinism and, more importantly, the denial of an objective reality. Such denials, even though they are often not well thought out, became the accepted wisdom concerning quantum mechanics and quantum reality, though there were some prominent exceptions such as Albert Einstein and Erwin Schrödinger.

Quantum philosophy provides a philosophical foundation for the claim that a fundamental physics focused on an objective reality is impossible. At the same time, quantum philosophy is hard to grasp. One of its most distinguished proponents was Niels Bohr, who insisted that a rational description of nature is impossible. Instead, he proposed the new concept of "complementarity," according to which

> A complete elucidation of one and the same object may require diverse points of view which defy a unique description. Indeed, strictly speaking, the conscious analysis of any concept stands in a relation of exclusion to its immediate application. [15, p. 102]

Schrödinger, who had intense discussions with Bohr on the meaning of quantum mechanics, in a letter to Wilhelm Wien says that

> Bohr's ... approach to atomic problems ... is really remarkable. He is completely convinced that any understanding in the usual sense of the word is impossible. Therefore the conversation is almost immediately driven into philosophical questions, and soon you no longer know whether you really take the position he is attacking, or whether you really must attack the position he is defending. [20, p. 228]

When all is said and done, the core of quantum philosophy is that physics is about measurement and observation, and not about an objective reality, about what seems and not about what is.

Along with Einstein and Schrödinger, John Stewart Bell was one of the few physicists who felt compelled to reject quantum philosophy. Bell preferred instead a "theory of local beables," writing:

> This is a pretentious name for a theory which hardly exists otherwise, but which ought to exist. The name is deliberately modelled on 'the algebra of local observables.' The terminology, be-able as against observ-able, is not designed to frighten with metaphysic those dedicated to realphysic. It is chosen rather to help in making explicit some notions already implicit in, and basic to, ordinary quantum theory. For, in the words of Bohr, 'it is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms'. It is the ambition of the theory of local beables to bring these 'classical terms' into the equations, and not relegate them entirely to the surrounding talk.
>
> The concept of 'observable' lends itself to very precise mathematics when identified with 'self-adjoint operator'. But physically, it is a rather wooly concept. It is not easy to identify precisely which physical processes are to be given status of 'observations' and which are to be relegated to the limbo between one observation and another. So it could be hoped that some increase in precision might be possible by concentration on the beables, which can be described in 'classical terms,' because they are there. The beables must include the settings of switches and knobs on experimental equipment, the currents in coils, and the readings of instruments.

‘Observables’ must be made, somehow, out of beables. The theory of local beables should contain, and give precise physical meaning to, the algebra of local observables. [3, p. 52]

This book is about a precise quantum theory in which the ‘classical terms’—the “local beables”—are brought into the equations. It is a book about an objective quantum description of nature. It is a book about quantum physics without quantum philosophy.

As soon as one frees oneself from the quantum philosophical notion that physics must not be about an objective reality and recognizes instead that a quantum theory must describe such a reality, the choice for its local beables, of particles moving in physical space, their motion being guided by waves, requires little imagination. This book is about such a theory, called Bohmian mechanics, the de Broglie-Bohm theory, or the pilot-wave theory.

In this theory particles move; how they move is determined by the wave function obeying Schrödinger’s equation. In Bohmian mechanics both the wave function and the particles are real physical objects “out there.” The structure of the theory is rather simple: The complete description of an N-particle system is provided by its configuration Q, defined by the positions $\mathbf{Q}_1, \ldots, \mathbf{Q}_N$ of its particles, together with its wave function $\psi = \psi(\mathbf{q}_1, \ldots, \mathbf{q}_N)$. The equations of motion are of the form

$$\begin{cases} \dfrac{dQ}{dt} = v^{\psi}(Q) & \text{(guiding equation)} \\[2ex] i\hbar \dfrac{d\psi}{dt} = H\psi & \text{(Schrödinger's equation),} \end{cases}$$

where H is the usual nonrelativistic Schrödinger Hamiltonian and v^{ψ} is a velocity field on configuration space determined by the wave function—its explicit form will be given later in this chapter and, in more detail, in the subsequent chapters of this book.

Bohmian mechanics is a counterexample to all claims that a rational account of quantum phenomena is impossible. It is also a counterexample to the claim that quantum mechanics proves that nature is intrinsically random—that there is no way that determinism can ever be reinstated in the fundamental description of nature. This book is about Bohmian mechanics, its applications, its prospects for relativistic extensions, and how it gives rise to the quantum mechanical rules one learns about in classes and textbooks.

2 What is Wrong with Quantum Mechanics?

It is often suggested that the fundamental problem with quantum mechanics is the *measurement problem*, or, more or less equivalently, Schrödinger’s cat paradox. However, these are but a dramatic symptom of a more fundamental problem: that the usual description of the state of a system in a quantum mechanical universe is of a rather unusual sort. The state of a quantum system is said to be given by a rather abstract mathematical object, namely the wave function or the quantum state vector (or maybe the density matrix) of the system, an object whose physical meaning is rather obscure in traditional presentations of quantum theory.

The measurement problem is this: if one accepts that the usual quantum mechanical description of the state of a quantum system is indeed the complete description of that system, it seems hard to avoid the conclusion that quantum measurements typically fail to have results. Pointers on measurement devices typically fail to point, computer printouts typically fail to have anything definite written on them, and so on. More generally, macroscopic states of affairs tend to be grotesquely indefinite, with cats seemingly both dead and alive at the same time, and the like. This is not good!

These difficulties can be largely avoided by invoking the measurement axioms of quantum theory, in particular the collapse postulate. According to this postulate, the usual quantum mechanical dynamics of the state vector of a system—given by Schrödinger's equation, the fundamental dynamical equation of quantum theory—is abrogated whenever measurements are performed. The deterministic Schrödinger evolution of the state vector is then replaced by a random collapse to a state vector that can be regarded as corresponding to a definite macroscopic state of affairs: to a pointer pointing in a definite direction, to a cat that is definitely dead or definitely alive, etc..

But doing so comes at a price. One then has to accept that quantum theory involves special rules for what happens during a measurement, rules that are in addition to, and not derivable from, the quantum rules governing all other situations. One has to accept that the notions of measurement and observation play a fundamental role in the very formulation of quantum theory, in sharp conflict with the much more plausible view that what happens during measurement and observation in a quantum universe, like everything else that happens in such a universe, is a consequence of the laws governing the behavior of the constituents of that universe—say the elementary particles and fields. These laws apply directly to the microscopic level of description, and say nothing directly about measurement and observation, notions that arise and make sense on an entirely different level of description, the macroscopic level.

We believe, however, that the measurement problem, as important as it is, is nonetheless but a manifestation of a more basic difficulty with standard quantum mechanics: it is not at all clear what quantum theory is about. Indeed, it is not at all clear what quantum theory actually says. Is quantum mechanics fundamentally about measurement and observation? Is it about the behavior of macroscopic variables? Or is it about our mental states? Is it about the behavior of wave functions? Or is it about the behavior of suitable fundamental microscopic entities, elementary particles and/or fields? Quantum mechanics provides us with formulas for lots of probabilities. What are these the probabilities of? Of results of measurements? Or are they the probabilities for certain unknown details about the state of a system, details that exist and are meaningful prior to measurement?

It is often said that such questions are the concern of the foundations of quantum mechanics, or of the interpretation of quantum mechanics—but not, somehow, of quantum mechanics itself, of quantum mechanics simpliciter. We think this is wrong. We think these and similar questions are a reflection of the fact that quantum mechanics, in the words of Bell, is "unprofessionally vague and ambiguous. Professional theoretical physicists ought to be able to do better. Bohm has shown us a way" [3, p. 160].

What is usually regarded as a fundamental problem in the *foundations* of quantum mechanics, a problem often described as that of *interpreting* quantum mechanics, is, we believe, better described as the problem of finding a sufficiently precise *formulation* of quantum mechanics, of finding a *version* of quantum mechanics that, while expressed in precise mathematical terms, is also clear as physics. Bohmian mechanics provides such a precise formulation.

3 History

Einstein never accepted quantum mechanics as the last word of physics on nature. He adhered to determinism and an objective description of the world, writing that

> I am, in fact, firmly convinced that the essentially statistical character of contemporary quantum theory is solely to be ascribed to the fact that this [theory] operates with an incomplete description of physical systems. [26, p. 666]

The random outcomes of quantum measurements had to arise, according to Einstein, from our ignorance about the values of certain variables that had not yet been introduced into quantum theory, variables describing the real state of affairs.

In the 1970s Bell introduced for such variables the notion of beables, and in particular of *local beables*, those beables describing the configuration of matter in space-time. A variety of choices for the fundamental local beables may be possible, and each choice may admit a variety of laws to govern the behavior of the fundamental local beables. Each such choice, of local beables and laws governing them, corresponds to a different theory, even when these different theories yield the same predictions for the results of experiments—even, that is, when they are *empirically equivalent*. When these predictions are those of orthodox quantum theory, the different theories are different *versions of quantum theory*.

The way such theories yield experimental predictions is via the fundamental local beables, in terms of which macroscopic variables are defined. Some of these macroscopic variables describe the results of experiments, so that the laws governing the behavior of the fundamental local beables have empirical implications. The orientation of a pointer, for example, is determined by the configuration of the fundamental local beables associated with the pointer, the configuration, say, of its particles, and the behavior of the pointer is determined by that of its particles.

This may seem rather obvious. We think it is. What is not so obvious, perhaps, and what, given the history of quantum mechanics and the surrounding controversy, is perhaps surprising, is that a choice of fundamental local beables and law for them yielding a version of quantum mechanics—yielding a theory empirically equivalent to quantum mechanics—should be possible at all.

But, as we've indicated, such a choice is possible and, insofar as nonrelativistic quantum mechanics is concerned, rather obvious. The local beables are the positions of the particles, and these move according to an equation of motion, the *guiding equation*, that involves the wave function of standard quantum theory. The resulting theory is the pilot-wave theory or Bohmian mechanics. It was discovered by Louis de Broglie not long after Schrödinger's creation, in 1926, of wave mechanics (Schrödinger's equation) [10]. In 1952, it was independently rediscovered and analyzed in measurement situations in two papers by David Bohm [4, 5], who showed that the theory is empirically equivalent to quantum mechanics.

Beginning in the 1960s, the theory was popularized by John Bell. Noting that the theory involved a manifestly nonlocal description of nature, Bell asked the question: Can one improve the theory so that it would involve only a local description? In an ingenious argument, in which Bell joined the famous EPR paradox with a simple probabilistic estimate, the famous Bell's inequality, he concluded in fact that any description of nature which predicts for experiments the quantum probabilities for their results must be nonlocal. The measured statistics for such nonlocality experiments do reproduce the quantum probabilities and therefore establish the fact that nature is indeed nonlocal. In other words, Bohmian mechanics is, arguably, just what the doctor ordered.

The pilot-wave approach to quantum theory was in fact initiated even before the discovery of quantum mechanics itself, by Einstein, who hoped that interference phenomena involving particle–like photons could be explained if the motion of the photons were somehow guided by the electromagnetic field—which would thus play the role of what he called a "Führungsfeld" or guiding field [28]. While the notion of the electromagnetic field as guiding field turned out to be rather problematical, the possibility that for a system of electrons the wave function might play this role, of guiding field or pilot wave, was explored by Max Born in his early papers founding quantum scattering theory [6, 7]—a suggestion to which Heisenberg was profoundly unsympathetic. Born presented in those papers the probability interpretation of the wave function, for which he eventually received the Nobel prize. He suggested that, guided by the wave function, particles move around in such a way that the distribution of their positions at a given time is given by the modulus squared of the wave function at that time.

Earlier still, the relation between geometric and wave optics had led to a reformulation of classical mechanics by Hamilton and Jacobi, in which the Hamilton-Jacobi function S, defined on the configuration space of a system of particles, defines via its gradient the momenta of the particles and acts like a guiding field. Bohmian mechanics can be regarded as a nonlocal generalization of the Hamilton-Jacobi theory, where S is replaced by the phase of the wave function. It turns out that the phase S, in Bohmian mechanics, obeys a modified Hamilton-Jacobi partial differential equation, modified by the addition of an extra potential term, called the *quantum potential* by Bohm, to the classical potential energy term of the usual Hamilton-Jacobi equation. However, while the classical Hamilton-Jacobi theory can be rephrased as Newtonian mechanics, with the Hamilton-Jacobi function S eliminated, this is not possible for Bohmian mechanics, for which a function S on configuration space plays an essential role.

The inclusion of local beables in quantum theory does not imply that the theory is thereby deterministic, though Bohmian mechanics happens to be so. Another version of quantum mechanics, called *stochastic mechanics* and invented by Edward Nelson around 1966 [21, 22], involves the same local beables as Bohmian mechanics—particles, described by their positions in space. But in this theory the particles evolve randomly, according to a diffusion process defined in terms of the wave function.

4 Impossibility

At the 1927 Solvay Congress, de Broglie presented his pilot-wave theory, which he regarded as an oversimplified version of what he hoped to be able to construct in the future, namely the so-called theory of the double solution, a theory about only waves and not particles. The reception of de Broglie's ideas was not enthusiastic. To begin with, the guiding law for a many-particle system was not formulated on three-dimensional physical space, but on the abstract high-dimensional configuration space: the law seemed to involve, as part of "physical reality," the wave function on that space, the meaning of which had been declared by Born to be a probability amplitude just before the Solvay Congress. While probability densities were acceptable as objects on configuration space, physical fields which determine the motion of particles were not.

Furthermore, de Broglie had not yet analyzed the theory for measurement situations and its relation to quantum mechanics remained unclear. In particular, he responded poorly to an objection of Wolfgang Pauli concerning an application to inelastic scattering, no doubt making a rather bad impression on the illustrious audience gathered for the occasion. After the Solvay Congress, de Broglie did not, for several decades, pursue these ideas further.

However in 1953, after the appearance of Bohm's rediscovery of the guiding equations, de Broglie returned to his idea of the double solution. Commenting on Bohm's revival of the pilot-wave theory, he expressed again his old doubts, which were shared by almost all physicists:

> The wave ψ used in wave mechanics cannot be a physical reality: it has arbitrary normalization, it is supposed to propagate in general in a visibly fictional configuration space, and, in conformity with the ideas of Mr. Born, it is merely a representation of probability depending on our state of knowledge and suddenly modified by information supplied by any new measurement. A causal and objective interpretation of wave mechanics cannot therefore be obtained on the sole basis of the pilot wave theory by assuming that the particle is guided by the wave. For this reason, I have been in full agreement with the purely probabilistic interpretation of Messrs. Born, Bohr, and Heisenberg since 1927. [11, p. 22][1]

Also Bohm, who had shown how the measurement rules of quantum mechanics emerge from his theory, in a discussion with Maurice Pryce in the 1960s described his theory as profoundly incomplete and preliminary [12].

Earlier on, it became widely accepted that quantum randomness could not be accounted for by averaging over additional variables—the so-called *hidden variables*.

[1] L'onde ψ utilisé en Mécanique ondulatoire ne peut pas être une réalité physique: Sa normalisation est arbitraire, sa propagation est censée s'effectuer en général dans un espace de configuration visiblement fictif, et, conformément aux idées de M. Born, elle n'est qu'une représentation de probabilité dépendant de l'état de nos connaissances et brusquement modifiée par les informations que nous apporte toute nouvelle mesure. One ne peut donc obtenir à l'aide de la seule théorie de l'onde-pilote une interprétation causale et objective de la mécanique ondulatoire en supposant que le corpuscule est guidé par l'onde ψ. Pour cette raison, je m'étais entièrement rallié depuis 1927 à l'interprétation purement probabiliste de MM. Born, Bohr et Heisenberg (English translation by S. Lyle).

The notion of hidden variables originated in 1932, in the book of the great mathematician John von Neumann, *On the Mathematical Foundations of Quantum Mechanics*, where he claimed that any attempt to introduce hidden variables into quantum mechanics and thereby restore determinism would inevitably lead to contradictions with quantum mechanical predictions and thus was hopeless. On the authority of von Neumann, this became common wisdom. In particular, the almost universal opinion arose that Bohmian mechanics must be flawed. After 1932 various other "no-go theorems," among them the celebrated paradox of Kochen and Specker [16], were obtained, all seeming to preclude the possibility of introducing hidden variables into quantum mechanics without contradictions.

The true status of the no-go theorems was clarified by Bell [2], who found that all such theorems involved unreasonable assumptions. About his personal discovery of Bohm's theory, Bell wrote:

> But in 1952 I saw the impossible done. It was in papers by David Bohm. Bohm showed explicitly how parameters could indeed be introduced, into nonrelativistic wave mechanics, with the help of which the indeterministic description could be transformed into a deterministic one. . . .
>
> . . . Moreover the essential idea was one that had been advanced already by de Broglie in 1927, in his "pilot wave" picture.
>
> But why then had Born not told me of this "pilot-wave"? If only to point out what was wrong with it? Why did von Neumann not consider it? More extraordinarily, why did people go on producing "impossibility'" proofs, after 1952, and as recently as 1978? [3, p. 160]

Bohmian mechanics is of course as much a counterexample to the Kochen-Specker argument for the impossibility of hidden variables as it is to the one of von Neumann. It is obviously a counterexample to any such argument. The assumptions of any such argument, however compelling they may seem to be, must fail for Bohmian mechanics.

Bell's clarification of the status of the no-go theorems had some impact on the accepted wisdom about the achievements of von Neumann and the others. It became somewhat less common for those no-go theorems to be used to justify the dismissal of Bohmian mechanics. Instead Bell's own theorem, his nonlocality proof, was more often used for that purpose. Bell's analysis became widely regarded as demonstrating that a realistic reformulation of quantum theory was impossible, or at least unacceptable.

But, as we have already indicated, Bell did not establish the impossibility of a realistic reformulation of quantum theory, nor did he ever claim to have done so. On the contrary, over the course of several decades, until his untimely death in 1990, Bell was the prime proponent, for a good part of this period almost the sole proponent, of the very theory, Bohmian mechanics, that he is supposed to have demolished.

It is worth noting that in Bohmian mechanics the positions of the particles are not really hidden. Again Bell:

> Absurdly, such theories are known as "hidden variable" theories. Absurdly, for there it is not in the wavefunction that one finds an image of the visible world, and the results of experiments, but in the complementary "hidden" (!) variables. Of course the extra variables are not confined

to the visible "macroscopic" scale. For no sharp definition of such a scale could be made. The "microscopic" aspect of the complementary variables is indeed hidden from us. But to admit things not visible to the gross creatures that we are is, in my opinion, to show a decent humility, and not just a lamentable addiction to metaphysics. In any case, the most hidden of all variables, in the pilot wave picture, is the wavefunction, which manifests itself to us only by its influence on the complementary variables. [3, p. 201]

5 The Dismissal

Bohmian mechanics is a version of quantum mechanics that, while expressed in precise mathematical terms, is clear as physics. Why isn't it taught? Why isn't it part of a quantum physics education, if only to point out that there is a version of quantum mechanics which is mathematically precise and physically clear? A possible answer could be that Bohmian mechanics is too contrived or that it is too complicated to be taken seriously as physics. This can be judged, of course, only by examining the formulation of the theory.

Many other reasons for dismissing Bohmian mechanics have been given, for example that because it is deterministic it is incompatible with free will, or that the notion of particles has been discredited or that it is a regression to classical modes of thought. If a clever person looks for reasons to dismiss something he does not like, he will find them.

At the end of the day, one of the fundamental reasons for the dismissal seems to be this: *Bohmian mechanics is against the spirit of quantum mechanics*. That is, an objective physical description, a return to determinism, a return to physical clarity clash with the tenets of quantum philosophy.

To students such an attitude must seem rather puzzling. But academia and scholarly behavior are not always rational. In the matter of Bohm, Robert Oppenheimer Bohm's PhD advisor, reportedly has said that "if we cannot disprove Bohm, then we must agree to ignore him" [23, p. 133].

6 Roads to Bohmian Mechanics

To the wave function and Schrödinger's equation of orthodox quantum theory, Bohmian mechanics adds the actual positions of particles and a first-order equation of motion for the positions, given by a velocity function $v = v^{\psi}$ depending on the wave function ψ and on the positions. By roads to Bohmian mechanics we mean basically ways of guessing a formula for v^{ψ}. There are many ways to do so. Here are some of them.

1. The simplest route, for particles without spin, is the following: Begin with the de Broglie relation $\mathbf{p} = \hbar\mathbf{k}$, a remarkable and mysterious distillation of the experimental facts associated with the beginnings of quantum theory—and itself a relativistic reflection of the first quantum equation, namely the Planck relation $E = h\nu$. The de Broglie relation connects a particle property, the momentum

$\mathbf{p} = m\mathbf{v}$, with a wave property, the wave vector $\mathbf{k}$. Understood most simply, it says that the velocity of a particle should be the ratio of $\hbar\mathbf{k}$ to the mass of the particle. But the wave vector $\mathbf{k}$ is defined only for a plane wave. For a general wave ψ, the obvious generalization of $\mathbf{k}$ is the local wave vector $\nabla S(\mathbf{q})/\hbar$, where S is the phase of the wave function (defined by its polar representation, see below). With this choice the de Broglie relation becomes $v = \nabla S/m$, the right hand side of which is our first guess for v^ψ.

We note that the de Broglie relation also immediately yields Schrödinger's equation, giving the time evolution for ψ, as the simplest wave equation that reflects this relationship. This is completely standard. In this simple way, the defining equations of Bohmian mechanics can be regarded as flowing in a natural manner from the first quantum equation $E = h\nu$.

2. Another route, for particles without spin, starts with writing Schrödinger's wave function in polar form $\psi = Re^{\frac{i}{\hbar}S}$, with R and S real. When one introduces this polar form into Schrödinger's equation for ψ, one obtains two equations: the modified Hamilton-Jacobi equation with the extra quantum potential for S and a continuity equation for $\rho = R^2 = |\psi|^2$. Since in classical Hamilton-Jacobi theory the momentum $mv = p = \nabla S$, it is natural to set $v = \nabla S/m$ in the quantum case as well.[2]

3. The quantum continuity equation is the key for a route which is meaningful also for particles with spin. This equation, an immediate consequence of Schrödinger's equation, involves a quantum probability density ρ and a quantum probability current J. Since densities and currents are classically related by $J = \rho v$, it requires little imagination to set $v^\psi = J/\rho$.[3]

4. Another way to arrive at a formula for v^ψ is to invoke symmetry. Since the space-time symmetry of the non-relativistic Schrödinger equation is that of rotations, translations, time-reversal, and invariance under Galilean boosts, it is natural to demand that this Galilean symmetry be retained when Schrödinger's equation is combined with the guiding equation. As described in Section 2.3, this leads to a specific formula for v^ψ as the simplest possibility.

5. We mention one last route. With any wave function $\psi(q)$, one can associate a Wigner distribution $W^\psi(q,p)$, a sort of quantum-mechanical joint distribution for position and momentum, which, however, need not be non-negative. Setting

$$mv^\psi(q) = \frac{\int pW^\psi(q,p)\,dp}{\int W^\psi(q,p)\,dp}$$

[2] This was basically the route followed by Bohm in his 1952 paper, though Bohm went further and recast the first-order theory defined by $v^\psi = \nabla S/m$ into a second-order theory involving accelerations. By differentiating the modified Hamilton-Jacobi equation with respect to time, Bohm obtained a modified Newton equation which involves, in addition to the usual force arising from the classical potential energy, an extra "quantum force" arising from the quantum potential. Bohm formulated his theory in terms of this second-order equation. We believe that the first-order form is preferable.

[3] This route to v^ψ could be called the Pauli-Bell "derivation" because this way of presenting Bohmian mechanics, Bell's favorite, originated with Pauli at the 1927 Solvay Congress ([9], page 134; English translation [1], page 365.)

yields our last formula for v^{ψ}.

There are several other natural routes to v^{ψ}, but we shall give no more. All these routes, those we've explicitly mentioned and those to which we've alluded, yield in fact exactly the same formula for v^{ψ}, though the explicit form may appear different in some cases.

7 Questions

Bohmian mechanics is not a particularly difficult theory. It is defined by two equations: the Schrödinger equation for the wave function ψ and the guiding equation for the positions of the particles. Moreover, the guiding equation is easy to find and is quite simple. There are no axioms about observers, observation, or measurement of observables. There are no axioms about the collapse of the wave function during measurement. This is in sharp contrast with orthodox quantum mechanics.

However, what is not at all easy to understand about Bohmian mechanics is its precise relationship with standard quantum theory. What is at first not at all clear is how the predictions of Bohmian mechanics are related, if at all, to those of that theory. For example:

1. Quantum interference for a particle in a two-slit experiment seems to show that the particle must go through both slits at once. But in Bohmian mechanics a single particles passes through only one of the two slits. Can Bohmian mechanics account for the two-slit experiment? (See below.)

2. Are Bohmian trajectories or velocities observable? (See Chapters 2, 3 and 7.)

3. How can the intrinsic randomness of quantum theory arise from a deterministic theory such as Bohmian mechanics? (See Chapter 2.)

4. How can the collapse rule for the wave function be compatible with Bohmian mechanics, one of whose axioms is Schrödinger's equation for the evolution of the wave function, which is incompatible with its collapse? (See Chapter 2, where, among other things, it is shown that the collapse rule is a theorem of Bohmian mechanics.)

5. In Bohmian mechanics a particle always has a well-defined position and velocity. How can this be compatible with Heisenberg's uncertainty principle? (See Chapter 2.)

6. Spin, unlike position, has no classical analogue. How can Bohmian mechanics deal with spin? (See Chapter 3.)

7. In the formulation of Bohmian mechanics there are no operators as observables, which play so prominent a role in quantum theory. How, if at all, do operators as observables arise in Bohmian mechanics? (See Chapter 3.)

8. There are many theorems precluding the possibility of hidden variables for quantum mechanics. These theorems seem to demonstrate that a deterministic reformulation of quantum mechanics is impossible, and that the values of quantum observables are normally created only by measurements and do not correspond to any pre-existing values for the observables. How then can Bohmian mechanics be compatible with the predictions of quantum theory? (See Chapter 3.)

9. Isn't the curious indistinguishability of quantum mechanical identical particles incompatible with particles having definite positions, and following definite trajectories? (See Chapter 8.)

10. Similarly, how can quantum entanglement be compatible with Bohmian mechanics? (See Chapter 2.)

11. A widely debated problem in quantum theory is that of time measurements, for example the dwell time of a particle within, and its time of escape from, a certain region of physical space. In quantum mechanics there is in fact no self-adjoint time observable of any suitable sort and there is a large and controversial literature on what to do about this. On the other hand, Bohmian mechanics makes specific predictions for the statistics of such times. How can Bohmian mechanics be then empirically equivalent to standard quantum mechanics? (See Chapter 6.)

12. In Bohmian mechanics the motion of even a single particle requires an infinite-dimensional system for its description—infinite-dimensional because it involves the wave function, whose specification requires an infinite collection of real numbers. How can this be compatible with the emergence of a classical regime, in which the motion of such a particle in physical space is determined by a 6-dimensional system, involving only position and momentum? (See Chapter 5.)

13. Given the prominent role that configuration space—a highly anti-relativistic structure—plays in its formulation, how can a relativistic version of Bohmian mechanics be possible? (See Chapters 9, 11 and 12.)

14. In quantum field theory particles are created and annihilated. Doesn't that preclude a description in terms of genuine particles? (See Chapter 10.)

15. In orthodox quantum mechanics the wave function of a system is usually regarded as a probability amplitude and thus is widely regarded as corresponding only to our knowledge of that system. But in Bohmian mechanics, in which it plays a crucial dynamical role, the wave function should be regarded as entirely objective. Is this a reasonable thing to do? (See Chapters 2, 11 and 12.) [4]

These are some of the questions that will be addressed in this book. Most will be addressed in the ensuing chapters, but the first will be dealt with right now.

According to Richard Feynman, the two-slit experiment for electrons is

[4] Very recently, Matthew Pusey, Jonathan Barrett, and Terry Rudolph have developed a nice new argument [24] showing that the Schrödinger wave function can't consistently be interpreted as merely statistical, i.e., subjective. (Rudolf's grandfather, incidentally, was Schrödinger.)

> a phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality it contains the only mystery. [25, p. 37.2]

As to the question:

> How does it really work? What machinery is actually producing this thing? Nobody knows any machinery. Nobody can give you a deeper explanation of this phenomenon than I have given; that is, a description of it. [13, p.145]

Now consider the Bohmian trajectories for the two-slit experiment (see Fig. 1).

One sees that Bohmian mechanics resolves the dilemma of the appearance, in one and the same phenomenon, of both particle and wave properties in a rather trivial manner. Notice that while each trajectory passes through but one of the slits, the wave passes through both, and the interference profile that therefore develops in the wave generates a similar pattern in the trajectories guided by this wave.

Finally, compare Feynman's presentation with that of Bell:

> Is it not clear from the smallness of the scintillation on the screen that we have to do with a particle? And is it not clear, from the diffraction and interference patterns, that the motion of the particle is directed by a wave? De Broglie showed in detail how the motion of a particle, passing through just one of two holes in screen, could be influenced by waves propagating through both holes. And so influenced that the particle does not go where the waves cancel out, but is attracted to where they cooperate. This idea seems to me so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored. [3, p. 191]

With regard to the second question, the following figure, similar to Figure 1, recently appeared in Science [17]:

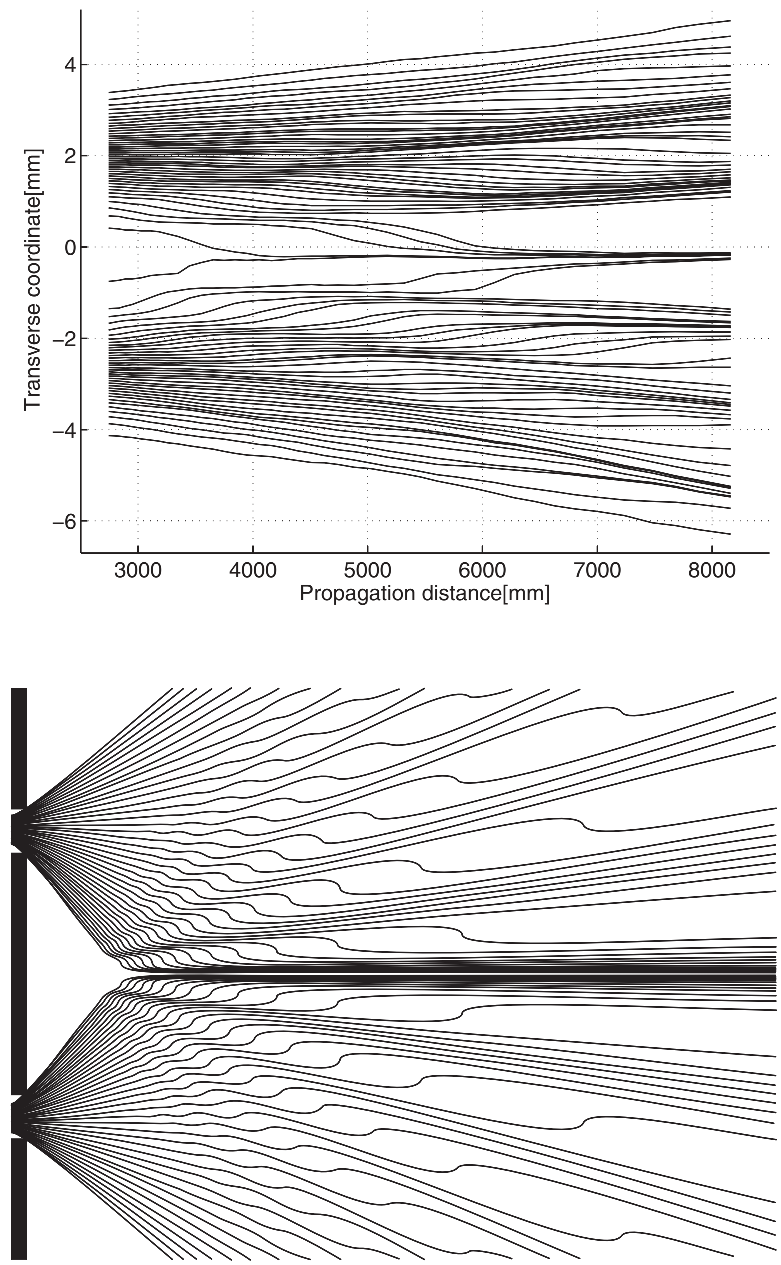

Figure 1: A family of Bohmian trajectories for the two-slit experiment. (Figure adapted by Gernot Bauer from B.J. Hiley, C. Philippidis, and C. Dewdney [8].)

In the accompanying article by Aephraim Steinberg et al., the trajectories depicted in the figure are reported to be the result of a suitable weak measurement on a quantum particle. According to the authors:

> Single-particle trajectories measured in this fashion reproduce those predicted by the Bohm-de Broglie interpretation of quantum mechanics, although the reconstruction is in no way dependent on a choice of interpretation. [17]

Chapter 7 provides a Bohmian analysis of this sort of experiment.

References

[1] G. Bacciagaluppi and A. Valentini. *Quantum Theory at the Crossroads: Reconsidering the 1927 Solvay Conference*. Cambridge Univ. Press, 2009.

[2] J. S. Bell. On the Problem of Hidden Variables in Quantum Mechanics. *Reviews of Modern Physics*, 38:447–452, 1966. Reprinted in [27] and in [3].

[3] J. S. Bell. *Speakable and Unspeakable in Quantum Mechanics*. Cambridge University Press, Cambridge, 1987.

[4] D. Bohm. A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables: Part I. *Physical Review*, 85:166–179, 1952. Reprinted in [27].

[5] D. Bohm. A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables: Part II. *Physical Review*, 85:180–193, 1952. Reprinted in [27].

[6] M. Born. Quantenmechanik der Stoßvorgänge. *Zeitschrift für Physik*, 37:863–867, 1926.

[7] M. Born. Quantenmechanik der Stoßvorgänge. *Zeitschrift für Physik*, 38:803–827, 1926. English translation (Quantum Mechanics of Collision Processes) in [19].

[8] B. J. Hiley C. Philippidis, C. Dewdney. Quantum Interference and the Quantum Potential. *Il Nuovo Cimento B*, 52:15–28, 1979.

[9] Solvay Conference. *Electrons et Photons: Rapports et Discussions du Cinquième Conseil de Physique tenu à Bruxelles du 24 au 29 Octobre 1927 sous les Auspices de l'Institut International de Physique Solvay*. Gauthier-Villars, Paris, 1928.

[10] L. de Broglie. La Nouvelle Dynamique des Quanta. In *Electrons et Photons: Rapports et Discussions du Cinquième Conseil de Physique tenu à Bruxelles du 24 au 29 Octobre 1927 sous les Auspices de l'Institut International de Physique Solvay*, pages 105–132, Paris, 1928. Gauthier-Villars.

[11] L. de Broglie. Scientific Papers Presented to Max Born. In *L'Interprétation de Mécanique Ondulatoire à l'Aide d'Ondes à Régions Singulières*, page 22, New York, 1953. Hafner Publishing Company Inc.

[12] D. Edge and S. Toulmin. *Quanta and Reality*. Hutchinson and Co, 1962.

[13] R. Feynman. *The Character of Physical Law*. MIT Press, Cambridge, MA, 1967.

[14] W. Heisenberg. *The Physicist's Conception of Nature*. Harcourt Brace, 1958. Trans. Arnold J. Pomerans.

[15] M. Jammer. *The Philosophy of Quantum Mechanics*. Wiley, New York, 1974.

[16] S. Kochen and E. P. Specker. The Problem of Hidden Variables in Quantum Mechanics. *Journal of Mathematics and Mechanics*, 17:59—87, 1967.

[17] S. Kocsis, B. Braverman, S. Ravets, M.J. Stevens, R. P. Mirin, L. K Shalm, and A. M. Steinberg. Observing the Average Trajectories of Single Photons in a Two-Slit Interferometer. *Science*, 332:1170–1173, 2011.

[18] L. D. Landau and E. M. Lifshitz. *Quantum Mechanics: Non-relativistic Theory*. Pergamon Press, Oxford and New York, 1958. Translated from the Russian by J. B. Sykes and J. S. Bell.

[19] G. Ludwig, editor. *Wave Mechanics*. Pergamon Press, Oxford and New York, 1968.

[20] W. Moore. *Schrödinger*. Cambridge University Press, New York, 1989.

[21] E. Nelson. Derivation of the Schrödinger Equation From Newtonian Mechanics. *Physical Review*, 150:1079–1085, 1966.

[22] E. Nelson. *Dynamical Theories of Brownian Motion*. Princeton University Press, Princeton, N.J., 1967.

[23] F. D. Peat. *Infinite Potential*. Addison-Wesley, 1997.

[24] M. F. Pusey, J. Barrett, and T. Rudolph. The Quantum State Cannot Be Interpreted Statistically. arXiv:1111.3328v1, 2011.

[25] M. Sands R. Feynman, R. B. Leighton. *The Feynman Lectures on Physics, I*. Addison-Wesley, New York, 1963.

[26] P. A. Schilpp, editor. *Albert Einstein, Philosopher-Scientist*. Library of Living Philosophers, Evanston, Ill., 1949.

[27] J. A. Wheeler and W. H. Zurek. *Quantum Theory and Measurement*. Princeton University Press, Princeton, N.J., 1983.

[28] E. P. Wigner. Interpretation of Quantum Mechanics. In [27], 1976.