Frequently Asked Questions about Bohmian Mechanics

Christian Beck, Robert Grummt, Florian Hoffmann,
Sören Petrat, Nicola Vona

November 26, 2013

You can find a video of this script at
http://www.mathematik.uni-muenchen.de/~bohmmech/videos.html

1. **Why should we care about Bohmian Mechanics at all?**

   Bohmian Mechanics is so important because it is a counterexample to all the claims that the quantum world and therefore all of nature is not understandable. The goal of physics, apart from applications, is to find out what is really going on in nature. It seems that the historical development of Quantum Mechanics led most physicists to give up on trying to understand the microscopic world, and to limit themselves to the description of experiments. New phenomena seemed to suggest that it was impossible to speak about nature objectively. This led not only to the general acceptance of many quantum mysteries, but some also considered them as true revelations.

   The lesson of Bohmian Mechanics is that this is not necessary. It is possible to understand quantum mechanics! Never give up on the goal of physics to try to understand what the world is made of and how this "stuff" behaves! Bohmian Mechanics brings us closer to the answer of the question what nature *is*!

   Let me finish with a quotation from John Bell, one of the really great physicists who very much appreciated this point. In his article "On the impossible pilot wave" he writes about his thoughts after he learned about Bohmian Mechanics (which he also called "Pilot wave" model):

   > But why then had Born not told me of this "pilot wave"? If only to point out what was wrong with it? Why did von Neumann not

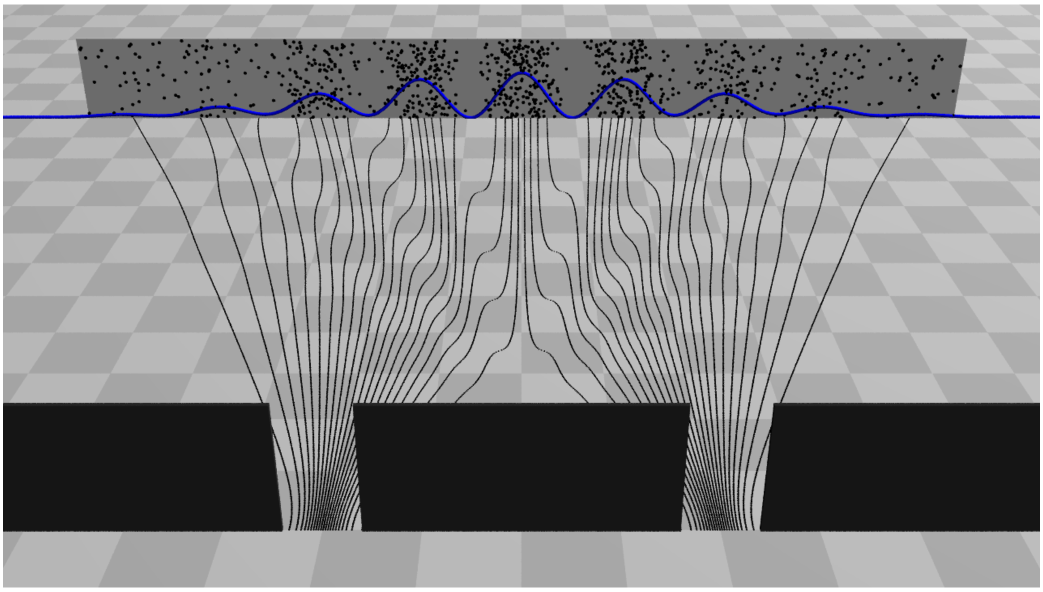


Figure 1. Double slit experiment. At every run of the experiment a single dot appears on the screen and a single Bohmian trajectory is followed by the relative particle; the set of dots and trajectories shown corresponds to the collection of many subsequent runs. In blue, the modulus squared of the wave function is shown.

> consider it? More extraordinarily, why did people go on producing "impossibility" proofs, after 1952, and as recently as 1978? When even Pauli, Rosenfeld, and Heisenberg, could produce no more devastating criticism of Bohm's version than to brand it as "metaphysical" and "ideological"? Why is the pilot wave picture ignored in text books? Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show that vagueness, subjectivity, and indeterminism, are not forced on us by experimental facts, but by deliberate theoretical choice?

2. **What about the particle wave dualism, how does it appear in BM?**

   BM is about particles and these particles are guided by Schrödinger's wave function. Thus in BM the situation is "wave and particle" rather than "wave or particle". This is illustrated by the Bohmian trajectories for the double slit experiment: Each particle follows its individual trajectory and the interference pattern of the wave function is reflected in the structure of the trajectories, which yields the characteristic distribution of dots on the screen.

3. **How does BM account for the double slit experiment? How can the interference pattern be there if we *know* that the particle went through just one of the slits?**

   Let us go through the experimental facts step by step. At each run of the experiment just one single spot appears on the detection screen; in BM one identifies the spot on the screen with a point particle. This particle runs from the source through one slit to the screen on a continuous path. Even though the particle goes through just one slit, the other slit being open or closed influences what happens, so the particle is in a sense not free: an additional physical entity must be considered in the description. The presence of the interference fringes suggests that this additional entity must be some kind of wave.

   These steps naturally lead to the Bohmian explaination of this experiment: each particle follows a continuous trajectory that can necessarily go through just one of the slits; on the contrary, the wave function that determines how the particle moves, travels across both slits leading to the appearance of the interference pattern (cf. Fig. 1).

   One important point in the analysis of the double slit experiment is usually considered to be the fact that if one determines through which of the slits the particle went, then the interference pattern disappears. Of course such a measurement needs an interaction between the wave and the detector, and you can easily imagine that it is the effect of this interaction that destroys the interference pattern. But very often the conclusion that is drawn from this observation is that our *knowledge* about the path followed by the electron causes the interference pattern to disappear. Now I ask you to carefully and critically consider this assertion: does it really make sense to think that simply getting to know something we are able to influence what actually happens? Do you really think that the electrons in an experiment behave differently than any other electron in the Universe because we know something about them? How can the electron know what we know? The final lesson here is that Physics is about Nature, not about us, even if we do the experiments and write the theories. We should always remember this.

4. **Isn't the Heisenberg uncertainty relation in contradiction to the existence of trajectories?**

   The predictions of Heisenberg's uncertainty relation are in accord with the existence of trajectories, there is no contradiction. Heisenberg's uncertainty relation reads

$$\Delta x \Delta p \geq \frac{\hbar}{2},$$

where Δx and Δp are the position and momentum variances respectively. Take the position variance for example, what does it tell us? It is a measure for how much a series of position measurements on identically prepared systems will fluctuate around the mean position $\langle x \rangle$. Thus Δx is not an uncertainty in the sense that the position of a single particle is uncertain or blurry. But it is an uncertainty in the sense that repeated position measurements will not yield the same results (not even when experimental measurement inaccuracies are taken into account). Instead repeated position measurements yield a position distribution centered around $\langle x \rangle$ with width Δx. Single position measurments yield single values and single particles move along well defined trajectories. The same holds for Δp.

Now having understood this you might ask, why repeated position measurements made by a perfectly accurate experimentator yield a distribution at all. Single particles move along well defined trajectories. Particles starting at different initial positions move along different well defined trajectories. Hence, the distribution obtained in repeated position measurements arises, because the particles start at different initial positions. Since we assumed that the experimentator works perfectly accurate, there must be another reason behind those differing initial conditions. BM gives such a reason and proves how the distribution of measurement results as well as Heisenberg's uncertainty relation arises from it. So BM – a theory about particles and their trajectories – proves Heisenberg's uncertainty relation.

5. **Why is the position representation special in BM? Aren't position and momentum equivalent in QM?**

   The position representation of the wave function is special, because Bohmian Mechanics is about particles and particles are described by their positions. Nevertheless, the wave function can also be expressed as function of momentum. As a matter of fact, the wave function in BM is exactly the same mathematical object as the wave function in standard QM. But even though the wave function can be expressed in different bases, positions are special. This is because BM is a theory about the world that surrounds us and this world is made out of particles and particles are described by their positions.

6. **What is the role of the observer in BM ?**

What is the role of the observer in electrodynamics? The theory is about fields and particles in space-time. After the theory is formulated, it can make sense to analyze e.g. which electric potential an observer would measure at a given capacitor. But for the theory the observer is irrelevant: the theory works well without ever mentioning an observer. The same goes for relativity: We can ask which length of a rocket is measured by an observer in a given Lorentz-frame, but the theory is only about space-time events and can be wholly formulated without ever mentioning an observer.

BM is about particles guided by a wave function and we do not need to mention observers in order to formulate or analyze the theory. Nevertheless, it makes a lot of sense to analyze the predictions of the theory for physical systems, which can be prepared and measured in laboratories. Such predictions predict what an observer observes. Thus the observer does not play any role in the formulation of Bohmian Mechanics.

7. **In QM the notion of a measurement seems to be central. What is a measurement in BM ? Are there special rules for the prediction of the outcomes of measurements derived from the Bohmian framework?**

A measurement is the interaction between two quantum mechanical systems, one of which is the measuring apparatus/device. This interaction is described in just the same way as every other interaction, this means by the Schrödinger equation. The characterizing property of a measuring device is that at the end of the experiment/interaction the measuring device is in one of several macroscopically distinguishable configurations, like for instance pointers pointing in different directions. Now, as is well known, including the measuring device in the quantum mechanical description, as we just did, produces the measurement problem, illustrated by the famous Schrödinger's cat gedankenexperiment. To understand this consider that if the system to be "measured" starts in a state a, the apparatus points to the right after the measurement, and to the left if the system started out in a diffenrent state b. Now we can prepare the system in a state $\frac{1}{\sqrt{2}}(a+b)$. Due to the linearity of the Schödinger equation the prediction for the wave function of the apparatus after the measurement is $\frac{1}{\sqrt{2}}$(pointing left + pointing right). This is obviously not in agreement with the experimental situation. In BM the resolution of the measurement problem comes about as we simply analyze the measurement situation. What we have ignored so far is the position of the particles. In our example the pointer positions left and right correspond to wave functions which have virtually disjoint support in configuration space. This is called decoherence. The Bohmian trajectories of the particles in the pointer are always well defined, so they are always somewhere. Now if the wavefunction consists of two parts of (almost) disjoint support, the particle positions will be in just one of them. So the Bohmian position of the particles in the pointer after the experiment will be located in the support of only the right or the left wave function in our example, while the other branch of the wave function does not play any role for the future state of affairs, and can be neglected. Thus we see that taking into account the particle positions effectively reproduces the collapse postulate of ordinary QM for measurement situations and gives a clear cut picture of what happens in during a measurement. In addition there is no more measurement problem in BM.

8. **Every measurable physical quantity should be represented by a self-adjoint operator. These operators are part of the axioms of QM, though they don't seem to appear in BM. How does one then describe measurements in BM?**

   The correspondence of measurable physical quantities to self adjoint operators on Hilbert-space does not have to be postulated as an axiom but can be deduced easily from the Born rule in quantum theory. This can also be achieved in ordinary QM and it was elaborated by Günter Ludwig. Though, the analysis involves a von-Neumann measurement, i.e. the superposition of different pointer positions together with the axiom that the system-plus-apparatus wave function collapses onto one state with a definite outcome at the end of the measurement. Therefore, the derivation of the operators is much more clear in BM, since there we understand very naturally how a definite outcome emerges, given an initial wave function in a superposition, without additional axioms.

   The mentioned analysis sheds light on the conceptual status of the "observable-operators": Given an experimental procedure such, that we can conclude from the outcome that the (effective) wave function of the system lives in a given subspace of Hilbert-space at the end of the experiment, there exists a self-adjoint operator on that Hilbert-space, which enables us to compute the statistics of outcomes for many repetitions of that experiment for some (arbitrary) given initial wave function. The measured quantity is an observable quantity, but to call also the operator an "observable" is more than misleading. The operator is not a fundamental object but rather a statistical bookkeeper of the associated experiment.

   Summing up it is important to note that the measurable physical quantities give us the selfadjoint operators not the other way round.

9. **In standard QM the wave-function of a system collapses upon observation. I always thought the collapse of the wave function was something essential. In BM the wave function always evolves according to Schrödinger's equation and never collapses. How can that be? What is the collapse in BM?**

   In BM the collapse of the wave function is not an axiom anymore but can be derived from the usual Schrödinger evolution. The wave function in BM always evolves according to the Schrödinger equation. But in addition to the wave function we have the positions of the particles. One actual configuration of the particles corresponds to one point in configuration space and this point is guided by the wave function. If now the wave function is in a superposition and if the different branches of the wave function do not overlap on configuration space, we can identify one unique branch which guides the actual configuration of the particles.

   Suppose now, that the branches, which evolve according to the Schrödinger equation, will for all practical purposes never again overlap. This is called decoherence, it happens in usual measurement situations. In this situation the actual configuration of the particles will stay in the same branch of the wave function (FAPP) forever. The other branches are no longer relevant for the dynamics of the particles. We do not lose information about anything of physical relevance, if we forget about them and treat only the remaining branch as the wave function of the system. This is called the *effective collapse* in BM and the remaining branch the *effective wave function*.

10. **Is BM not a step BACK to a classical theory?**

    BM does not at all provide a classical picture of the world. Initial positions together with the initial wave function uniquely determine the motion of the particles. This might seem reminiscent of a classical theory, but the motion of the particles is not at all classical, as it is determined by the wave function which is a solution of the Schrödinger equation. This central role of the wave function renders BM a completely non-classical theory.

11. **Is BM not a hidden variable theory? There are various no-go theorems, ruling out hidden variable approaches. Does not for example Bell's Theorem or the Kochen-Specker Theorem prove BM wrong?**

    The name Hidden Variable Theories refers to theories that substitute or supplement the wave function of QM by some other variable. This definition also applies to BM, where the wave function is supplemented by the actual position of the particle. Now, you can decide by yourself whether the term *hidden* is appropriate or not for particle positions.

No-go theorems are not general theorems about hidden variables as defined above, even if they are often invoked when speaking in general terms about hidden variables. To really understand if they say something about BM or not, general terms are not sufficient, and we have to look at the theorems closer.

For example, Kochen-Specker theorem says that it is not possible to describe quantum mechanical observables by variables independent of the experimental set-up. But in BM the outcomes of experiments are described precisely by quantum mechanical observables, not by classical variables, in perfect agreement with the theorem. Only positions are described by usual variables in addition to the wave function, but the Bohmian positions are the actual positions occupied by the particles during the whole evolution, and not results of position measurements, that are also described by quantum observables.

In contrast, Bell's Theorem can be formulated without even speaking about hidden variable theories: the theorem states that some predictions of QM, well confirmed by several experiments, can not be explained by *any local theory*. And BM is nonlocal, as well as QM is. In fact BM inspired Bell to investigate non-locality, finally leading him to discover his famous inequalities. Bell was one of the most prominent proponents of BM and wrote many articles explaining it in great detail.

Bell's Theorem is often misunderstood and reduced to a mere statement that excludes the possibility of substituting QM with a local classical theory. Conversely, it is an extremely important result, that requires us to change drastically our conception of the world, and that is the source of many difficulties in the reconciliation between QM and Relativity.

12. Can one observe the positions of the Bohmian particles?

Yes, of course. According to Bohmian Mechanics the world is made out of particles. Therefore, the particles are in fact the *only* thing that you can observe.

Let me stress again that according to Bohmian Mechanics the world that you see around you with your naked eye is really composed of Bohmian particles and they are really there where you see them. Tables, chairs and stones are made of particles. If you make a double slit experiment with single particles and there appears a black dot on the screen, then the particle has really hit the screen where the black dot is. In Bohmian Mechanics it is clear what observing a particle's position in a single experiment means because there *are* particles in the theory.

If you apply Bohmian Mechanics to an ensemble of subsystems, i.e., if you are interested in the statistical behavior of many identically prepared systems, then you find that the statistics of position measurements is described by the usual position operator, which is of course the same position operator as in standard quantum mechanics. That means that if you observe a particle's position many times in identically prepared systems then the position is $|\psi|^2$ distributed.

One more thing should be added. In many physical situations it's not really necessary to know or to measure the positions of the Bohmian particles. In the ground state of a hydrogen atom the electron does not move at all for example. This is not very interesting, right? What's interesting is, for example, the spectral lines of a hydrogen atom, i.e., the difference of the energy eigenvalues of the ground state wave function and excited states. In other situations you might only want to know the distribution of many particles which, as mentioned before, is given by the absolute square of the wave function. So most of the times all you need to know is the wave function of a system. Most of the times this gives you all you want.

13. Can one observe the trajectories of the Bohmian particles?

In general the trajectories cannot be observed in the usual sense of measuring the particle's position in short time intervals. This interaction with the measurement apparatus changes the original trajectory, so one doesn't observe the trajectory the particle would have taken if one had not measured it. In other words, by the process of measuring the position of a particle you have to somehow interact with it and with this interaction you always disturb its motion.

In some situations, when one can neglect the interference effects of the wave function, for example when one deals with a very heavy particle or an object composed of very many particles, one can actually show that the Bohmian trajectories become classical and then you can of course observe them. If you throw a stone you can observe its trajectory with your naked eye because the interaction of photons with the stone don't disturb its motion very much.

Recently a lot of progress in observing trajectories of single particles has been made by so called weak measurements. These are measurements that disturb a particle trajectory only very little. On the downside, in a single run, a weak measurement gives you only very little information about the trajectory, so you have to repeat it many times. This sort of measurement has recently been done for photons in a double slit experiment and the measured trajectories are exactly the Bohmian ones.

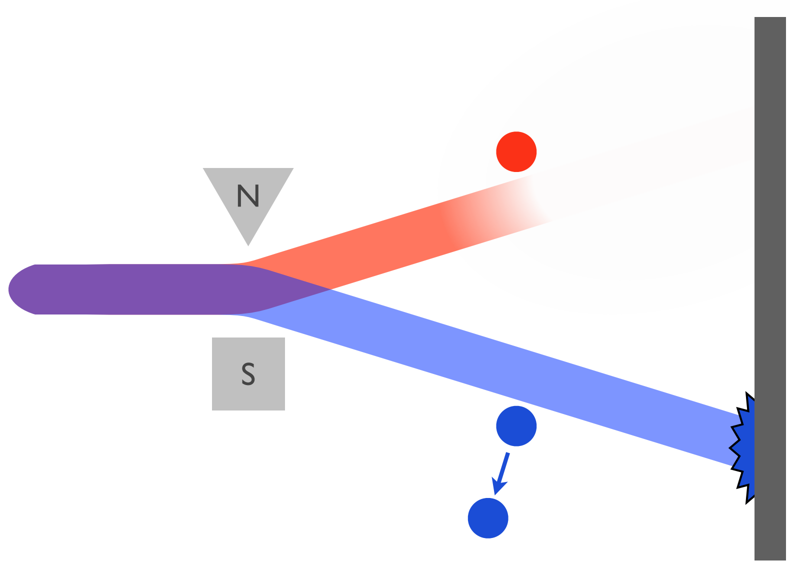
14. **I have heard about surrealistic trajectories in Bohmian Mechanics, what are these trajectories and why are they called surrealistic?**

The term *surrealistic trajectories* was used the first time by Englert, Scully, Süssmann, and Walther, in a paper dated 1992.[1] Their goal was to show that Bohmian Mechanics does not make sense, by providing an example in which the trajectory predicted by Bohmian Mechanics is clearly different from the actual trajectory followed by a physical particle. In facts, what they found is a remarkable example in which the Bohmian trajectories are completely different from what *we would expect*. Textbooks are full of examples in which a quantum system behaves in a way completely against what we would have expected based on our classical daily-life. Among these examples, the surrealistic trajectories are a special case because the result of the experiment is what one would expect, just the shape of the trajectories is unexpected from a classical perspective, and therefore might seem implausible.

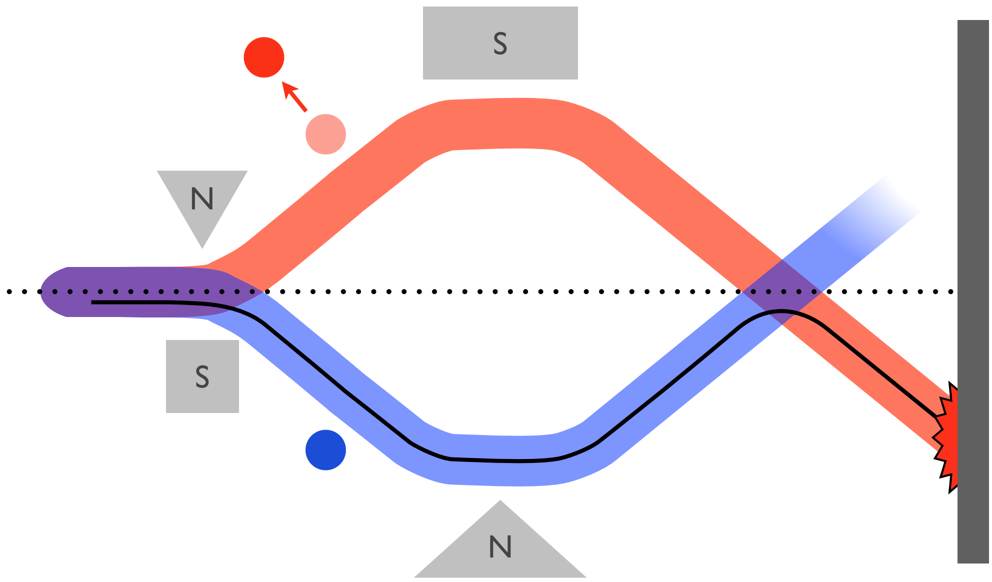
But now let's look at this example explicitly (see Fig. 2). Consider an ion with positive charge and spin one half, that passes through a Stern-Gerlach magnet, and then reaches a detecting screen. The support of the wave function gets split in two parts, that travel along two different paths, let's call them Red and Blue, that end at two distant places on the screen. Consider now two particles also with positive charge, sitting respectively close to the Red and Blue paths. Let's call them the Red and the Blue particle. When the ion passes close to such a particle, the electric repulsion causes the ion to deviate and puts the colored particle in motion. Of course, if the ion had only spin up, then the Red particle starts moving, and the screen gets hit in the upper region, and similarly for spin down. We want to consider a superposition of spin up and down with equal weights. In this state both results are possible, but it is still true that the colored particle that starts moving is that on the same side where the ion hits the screen.

To make the experiment more interesting, suppose that before the screen a second Stern-Gerlach magnet is present, reversed with respect to the first one, so that the Red and the Blue supports are brought back together and cross before reaching the screen. Now the Red wave function ends up in the lower region on the screen, therefore when the screen flashes at this

[1] B.G. Englert, M. O. Scully, G. Süssmann, and H. Walther. Surrealistic Bohm trajectories. *Zeitschrift für Naturforschung. A*, vol. 47a(12):1175–1186, 1992.



(a)



(b)

Figure 2. First (a), and (b) second Stern-Gerlach setup for the surrealistic trajectories experiment; see text for details. The dotted line is the symmetry axis, the solid one is a typical Bohmian trajectory.

spot the upper colored particle moves, while when the screen flashes at the upper spot, then the lower colored particle gets the kick.

Until now, nothing surprising. But let's have a closer look at the Bohmian trajectories. In particular, it is interesting to look at the case in which the two colored particles have a very small charge. In this case the kick that the ion gives to the colored particle is very small, nevertheless a long time after the ion reached the screen, the position of the colored particle will have changed by a detectable quantity, no matter how gentle the kick was. On the contrary, the recoil of the kick on the ion can be neglected, and its trajectory remains almost indistinguishable from that of an ion alone, without the colored charges. These trajectories can be calculated, and are such that they always stay either in the upper half-space, or in the lower one, never crossing the symmetry plane. This means that the trajectories that end in the lower region on the screen were initially guided by the Blue wave packet, and later by the Red one. Therefore, if the Red particle moves, the Bohmian trajectory of the ion was in the Blue support! This at first seems rather surprising, and is the reason why the authors of the respective paper chose to call them *surrealistic*.

Apart from the surprise, can we learn something from this example? Yes, something very important. Our classical intuition suggests us to think of a Bohmian particle in a way similar to how we think of a classical particle, so for example we expect the ion to bring its charge along with its Bohmian position, and the interaction between the ion and the colored particles to depend on the Bohmian position of the ion. But this is completely wrong! The only property of the Bohmian particle is its position, every other property resides in the wave function. Therefore, every interaction happens at the wave function level, and is independent of the actual Bohmian position. That's what the theory tells us.

Bohmian Mechanics is very different from Classical Mechanics!

15. Niels Bohr always insisted upon the statement that a measurement does not reveal any preexisting property of the system, but rather produces it. Does Bohmian mechanics disprove this statement?

Bohr's statement actually applies to BM, except for the Bohmian position that is the only preexisting property. In BM the result of every measurement is predetermined by the actual positions of the particles. In fact there are several so called no go theorems in QM like the one of Kochen and Specker, showing that for a given quantum system there does not exist an assignment of a unique value to each observable. Thus speaking about the observables as properties of the system can be called naive realism about operators. Bohr was already aware of this. What is actually produced by the measurement is, for instance, the position of a pointer. These no go theorems can in just the same way be derived in BM.

16. **In BM the electron in the Hydrogen atom, when in its ground state, stands still. Doesn't it fall into the nucleus?**

    This question arises when we apply the intuition from our daily life to atoms. However, the world perceived in our daily life is governed by Newtonian Mechanics. A priori it is of course possible that the Newtonian laws also describe atoms, but it turns out that atoms follow different (Bohmian) laws. Taking these laws from Bohmian Mechanics seriously we arrive at the prediction that the ground state electron in Hydrogen stands still and does not fall into the nucleus. This just shows that BM is not a classical theory.

17. **One of the main devices for measuring particle trajectories are cloud chambers and the like. Do we see the Bohmian trajectory of a single particle there?**

    Cloud chambers are usually used to detect the trajectory of a scattered particle. In this case the particle really passes where you see the trajectory. But you should note that the track the particle leaves in the cloud chamber is much bigger than the diameter of an atom, so big you can see it with the naked eye, therefore the accuracy is not very good. Moreover, one could ask if the tracks one sees in the cloud chamber is close to the trajectory the particle would follow if the cloud chamber was not there. The biggest influence of the cloud chamber on the particles is to disturb interference phenomena. Thus, in situations where interference plays no essential role the effect of the cloud chamber on the motion of the particles is negligible. For a scattered particle far from the scattering target this condition is satisfied and you really observe the Bohmian trajectory with just a negligible disturbance.

    On the contrary, in a situation where interference matters, the observed tracks will differ from the Bohmian trajectories one would have without the cloud chamber. So for example performing a double slit experiment in a cloud chamber you would completely destroy the interference pattern.

18. **Is BM not much more complicated than the easy formalism and the easy to apply rules of QM?**

    If you are accustomed to standard QM it might seem so. But taking a step back and reexamining we can realize that we basically have a quite minimal theory in BM. There is the wave function with the Schrödinger equation and there are the particle positions guided by the wave function. From these simple ingredients all postulates of orthodox QM follow as theorems, there is no place for extra axioms. The $|\psi|^2$−distribution, the collapse of the wave function just as the collapsed wave function itself, the description of the measurement process, the description of the statistics of experiments by self adjoint operators as observables, etc. I would say therefore BM is pretty minimal and most importantly very easy to understand. It is a precise mathematical theory that does in its very formulation not rely on vague concepts such as measurement, observer, the distinction between the classical and the quantum world and so on. I believe that this is actually much closer to the ideal of a physical theory (such as Electrodynamics, SR, GR, etc.) than any formulation of QM.

19. **How does BM account for spin and what's its role?**

BM accounts for spin in the same way standard QM does: it uses spinor-valued wave functions. The guiding equation is modified straight-forwardly to take into account the spinor-valuedness. Thus, in BM, spin is a property of the wave function, not a property of the particle itself. Recall here that the particles are described by their respective positions and the wave function guides the particles.

20. **In Bohmian Mechanics one talks about the wave function of the universe. Why so? This seems like an inaccessible object.**

First of all, it is very natural to ask about the wave function of the universe. If one takes quantum mechanics seriously, then not just atoms and small systems but also the whole universe has a wave function. The question of how this wave function looks like is a very interesting one. It brings together quantum mechanics and cosmology and also has to do with the nature of time. Many physicists have thought about what this wave function could look like, even though we have very little access to it. An example for an equation for the wave function of the universe is the Wheeler-deWitt equation.

In Bohmian Mechanics there is another reason why one talks about the wave function of the universe. Namely, in order to justify that you can apply Bohmian Mechanics to small subsystems without taking into account what's going on in the rest of the universe you need to use an analysis in which the wave function of the universe enters. That is because a priori Bohmian Mechanics is a theory about the whole universe, i.e., all particles are guided by the wave function of the universe. The question now is: How and why can you apply this theory to small subsystems?

I should elaborate a little more on this, since it is a very important point. Bohmian Mechanics is in principle a *fundamental* theory, i.e., one whose basic laws apply to the whole universe. Of course it is not the final theory of our universe but you could imagine a world that behaves according to that theory. All fundamental theories like Newtonian Mechanics, Electromagnetism and General Relativity are theories of the whole universe. In these theories you can *derive* the behavior of subsystems. You cannot derive the behavior of small subsystems without talking about a bigger system that contains many of these small subsystems, i.e., the universe.

Now in Newtonian Mechanics it's mostly easy to talk about small subsystems like the earth, a laboratory or a stone I'm throwing. It's easy for several reasons: for instance the gravitational force decreases with one over the distance squared, i.e., it gets very weak on large distances. Also the galaxies around the earth are distributed very homogeneously, so the net gravitational force on the earth mostly cancels out. That's why we can describe physics here on earth without having to take into account, say, the gravitational force from the Andromeda galaxy. In fact this works so well that one might forget that in principle Newtonian Mechanics is indeed a theory about the whole universe.

In Quantum Mechanics the issue becomes more subtle. That's because you have entanglement. Entanglement is nothing that becomes in any way weaker at large distances. It is independent of the spatial distance, it's a property of the wave function. If you now start with the wave function of the universe, it is not immediately clear how to *separate* it into different parts that describe different small subsystems. This is of course also a problem in standard quantum mechanics. In Bohmian Mechanics there is a simple solution: you can easily define something called a conditional wave function. More or less you simply use the fact that all particles outside of your subsystem have some definite position. In this way you have a clear notion and justification of a wave function of a subsystem. For special physical situations, for example measurements, this wave function becomes an effective wave function, i.e., one that can be handled without explicit reference to the rest of the universe. These are the usual wave functions one uses in applications.

So to summarize, a fundamental theory of nature is always about the whole universe and you have to use reasoning in order to apply such a theory to smaller subsystems. In principle it could be that in your theory it is not possible to describe the behavior of subsystems independent of the rest of the universe. This would be a very complicated theory then, one that is hard to analyze and verify. For other theories, namely most theories we know so far, it is sometimes easy and sometimes hard to justify why we can apply them so successfully to subsystems, disregarding the rest of the universe. But this justifications gives you more insight into what's going on. Bohmian Mechanics is a good example for that.

21. **What's the role of randomness in BM? QM is a probabilistic theory while BM is deterministic, how does randomness come about in BM ?**

    The role of randomness in BM is essentially the same as the role of randomness in classical statistical mechanics as understood by Boltzmann. The microscopic dynamics is a deterministic one. Limited access to microscopic initial conditions requires a coarse-grained description of physical systems. The coarse-grained description must emerge naturally from the underlying microscopic dynamics and has to tell us what typically happens in the physical world, that means, what happens for the overwhelming majority of initial conditions in a given situation. Thus, we need some measure of typicality on the space of microscopic description (the space of initial conditions), in order to understand which sets of microstates are large and which are not. The proper measure of typicality must be singled out by the underlying dynamics. It must be guaranteed that typicality is preserved under the dynamics, what is typical today must be typical tomorrow. In classical statistical mechanics, this is realized by a stationary measure on phase-space, which is independent of time, like the microcanonical Lebesgue-measure or the canonical measure for subsystems. In BM, due to the general time dependence of the wave function, this is realized by an equivariant measure on configuration space $\rho = |\psi|^2$. This is the measure naturally given by the Bohmian dynamics, such that if we evolve it along the Bohmian trajectories, then at later times the relation $\rho(t) = |\psi(t)|^2$ still holds.This measure can be read off from the continuity equation of quantum mechanics: Since the current therein is the Bohmian current, the corresponding density, namely $\rho = |\psi|^2$, is preserved under the Bohmian flow and thereby the corresponding measure $\rho\, d^{3N}x$ on configuration space. Taking this as a measure of typicality, i.e., typical configurations are $|\psi|^2$ distributed (the so called quantum equilibrium hypothesis), what is typical today will be typical tomorrow.

22. **Is there any way to extract experimentally testable statements from BM that differ from those of standard QM, or are there new results of BM one could test?**

    BM is a theory about Nature, and therefore contains also the description of the outcomes of experiments. If we restrict BM to these phenomena, we can get an effective theory that does not anymore speak about Nature, but just about the results of experiments. The theory that we get in this way is exactly QM. Therefore, QM and BM are completely equivalent at the level of the experimental predictions. You should not think to them as alternative theories: BM explains the empirical content of QM on a deeper level, as for example Statistical Mechanics does with Thermodynamics.

Having an underlying picture has consequences on the experimental level, but just in the sense that we are provided with new ways of thinking, that can be useful in the analysis of complicated situations. An example of this are time measurements: it is very difficult to say *when* a particle will arrive at a detector using QM, while it is straightforward if we use BM, it will arrive simply when its trajectory crosses the detector! Nevertheless, the final predictions of BM and QM for the outcomes of any experiment always agree.

One more example of how one can benefit from the additional thinking tools that Bohmian Mechanics provides is the possibility of using the Bohmian trajectory to build accurate and efficient numerical approximations of complicated processes, as found for example in Chemistry or in Solid State Physics.

We can of course still imagine the possibility that one day somebody will come up with an idea that transcends our present notion of experiment and that allows to derive new predictions from BM, but we should not be disappointed if this never happens.

23. Isn't the choice between BM and QM a philosophical problem, as both theories make the same empirical prediction?

To some extend the question what you believe to be true might be called a philosophical one. But that is not the way we do physics and not the point at issue here. In physics a theory is considered superior to another if it is able to account for the same or a wider array of phenomena with less assumptions, axioms and problems. Clearly BM does that job as opposed to orthodox QM. Moreover it is crucially important for the further development of a theory and for the unification with other, incompatible theories to fully understand the structure of a physical theory, and most of all to know what it is about. This is one of the main merits of BM. It is a mathematical theory that does not refer to blurry philosophical and vague concepts and that is based on a very clear foundation: It's a theory about the motion of particles. There is not much philosophy in here.

24. What exactly are the "empty" branches of the wave function in Bohmian Mechanics?

In Bohmian Mechanics the “empty” branches of the wave function are certain “parts” of the wave function that do not influence the dynamics of the particles. Take the example of a Schrödinger cat like experiment. You know that if the wave function of the atom is in a superposition of decayed and not decayed then the wave function of the cat will be in a superposition of live and dead. But in Bohmian Mechanics the particles that the cat is actually made of are either in the support of the “live” part of the wave function or the “dead” part of the wave function. So the wave packet where the particles are not in, you could call “empty”, since it doesn’t play any role anymore for the dynamics of the particles, at least for a very long time. That is so because both wave packets are so far separated in configuration space that it’s practically impossible for them to interfere again. This is usually called decoherence. Of course there are also instances where the “empty” branches can become important again, namely when after a while the wave packets come closer and start to interfer again. However, as a rule of thumb, for “big” systems this is prevented by decoherence.

The situation that parts of a field are “empty” is very common in physics. A good example is the gravitational Coulomb field in Newtonian Mechanics. At places where there is no matter its value doesn’t play any role for the dynamics of the actual matter. The Newtonian gravitational field in empty parts of space is so to say also “empty”. The same is true for the electric Coulomb field which doesn’t play any role where there is no charged matter.

25. Is there a relativistic version of BM ?

The reconciliation of Quantum Theory with Relativity and Gravity is the biggest challenge that physics faces nowadays. The main attempt to solve this problem is Quantum Field Theory, whose history is almost one century long, involved a huge number of people – among which some of the most brilliant minds ever – and produced very precise predictions and huge advancements of our knowledge. But even all this was not enough to get a clear theory where QM and Relativity peacefully live together.

QFT represents some progress in this direction, but it is far from being a complete solution of the problem. The same results can be obtained also in a tentative relativistic BM, what is still missing is a relativistic BM with the same high level of clarity of the non relativistic theory.

The complete reconciliation of Quantum Theory with Relativity and Gravity is the most urgent problem of contemporary Physics, not only of BM.

26. How does Bohmian Mechanics treat indistinguishable particles?

In Bohmian Mechanics each particle has a well defined trajectory. The way one usually writes down the equation of motion for the particles is by giving each particle a label, i.e., one numbers the particles from 1 to N. Indistinguishability of particles then means that the equation of motion doesn't change under exchange of particle labels. If you look at the Bohmian law for the motion of the particles this means that the wave function has to be either symmetric or antisymmetric under exchange of particle labels.

This is not the end of the story though. Let us take a closer look at the space on which the wave function is defined. If the Bohmian particles are really indistinguishable then the wave function should not depend on N different particle variables but rather on N points in three dimensional space. A set of N points in three dimensional space is free of any label. If you take this into account then you see that the configuration space of the wave function has a more complicated topological structure. If you analyze it you find that in three dimensions there can be only Bosons and Fermions. Translated into the picture where the wave function depends on N different variables this means that it can either be symmetric or antisymmetric. In two space dimensions you find that the topological structure is much richer and that there are not only Bosons and Fermions but also so called Anyons that correspond to other symmetry properties of the wave function. Those Anyons are important in explaining many two dimensional quantum phenomena. So you see that the symmetry properties of the wave function are really a topological effect. In Bohmian Mechanics this follows from straightforward reasoning: If there are really particles and if they are really indistinguishable then the correct configuration space for the wave function is that of N unlabeled points in normal space.

27. Is BM a fully developed theory or are crucial things still to be shown? Could it happen that it would be found inconsistent at some later time? What needs to be done in the future?

BM is today a fully mature theory, well developed in all its parts, from its fundamental equations to the statistical analysis of the experimental results, and in all of these parts it maintains a very high rigor and an astonishing simplicity. It is internally consistent, and in accordance with the results of all quantum experiments so far performed. This does not mean that BM is the ultimate theory of Nature and that nothing can be added to it! On the contrary, the hardest work is still to be done, the work that will probably need a huge leap in our understanding of Nature: the formulation of a consistent Relativistic Quantum Theory. This is needed in BM, as in any other quantum theory, but this deserves a discussion on its own.