What is and to which end does one study Bohmian Mechanics?

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- particles move

The LAW of motion

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- respects Galilean symmetry but is non-Newtonian. It is a mathematically consistent simplification of the Hamilton Jacobi idea of mechanics:

$$Q(t) = (\mathbf{Q}_1(t), \ldots, \mathbf{Q}_N(t)), \quad \nabla = \frac{\partial}{\partial q} \quad \text{configuration}$$

obeys (time reversal invariance in "first order" theory achieved by complex conjugation)

$$\frac{\mathrm{dQ}}{\mathrm{dt}} = v^{\Psi}(Q(t), t) = \alpha \mathrm{Im} \frac{\Psi^* \nabla \Psi}{\Psi^* \Psi}(Q(t), t) \quad \text{guiding equation}$$

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- the "universal" wave function

$$\Psi : \mathbb{R}^{3N} \times \mathbb{R} \mapsto \mathbb{C}^{(n)} \quad (q = (\mathbf{q}_1, \ldots, \mathbf{q}_N), t) \mapsto \Psi(q, t)$$

ψ

Ψ

- solves the Schrödinger equation

$$\mathrm{i}\frac{\partial \Psi}{\partial t}(q,t) = H\Psi(q,t) \quad \text{“Schrödinger“ equation}$$

$H = -\sum_{k=1}^{n} \frac{\alpha}{2}\Delta_k + W$ (Galilean invariant operator)

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- $v = m^{-1}\nabla S$ for a Newtonian particle with mass m
- $v^{\Psi} = \frac{\alpha}{\hbar}\nabla S \Longrightarrow$ identify $\alpha = \frac{\hbar}{m}$ and $\frac{W}{\hbar} =: V$ as the "Newtonian potential"
- Newtonian Bohmian motion for "Quantum Potential" $\frac{\hbar^2}{2m}\frac{\Delta R}{R} \approx 0$

Bohmian mechanics with Newtonian identification of parameters[1]

$$\frac{\mathrm{dQ}}{\mathrm{dt}} = v^{\Psi}(Q(t), t) = \hbar m^{-1} \mathrm{Im} \frac{\Psi^* \nabla \Psi}{\Psi^* \Psi}(Q(t), t)$$

where m is a diagonal matrix with mass entries m_k

$$\mathrm{i}\hbar \frac{\partial \Psi}{\partial t}(q, t) = \left(- \sum_{k=1}^{n} \frac{\hbar^2}{2m_k} \Delta_k + V(q) \right) \Psi(q, t)$$

[1] Analogy: Boltzmann's constant k_B relates thermodynamics to Newtonian mechanics, $\hbar$ relates Newtonian mechanics to Bohmian Mechanics

simplest way to Bohmian mechanics

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- $v^\Psi = \frac{j^\Psi}{|\Psi|^2}$ (Pauli 1927, J.S. Bell 1964)

"Bohmian Mechanics agrees with Quantum Predictions"

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- quantum flux equation means $\rho(t) = |\Psi(t)|^2$ is equivariant: Assume Q is distributed according to $\rho = |\Psi|^2$ then $Q(t)$ at any other time is distributed according to $\rho(t) = |\Psi(t)|^2$

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- particle creation and annihilation contradicts the existence of particles
- solution: standard birth and death process

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for entangled wave function influenced by all particles at $t \Longrightarrow$ manifestly not local, against the spirit of relativity

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- the derivation of Bell's inequalities and the experimental results establish that nature is nonlocal (Jean Bricmont's talk)
- Ψ is that nonlocal agent, which produces nonlocal correlations
- BM is just what the doctor ordered.

S. Goldstein, N. Zanghì and I started this analysis 25 years ago here at IHES

universal Ψ and subsystem's φ

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- Boltzmann's statistical analysis of BM ($\rho = |\varphi|^2$) based on typicality measure $\mathrm{d}\mathbb{P}^\Psi = |\Psi|^2 \mathrm{d}q^{3N}$ which is equivariant (cf. quantum flux equation)
  Bohmian flow $T_t^\Psi : Q \mapsto Q(t)$ commutes with Schrödinger evolution $\Psi \mapsto \Psi_t$:
  $$\mathrm{d}\mathbb{P}^\Psi \circ (T_t^\Psi)^{-1} = \mathrm{d}\mathbb{P}^{\Psi_t}$$

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- φ is wave function of subsystem

conditional wave function φ of subsystem

$$X = (\mathbf{X}_1, \ldots, \mathbf{X}_n) \quad \text{system's particles}$$

$$Q = (X, Y) \quad \text{splitting in system and rest of universe}$$

$$\Downarrow$$

$$\varphi^Y(x) := \Psi(x, Y)/\|\Psi(Y)\|$$

normalized *conditional* wave function of subsystem guides X

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crucial "conditional measure" formula

$$\mathbb{P}^{\Psi}(X \in \mathrm{d}x | \varphi^Y = \varphi) = |\varphi(x)|^2 \mathrm{d}x$$

Autonomous subsystem: effective wave function

If wave function of universe $\Psi(x, y) = \varphi(x)\Phi(y) + \Psi(x, y)^{\perp}$

where

$\mathrm{supp}\Phi \cap \mathrm{supp}\Psi^{\perp} = \emptyset$ macroscopically disjoint

and if $Y \in \mathrm{supp}\Phi$ e.g. preparation of φ

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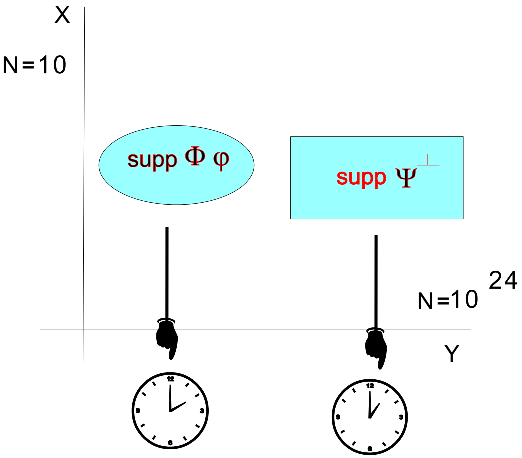
decoherence sustains disjointness of supports

$$\Downarrow$$

Schrödinger equation for φ for some time

$$\mathrm{i}\hbar\frac{\partial\varphi}{\partial t}(x,t) = -\sum_{k=1}^{n}\frac{\hbar^2}{2m_k}\Delta_k\varphi(x,t) + V(x)\varphi(x,t)$$

macroscopically disjoint Y- supports



Bohmian Subsystem

$$(X, \varphi) \quad \text{physical variables}$$

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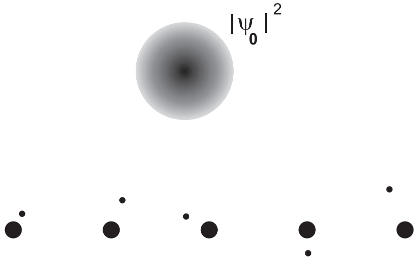
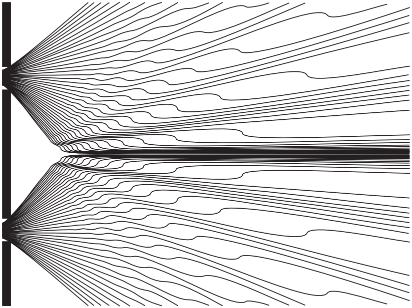
- In short: Quantum Equilibrium holds!

Hydrogene ground state: $\rho = |\psi_0|^2, \quad v^{\psi_0} = 0$

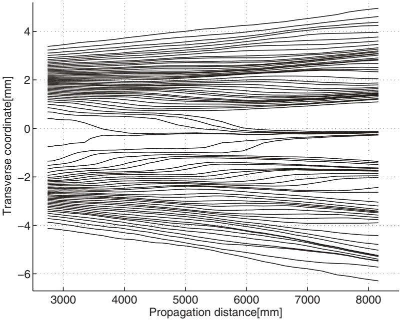


two slit experiment, computed trajectories



computer simulation of Bohmian trajectories by Chris Dewdney

two slit experiment: weak measurement of phase, trajectories reconstructed



S.Kocsis et al: Observing the Average Trajectories of Single Photons in a Two-Slit Interferometer. Science 2011

operational analysis of BM: PVM's

system (X, φ) and apparatus (Y, Φ) with pointer positions Y_α pointing towards value α. Suppose

$$\varphi_\alpha \Phi \xrightarrow{\text{Schrödinger evolution}} \varphi_\alpha \Phi_\alpha$$

then for $\varphi = \sum_\alpha c_\alpha \varphi_\alpha, \quad \sum_\alpha |c_\alpha|^2 = 1$

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- PVM $\Rightarrow$ self adjoint $\hat{A} = \sum \alpha |\varphi_\alpha\rangle\langle\varphi_\alpha|$ encodes all relevant data for the experiment

operational analysis: POVMs

Suppose not $\varphi_\alpha \Phi \overset{\text{Schrödinger evolution}}{\longrightarrow} \varphi_\alpha \Phi_\alpha$

$$\text{but apparatus } (Y, \psi) \text{ with values} \quad F(Y) = \lambda \in \Lambda$$

then probability for pointer position if system's wave function is φ

$$\mathrm{Prob}^{\varphi}(A) := \mathbb{P}^{\Phi_T}(F^{-1}(A))\,, A \subset \Lambda$$

can be written as

$$= \langle \varphi | \int_A d\lambda |\phi_\lambda\rangle\langle\phi_\lambda| |\varphi\rangle$$

where in general $\langle\phi_\lambda|\phi_\nu\rangle \neq \delta_{\lambda,\nu}$ (overcomplete set)

$$\int_A d\lambda |\phi_\lambda\rangle\langle\phi_\lambda|, \quad A \subset \Lambda$$

is called POVM or generalised observable

Heisenberg's uncertainty relation follows from BM

Equivariance of $\rho = |\varphi|^2$

$$\frac{\partial |\varphi(x,t)|^2}{\partial t} = -\mathrm{div} v^{\varphi}(x,t)|\varphi(x,t)|^2 \Longrightarrow$$

$$\mathbb{E}^{\varphi}(f(X(t)) = \mathbb{E}^{\varphi(t)}(f(X))$$

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$$\Downarrow \text{ by analysis}$$

$$\frac{m}{\hbar}V_\infty :\overset{\mathcal{L}}{=} \lim_{t\to\infty}\frac{m}{\hbar}\frac{X(t)}{t} \quad \text{is distributed according to } |\hat{\varphi}|^2$$

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$$\Downarrow$$

$$\hat{P} = \int dk k |k\rangle\langle k| \quad \text{momentum observable}$$

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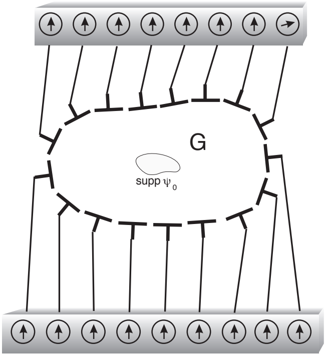
empirical import: $(X(t), \varphi)$ for interesting φ

- classical limit Bohmian trajectories approximately Newtonian
- measurement of φ $|\varphi|^2$ through measuring X, phase by weak measurement
- statistics of (arrival) time for good wave functions good statistics

arrival time statistics



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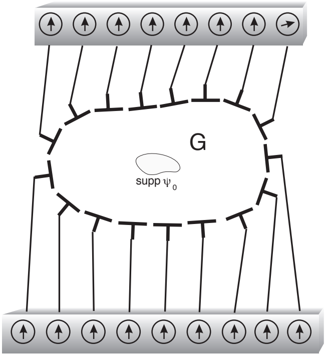
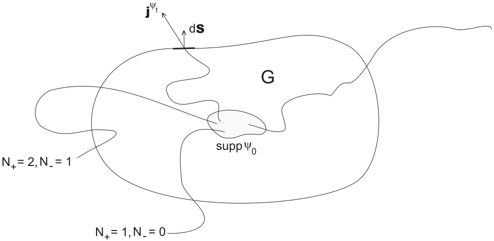


when and where does a counter click?

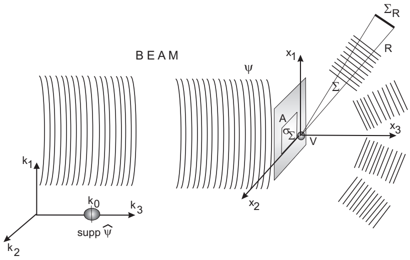
time statistics for Bohmian flow



$$\mathbb{P}^{\psi}(X(\tau) \in dS, \tau \in dt) = v^{\psi}|\psi|^2 \cdot dSdt = j^{\psi} \cdot dSdt$$

scattering formalism and scattering cross section

Born's scattering formula for single particle



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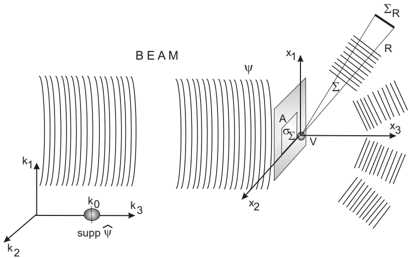
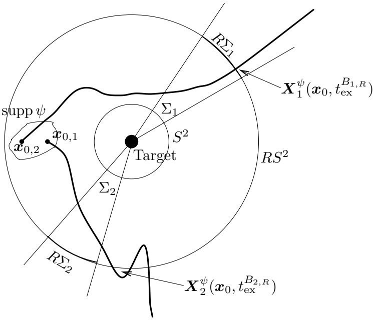


$$\mathbb{P}^{\psi}(X(\tau) \in \Sigma_R, \tau \in [0,\infty)) \overset{\text{R large}}{\approx} \int_{C_\Sigma} dk \langle k | S\psi_{\text{in}} \rangle^2$$

many particle scattering



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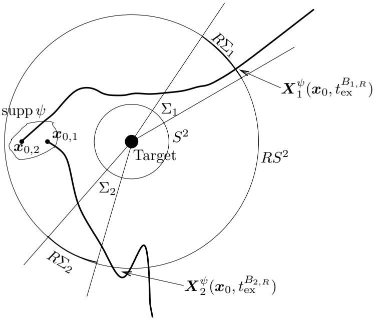

"genuine" Bohmian analysis

Gretchen Frage: Wie hältst du es mit der Relativität?

Relativistic Bohmian Theory

Weinberg's challenge

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Relativistic Bohmian Theory

Weinberg's challenge

It does not seem possible to extend Bohm's version of quantum mechanics to theories in which particles can be created and destroyed, which includes all known relativistic quantum theories. (Steven Weinberg to Shelly Goldstein, 1996)

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- technically, i.e. mathematically not possible?
- not possible in a deterministic theory of particles in motion?

Creation and Annihilation, the configuration space

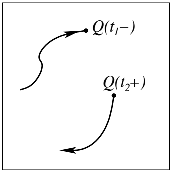
$\mathcal{Q}$: configuration space $\quad \mathcal{Q} = \bigcup_{n=0}^{\infty} \mathcal{Q}^{(n)}$ (disjoint union)

a) $\mathcal{Q}^{(0)}$ no particle $\quad$ b) $\mathcal{Q}^{(1)}$ one particle

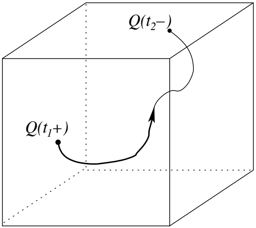
c) $\mathcal{Q}^{(2)}$ two particles $\quad$ d) $\mathcal{Q}^{(3)}$ three particles

(a)

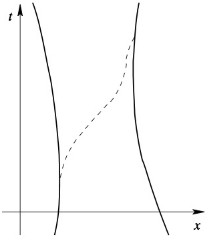
(b)



(c)



(d)



The LAW: equivariant Markov Process

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- $P(dq)$: positive-operator-valued measure (POVM) on $\mathcal{Q}$ acting on $\mathcal{F}$ so that the probability that the systems particles in the state Ψ are in dq at time t is

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- Find "minimal" generator so that (rewrite left hand side, so that)

$$\frac{d\mathbb{P}_t(dq)}{dt} = \mathcal{L}_t \mathbb{P}_t(dq) .$$

(Minimal) Markovian Process: Flow, (No) Diffusion, (Only as much as necessary) Jumps

Quantum field Hamiltonians provide rates for configuration jumps

Generator for pure Jump-Process

$$(\mathcal{L}\rho)(dq) = \int\limits_{q' \in \mathcal{Q}} \Big(\sigma(dq|q')\rho(dq') - \sigma(dq'|q)\rho(dq)\Big)$$

$$H = H_0 + H_I$$

$$L = L_0 + L_I$$

H_I is often an Integral-Operator $\longrightarrow$ Jump-Generator given by rates

$$\sigma(dq|q') = \frac{[(2/\hbar)\,\mathrm{Im}\,\langle\Psi|P(dq)H_I P(dq')|\Psi\rangle]^+}{\langle\Psi|P(dq')|\Psi\rangle}\,.$$

The tension with relativity challenge: Einstein's criticism of QM

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Possible relief: The foliation $\mathscr{F}^{\Psi}$ is given by the wave function, e.g. defined by a time like vector field induced by the wave function. Covariance is expressed by the commutative diagram

$$\begin{array}{ccc} \Psi & \longrightarrow & \mathscr{F}^{\Psi} \\ {\scriptstyle U_g}\big\downarrow & & \big\downarrow{\scriptstyle \Lambda_g} \\ \Psi' & \longrightarrow & \mathscr{F}^{\Psi'}. \end{array} \tag{1}$$

Here the natural action Λ_g on the foliation is the action of Lorentzian g on any leaf Σ of the foliation $\mathscr{F}^{\Psi}$.

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  - a guiding example is Gauss-Weber-Tetrode-Fokker-Schwarzschild-Wheeler-Feynman direct interaction theory. Fully relativistic and without fields (my friends Shelly and Nino are not enthusiastic about that theory, my young friends are and future is theirs)

the end: perhaps more on the solutions of second class difficulties in 25 years