A moduli problem for the non-split Cartan modular curve

Christian Wuthrich, Nottingham

Modular curves like $X_0(N)$ have a nice moduli interpretation; they classify elliptic curves together with extra structure in the N-torsion part. For instance, $X_0(N)$ classifies cyclic subgroup of order N. Among the important modular curves, important to Serre's question for a uniform bound on the surjectivity of the Galois representation of an elliptic curve over $\mathbb{Q}$ for example, among these curves there is one $X_{nonsplit}(N)$ that did not yet admit a simple moduli interpretation. In joint work with M. Rebolledo, we found that this curve parametrises necklacesön the cyclic subgroups of order N. It leads us to a simplifed proof of Chen's isogeny linking the various modular curves.