The Generalized Fermat Equation $x^2 + y^3 = z^{11}$

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Generalizing Fermat's original problem, equations of the form $x^p + y^q = z^r$, to be solved in coprime integers, have been quite intensively studied. It is conjectured that there are only finitely many solutions in total for all triples (p, q, r) such that $1/p + 1/q + 1/r < 1$ (the 'hyperbolic case'). The case $(p, q) = (2, 3)$ is of special interest, since several solutions are known. To solve it completely in the hyperbolic case, one can restrict to $r = 8, 9, 10, 15, 25$ or a prime ≥ 7. The cases $r = 7, 8, 9, 10, 15$ have been dealt with by various authors. In joint work with Nuno Freitas and Bartosz Naskrecki, we are now able to solve the case $r = 11$ and prove that the only solutions (up to signs) are $(x, y, z) = (1, 0, 1), (0, 1, 1), (1, -1, 0), (3, -2, 1)$. We use Frey curves to reduce the problem to the determination of the sets of rational points satisfying certain conditions on certain twists of the modular curve $X(11)$. A study of local properties of mod-11 Galois representations cuts down the number of twists to be considered. The main new ingredient is the use of the 'Selmer group Chabauty' techniques developed recently by the speaker to finish the determination of the relevant rational points.