From scaffolds for local Galois module structure to Hopf orders in group rings

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The structure of the ring of integers over its associated order in a Galois extension of local fields is an old, difficult (even intractable?) question. But given a Galois scaffold, this question in totally ramified p-extensions of local fields is no more difficult a problem than when the extension has degree p. To understand the definition of a Galois scaffold we, interestingly enough, need to leave the setting of Galois extensions, even separable extensions. For it turns out that scaffolds arise more naturally (most naturally?) in the setting of purely inseparable extensions with the divided power Hopf algebra taking the role classically played by the group algebra. I will describe the status of scaffolds, based upon joint work with Nigel Byott and Lindsay Childs. I will then use scaffolds to produce Hopf orders in group algebras. Recall that Hopf orders have provided another approach to simplify Galois module theory, namely by "taming wild ramification". I will close with a family of Hopf orders in the elementary abelian group algebra KC_p^n that conjecturally comprises the complete classification.