12. und letztes Übungsblatt Elliptische Kurven

28. Jan. 22

Aufgabe 1)

8.5. Let $\xi \in H^1(G_{\bar{K}/K}, M)$ be unramified at v. Prove that the cohomology class of ξ contains a 1-cocycle $c : G_{\bar{K}/K} \to M$ satisfying $c_\sigma = 0$ for all $\sigma \in I_v$. (*Hint.* Use the inflation–restriction sequence (B.2.4) for $I_v \subset G_{\bar{K}/K}$.)

Erläuterung: ξ ist unverzweigt, falls ξ im Kern der Restriktionsabbildung

$$\mathrm{res} : H^1(G_K, M) \to H^1(I_v, M)$$

ist.

Aufgabe 2)

8.6. Prove *Kronecker's theorem*: Let $x \in \bar{\mathbb{Q}}^*$. Then $H(x) = 1$ if and only if x is a root of unity. (This is the multiplicative group version of (VIII.9.3d).)

Aufgabe 3)

8.10. Let F be the rational map

$$F : \mathbb{P}^2 \longrightarrow \mathbb{P}^2, \qquad [x, y, z] \longmapsto [x^2, xy, z^2],$$

from (I.3.6). Note that F is a morphism at every point except at $[0, 1, 0]$, where it is not defined. Prove that there are infinitely many points $P \in \mathbb{P}^2(\mathbb{Q})$ such that

$$H\big(F(P)\big) = H(P).$$

In particular, (VIII.5.6) is false if the map F is merely required to be a rational map.