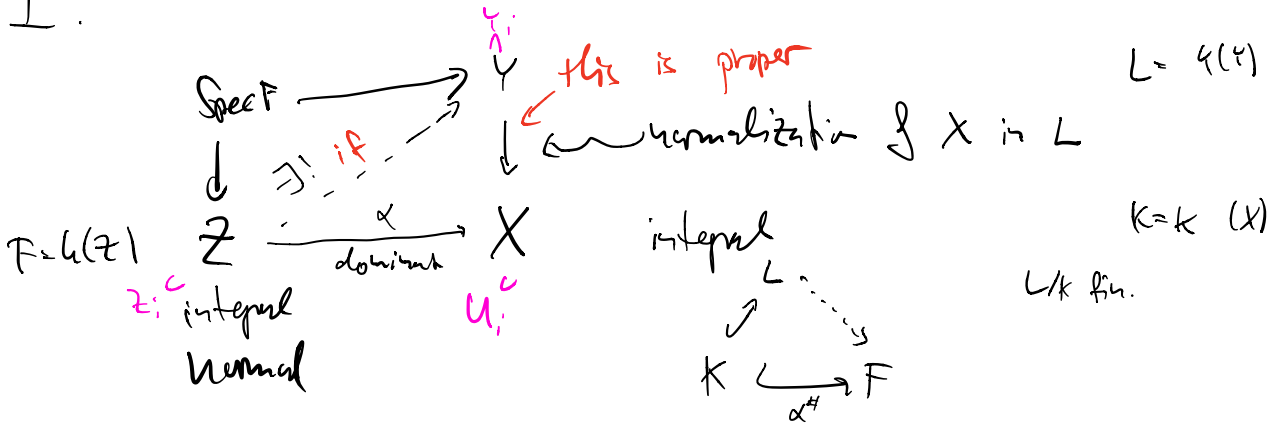


1. all p.c. k

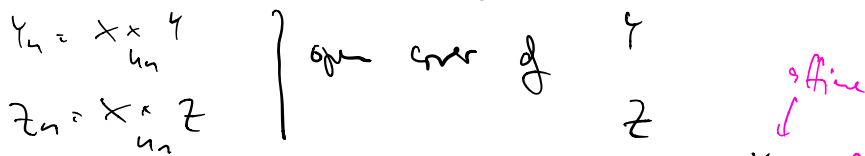


Suppose first $Z = \text{Spec } C$, C dom.

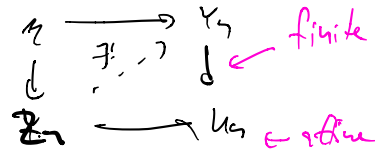
Know that $Z \rightarrow X$ is finite hence proper.
 $\therefore \exists!$ lift in this case.

We may assume that $Z = \text{Spec } C$ is affine, s.t. glue morphisms in open cover.

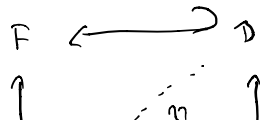
Suppose U_1, \dots, U_n is an open affine cover of X .



By prev. reasoning, sufficient to treat



Pick affine open cover of $Z \rightsquigarrow$ w.h.a $Z = \text{Spec } C$, $Y = \text{Spec } B$, $X = \text{Spec } A$.



$$\begin{array}{ccc}
 \cup & \xrightarrow{\exists!} & 1 \\
 C & \longleftrightarrow & A
 \end{array}$$

This exists (uniquely) $\iff B \subset C$ as subsets of A

$$\iff B \subset \bigcap_{\text{ht}(p)=1} C_p \quad \forall \text{ht}(p)=1$$

\iff the same problem works for $Z' = \text{Spec } C_p$ \leftarrow already dealt with. \square

\swarrow Many properties of mod. (f.e., proper, open imm. ...)

are stable under base change

But normalization is not. But under dense open

$$\begin{array}{ccc}
 \tilde{U} & \xrightarrow{\quad} & \tilde{X} \\
 \downarrow & \searrow & \downarrow \text{normalization} \\
 U & \xrightarrow{\quad} & X \\
 & \text{open imm.} &
 \end{array}$$

2. K field $A_i \subset K$ i.c. $\forall i \in I$

$$\text{Frac}(A_i) = K$$

- 1) Show $\bigcap_{i \in I} A_i \subset K$ is i.c. 2) Ex. where $\text{Frac}(\bigcap_i A_i) \neq K$?

1) $x \in K$ integral over A

$$\Leftrightarrow x^n + a_1 x^{n-1} + \dots + a_n = 0 \quad \text{for some } a_i \in A$$

$$\Rightarrow a_i \in A_i \quad \forall i$$

$$\Rightarrow x \text{ integral over } A_i \quad \forall i$$

$$\Rightarrow x \in A_i \quad \forall i$$

$$\Rightarrow x \in A.$$

2) $K = k(x) = k(A^2)$

$$k[x] \hookrightarrow k(x) \text{ is i.c.}$$

$$\cong \mathcal{O}_{\mathbb{P}^2}(A^2)$$

The $k[x'] \hookrightarrow k(x)$ is also i.c.

$$\text{But } k[x] \cap k[x'] = k \dots$$

Theory of complete un-
singular curves \leftrightarrow
function fields

C/k smooth complete curve

$$k = k(C)$$

$$c \in C \quad \mathcal{O}_{C,c} \subset K$$

closed div
cf. k

FACT: all divs of k ("constant" divs)
arise in this way.

$$\mathcal{O}_{\mathbb{P}^2}(A^2) = \mathcal{O}_{\mathbb{P}^2}(A^2, \omega) \cap \mathcal{O}_{\mathbb{P}^2}(A^2, 0)$$

$k[x]$ $k[x']$

3. $A \subset B$ divs

$$\text{Frac}(A) = \text{Frac}(B) =: K$$

$$b \in B \setminus A \Rightarrow b^{-1} \in A \Rightarrow b \in B^\times$$

Valuation of $A: K^\times \rightarrow \mathbb{Z}$
 $a \cdot t^n \mapsto n$

$$t \in B.$$

Show: $A = B.$

Choose $t \in B \setminus A$
s.t. $v_A(t) = 1$

Can write $x \in K^\times$ uniquely as
 $x = u \cdot t^n \quad u \in B$

Claim: $t \in B \setminus B^x$
 $= {}^u B$

olw $B \supset A \langle t, t^{-1} \rangle = K$
 ~~$\cdot \cdot$~~

$u \in A^*$

Let $x = u \cdot t^n \in K$, $u \in A^*$.

Suppose $x \in B$. Then $u \neq 0$: $t^n = x \cdot u^{-1} \in B$

So if $u < 0$ then $K = B \cdot x$.

$\therefore x = u \cdot t^n \in A$.