

Algebraic Geometry 1

Exercises 7

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Exercise 1. Given an example of an integral domain A with fraction field K and a morphism of ringed spaces $\mathrm{Spec} K \rightarrow \mathrm{Spec} A$ which is not *locally* ringed.

Exercise 2. Let X be a scheme and K a field. Describe a bijection

$$\mathrm{Hom}_{Sch}(\mathrm{Spec} K, X) \simeq \{(x, \alpha) \mid x \in X, \alpha : \kappa(x) \hookrightarrow K\}.$$

Exercise 3. Let X be a scheme and A a commutative ring. Show that the canonical map

$$\mathrm{Hom}_{Sch}(X, \mathrm{Spec} A) \rightarrow \mathrm{Hom}_{Ring}(A, \mathcal{O}_X(X))$$

is a bijection.

Exercise 4. Let X be a scheme and $A = \mathcal{O}_X(X)$. Show that X is affine if and only if the canonical morphism $X \rightarrow \mathrm{Spec} A$ is an isomorphism.