Algebraic Geometry 1

Exercises Tutorium 2

Dr. Tom Bachmann Winter Semester 2020–21

Throughout k is an algebraically closed field.

Exercise 1. Let X be a topological space, $U \subset X$ a subspace and $Y \subset U$ closed (in the induced topology on U). Let $\overline{Y}$ be the closure of Y in X. Show that $\overline{Y} \cap U = Y$.

Exercise 2. Let X be a topological space and $Y \subset X$ a subspace. Show that $\dim Y \leq \dim X$, where dim denotes the Krull dimension.

Hint: You may use Exercise 3(2) of this week's Zentralübung.

Exercise 3. Let X be a topological space.

(1) Show that X is Noetherian if and only if every subspace of X is compact.
(2) Let X be Noetherian. Show that subspaces of X are Noetherian.

Exercise 4. Let X be a Noetherian topological space. Show that X is a finite union of irreducible subspaces.