Algebraic Geometry 1

Exercises Tutorium 11

Dr. Tom Bachmann Winter Semester 2020–21

Exercise 1. Let S be a scheme. Show that $\mathbb{P}^1_S := S \times_{\operatorname{Spec}\mathbb{Z}} \mathbb{P}^1_{\mathbb{Z}}$ can be obtained by gluing two copies of $\mathbb{A}^1_S$.

Exercise 2. Let k be an algebraically closed field and $S = \operatorname{Spec} k(t)$. Describe $S \times_k S$.

Exercise 3. Let S be a scheme of finite type over a field. Show that the closed points of S are dense. Exhibit a scheme (necessarily not of finite type over a field) for which this fails.

Exercise 4. Let $T \to S$ be a finite type morphism of schemes. Show that if S is noetherian then so is T.

Exercise 5. Show that the morphism $\mathbb{A}^1_{\mathbb{Z}} \to \operatorname{Spec}\mathbb{Z}$ is not closed. What about $\mathbb{P}^1_{\mathbb{Z}} \to \operatorname{Spec}\mathbb{Z}$?