Algebraic Geometry 1

Exercises Tutorium 1

Dr. Tom Bachmann Winter Semester 2020–21

Throughout k is an algebraically closed field.

Exercise 1. For which n do the classical and Zariski topologies on $\mathbb{A}^n(\mathbb{C})$ coincide?

Extra question: Let $Z \subset \mathbb{A}^n(\mathbb{C})$ be an algebraic subset. When do the classical and Zariski topologies on Z coincide?

Exercise 2. Is the Zariski topology on $\mathbb{A}^{n+m}(k) \simeq \mathbb{A}^n(k) \times \mathbb{A}^m(k)$ the same as the product topology?

Exercise 3. Recall that a topological space X is T_1 if for any $x \neq y \in X$ there exists an open subset $U \subset X$ with $x \in U$ but $y \notin U$. Show that algebraic sets are T_1.

Extra question: Which algebraic sets are Hausdorff?

Exercise 4. (1) Show that a topological space X is Hausdorff if and only if $\Delta(X) \subset X \times X$ is a closed subset. (2) Show that the canonical map of algebraic sets $\Delta : \mathbb{A}^n(k) \to \mathbb{A}^{n+n}(k)$ identifies the source with a closed subset of the target.

Does this show that $\mathbb{A}^n(k)$ is Hausdorff?

Extra question: Show that the category of algebraic sets has binary products, and if Z is an algebraic set then the diagonal $\Delta : Z \to Z \times Z$ identifies the source with a closed subset of the target.