Prof. Dr. Werner Bley (LMU)
Prof. Dr. Cornelius Greither (UniBWM)
Prof. Dr. Gregor Kemper (TUM)
Prof. Dr. Christian Liedtke (TUM)
Prof. Dr. Andreas Rosenschon (LMU)

Sommersemester 2022

Arithmetische und Algebraische Geometrie

Dienstag 16-18, [P]=Präsenz: LMU Theresienstr. 39, B 134; TUM Math. Inst. 03.06.011;
[V]=Virtuell über zoom.

10.5.2022 Ihsen Yengui (Sfax, Tunisia); TUM [P].
Title: "Making the use of maximal ideals constructive "
Abstract: I will explain how to make the use of maximal ideals constructive by treating the two main aspects of utilization of maximal ideals, namely, localizing at a generic maximal ideal or quotienting by a generic maximal ideal. For the first aspect, as guiding example, I will give a dynamical method for computing the syzygy module of polynomials over Dedekind rings using the notion of dynamical Groebner bases. For the second aspect, as academic example, I will decipher constructively a lemma of Suslin which played a central role in his solution to Serre's conjecture.

24.5.2022 Martin Lorenz (Temple University); TUM [P].
Title: "Prime ideals and group actions in noncommutative algebra "
Abstract: Having originated from number theory, the notion of a prime ideal has become central in many different branches of algebra. This talk will focus on the role of prime ideals in the representation theory of noncommutative algebras and the use of group actions as an efficient tool in organizing the spectrum of all prime ideals of a given noncommutative algebra.

The emphasis will be on the case where an affine algebraic group G acts rationally by automorphisms on R. Then the prime spectrum of R is partitioned into "G-strata", each of which can be described in terms of the prime spectrum of a certain *commutative* algebra. Recent work on quantized coordinate algebras R of algebraic varieties has revealed that, in many cases, there is a suitable choice of the acting group G, usually an algebraic torus, such that there are only finitely many G-strata. The resulting finite set tends to carry an interesting combinatorial structure.

7.6.2022 Cinzia Casagrande (Turin); [V].
Title: "Fano manifolds with Lefschetz defect 3"
Abstract: We will talk about a structure result for some (smooth, complex) Fano varieties X, which depends on the Lefschetz defect $\delta(X)$, an invariant of X defined as follows. Consider a prime divisor D in X and the restriction $r : H^2(X, R) -> H^2(D, R)$. Then $\delta(X)$ is the maximal dimension of $\ker(r)$, where D varies among all prime divisors in X. If $\delta(X) > 3$, then X is isomorphic to a product $S \times T$, where S is a surface. When $\delta(X) = 3$, X does not need to be a product, but we will see that it still has a very explicit structure. More precisely, there exists a smooth Fano variety T with $\dim T = \dim X - 2$ such that X is obtained from T with two possible explicit constructions; in both cases there is a $\mathbb{P}^2$-bundle Z over T such that X is the blow-up of Z along three pairwise

disjoint smooth, irreducible, codimension 2 subvarieties. This structure theorem allows to complete the classification of Fano 4-folds with Lefschetz defect at least 3. This is a joint work with Eleonora Romano and Saverio Secci.

14.6. Markus Land (LMU); LMU [P]. Title: "Some advances on the K-theory of $\mathbb{Z}[C_p]$"
Abstract: I will report on some recent advances on the K-theory of $\mathbb{Z}[C_p]$. First, I will recall the what was known previously: I will explain the Rim square for $\mathbb{Z}[C_p]$ - a particular Milnor square - and what the consequences for the K-theory of $\mathbb{Z}[C_p]$ are. As a result, it is the p-primary torsion information of $K(\mathbb{Z}[C_p])$ which was previously inaccessible. I will then explain some recent advances in K-theory of Milnor squares which allows to also say something about the p-primary torsion of $K(\mathbb{Z}[C_p])$. Along the way, I will recall some necessary result from the K-theory of number rings and end with the remaining open problems.

21.6. Marco DAddezio (Paris); [V].
Title: "Boundedness of the p-primary torsion of the Brauer group of an abelian variety"
Abstract: I will present a new finiteness result for the p-primary torsion of the transcendental Brauer group of abelian varieties defined over finitely generated fields of positive characteristic p. During the talk I will start out by recalling what it is known on the prime-to-p torsion of the Brauer group and it's relation with the Tate conjecture for the tale cohomology. Then I will explain what fails and what remains true for the p-primary part. In particular, I will talk about an fppf variant of the Tate conjecture.

28.6. Domenico Valloni (Hannover); [V].
Title: "Neighbors and arithmetic of isogenous K3 surfaces"
Abstract: In this talk, I will show how a classical lattice theoretical construction due to Kneser can be applied to construct isogenies of K3 surfaces via isotropic Brauer classes. This construction turns out to be very natural and it allows one to study the isogeny class of the K3 surface via its Brauer group. I will apply the result to show how a conjecture of Shafarevich about the finiteness of the Nron-Severi lattices implies uniform bounds on the level structures of Brauer groups of K3 surfaces over number fields.

5.7. Timo Keller (Bayreuth); LMU [P]. Title: "Exact verification of the strong BSD conjecture for some absolutely simple modular abelian surfaces"
Abstract: The strong Birch–Swinnerton-Dyer conjecture and in particular the exact order of the Shafarevich–Tate group for abelian varieties over the rationals has only been known for elliptic curves (dimension 1) or in higher dimension where the conjecture could be reduced to dimension 1. We give the first absolutely simple examples of dimension 2 where the conjecture can be verified:

Let X be (1) a quotient of the modular curve $X_0(N)$ by a subgroup generated by Atkin–Lehner involutions such that its Jacobian J is an absolutely simple modular abelian surface, or, more generally, (2) an absolutely simple factor of $J_0(N)$ isomorphic to the Jacobian J of a genus-2 curve X. We prove that for all such J from (1), the Shafarevich–Tate group of J is trivial and satisfies the strong Birch–Swinnerton-Dyer conjecture. We further indicate how to verify strong BSD in the cases (2) in principle and in many cases in practice.

To achieve this, we compute the image and the cohomology of the mod-$\mathfrak{p}$ Galois representations of J, show effectively that almost all of them are irreducible and have maximal image, make the Heegner points Euler system of Kolyvagin–Logachev effective, compute the Heegner points and Heegner indices, compute the $\mathfrak{p}$-adic L-function, and perform $\mathfrak{p}$-descents. Since many ingredients are involved in the proof, we will give an overview of the methods involved and give more details regarding the computation of the Galois representations and Heegner indices.

This is joint work with Michael Stoll.

12.7. Dominik Bullach (King's College); LMU [P].
Titel: "Dirichlet L-series at s = 0 and the scarcity of Euler systems"
Abstract: In 1989 Coleman conjectured that Euler systems for Q are 'scarce' in a precise sense. I will present a natural generalisation of Coleman's Conjecture to arbitrary number fields, explain how this gives a more elementary approach to the equivariant Tamagawa Number Conjecture (eTNC) for Dirichlet L-series at s = 0, and report on progess towards these conjectures. This is joint work with David Burns, Alexandre Daoud, and Soogil Seo.

19.7. Claudia Stadlmayr (TUM); TUM [P].
Titel: "Del Pezzo surfaces with global vector fields"
Abstract: If X is a del Pezzo surface (or a weak del Pezzo, or an RDP del Pezzo), then its automorphism scheme Aut_X is a, possibly non-reduced, affine group scheme of finite type. In particular, X has infinitely many automorphisms if and only if Aut_X is positive-dimensional and then X admits global vector fields (since the space of global vector fields on X is the tangent space to the automorphism scheme). The last implication is an equivalence in characteristic 0, but its converse can fail in positive characteristic. Over the complex numbers, a del Pezzo surface with rational double point singularities admits global vector fields if and only if its minimal resolution, the corresponding weak del Pezzo surface, does. In small characteristics, one implication of this equivalence breaks down due to the existence of non-lifting vector fields on rational double points. I will explain how to overcome these obstacles in order to classify weak and RDP (if $p \neq 2$) del Pezzo surfaces with global vector fields. Further, I will show examples displaying interesting behaviour of such surfaces in small characteristics. This is joint work with G. Martin.

26.7. Jonathan Elmer (Middlesex University); TUM [P].
Title: "Separating invariants for n-tuples of 2x2 matrices"
Abstract: Separating invariants are a new (-ish) trend in invariant theory, and in some sense a return to its roots: a separating set is a subset of a ring of invariants with the same point-separation properties as the invariant ring. A separating set S is minimal if no proper subset of S is a separating set and minimum if it has the smallest cardinality among all separating sets.

The ring of invariants of n-tuples of 2x2 matrices under simultaneous conjugation is a classical object in invariant theory. A generating set for this ring was discovered in 1990 by LeBruyn and Procesi. Recently, Kaygodorov, Lopatin and Popov showed that this generating set is a minimal separating set. In this talk we will set out to discover whether or not it is a minimum separating set.

1.9. Katharina Müller (Laval University); LMU [P].
Title: Iwasawa Theory for $\mathbb{Z}_p^l$-covers of Graphs.
Abstract: We will give a short introduction to the Iwasawa Theory of $\mathbb{Z}_p$-covers of Graphs and then show how one can generalize work of Gonet to prove an asymptotic formula for the number of spanning trees along a $\mathbb{Z}_p^l$-cover. If time allows we will also discuss an Iwasawa main conjecture for graphs. This is joint work in progress with Sören Kleine.