When do the spectra of self-adjoint operators converge?

(Siegfried Beckus)

Given a self-adjoint bounded operator A, its spectrum $\sigma(A)$ is a compact subset of $\mathbb{R}$. The space $\mathcal{K}(\mathbb{R})$ of compact subsets of $\mathbb{R}$ is naturally equipped with the Hausdorff metric d_H induced by the Euclidean metric. Let T be a topological (metric) space and $(A_t)_{t\in T}$ be a family of self-adjoint, bounded operators. In the talk, we study the map $t \mapsto \sigma(A_t)$. More precisely, the (Hölder-)continuity of this map is characterized.
As application, we study Schrödinger operators over dynamical systems. Using the previous characterization, we show that the spectra converges if and only if the underlying dynamical systems converge in a suitable topology. This has a wide range of applications for numerical as well as analytic questions. A particular focus is put on Schrödinger operators arising by quasicrystals.