Mathematisches Oberseminar *PDG und Spektraltheorie* (SoSe 2016).

Date: 21.04.2016.
Time and place: 14:15 in B 134.

Speaker: Marco Baffetti (LMU).
Titel: *The proof of the Fundamental Gap Conjecture.*
Abstract:

In 2011 Ben Andrews and Julie Clutterbuck proved the following Fundamental Gap Conjecture, which had been suggested for the first time by M. van den Berg in 1983:

Theorem (Fundamental Gap Conjecture). *Let $\Omega \subset \mathbb{R}^n$ be a bounded convex domain with diameter D and $V : \overline{\Omega} \to \mathbb{R}$ a convex potential. Then the first two eigenvalues λ_0 and λ_1 of the Schrödinger operator $-\Delta + V$ on Ω with Dirichlet boundary condition satisfy:*

$$\lambda_1 - \lambda_0 \geq \frac{3\pi^2}{D^2}.$$

The crucial result proved by Andrews and Clutterbuck is a log-concavity estimate for the first eigenfunction, which implies a comparison theorem for the fundamental gap. The proof involves a study of moduli of concavity for positive solutions to a parabolic Schrödinger equation and of moduli of continuity for solutions to Neumann heat equations with drift. In order to simplify some of the arguments given by Andrews and Clutterbuck, Yue He in 2013 gave an alternative proof of the theorem which uses different methods, although it preserves the structure of the original proof. He employs elliptic and parabolic maximum principles, along with geometrical properties of convex domains. In my talk I will discuss the Fundamental Gap Conjecture and present an outline of the two proofs.

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