$\boxed{\text{Note}}$ On claim in Pf. of 2-9:

"If $\Psi \in D(H_N^{in})$ is a unit vector, we may write

$$\Psi = \psi_1 \otimes \Psi_1 + \dots + \psi_K \otimes \Psi_K$$

where $\psi_1, \dots, \psi_K \in D(h_1)$ and $\Psi_1, \dots, \Psi_K \in h_2 \otimes \dots \otimes h_N$ are orthonormal. Since Ψ is a unit vector we have $\|\psi_1\|^2 + \dots + \|\psi_K\|^2 = 1$."

$\boxed{\text{Proof:}}$ We denote $\mathcal{H} := h_1$, $\mathcal{K} := h_2 \otimes \dots \otimes h_N$

Let $\Psi \in \operatorname{span}\{\phi \otimes \psi \mid \phi \in \overset{D(h_1)}{\cancel{\mathcal{H}}}, \psi \in \mathcal{K}\}$,

i.e. $\Psi = \sum_{i=1}^{M} a_i \tilde{\phi}_i \otimes \tilde{\psi}_i$, $\tilde{\phi}_i \in \underset{D(h_1)}{\cancel{\mathcal{H}}}$, $\tilde{\psi}_i \in \mathcal{K}$

Let $\{\Psi_1, \dots, \Psi_K\} \subseteq \mathcal{K}$ be an ONB for the finite dim. subspace $\operatorname{span}\{\tilde{\psi}_1, \dots, \tilde{\psi}_M\} \subseteq \mathcal{K}$.

Then $\tilde{\psi}_i = \sum_{j=1}^{K} b_{ij} \Psi_j$ for certain $b_{ij} \in \mathbb{C}$.

Hence, by ~~multi~~ bilinearity,

$$\tilde{\phi}_i \otimes \tilde{\psi}_i = \tilde{\phi}_i \otimes \left(\sum_{j=1}^{K} b_{ij} \Psi_j\right) = \sum_{j=1}^{K} b_{ij} \tilde{\phi}_i \otimes \Psi_j,$$

and so
$$\Psi = \sum_{i=1}^{M} a_i \tilde{\phi}_i \otimes \tilde{\psi}_i = \sum_{i=1}^{M} \sum_{j=1}^{K} (a_i b_{ij}) \tilde{\phi}_i \otimes \Psi_j$$

$$= \sum_{j=1}^{K} \left(\sum_{i=1}^{M} a_i b_{ij} \tilde{\phi}_i\right) \otimes \Psi_j = \sum_{j=1}^{K} \psi_j \otimes \Psi_j$$

$\left(\psi_j := \sum_{i=1}^{M} a_i b_{ij} \tilde{\phi}_i \in D(h_1)\right)$, with $\{\Psi_1, \dots, \Psi_K\} \subseteq \mathcal{K}$

ONS. Furthermore,

$$
\begin{aligned}
1 = \| \Psi \|_{H \otimes K} &= (\Psi, \Psi)_{H \otimes K} \\
&= \sum_{j=1}^{k} \sum_{i=1}^{k} (\psi_j \otimes \Psi_j, \psi_i \otimes \Psi_i)_{H \otimes K} \\
&= \sum_{j=1}^{k} \sum_{i=1}^{k} (\psi_j, \psi_i)_H \cdot \underbrace{(\Psi_j, \Psi_i)_K}_{\delta_{ij}} \\
&= \sum_{j=1}^{k} \| \psi_j \|_H^2 . \qquad \blacksquare
\end{aligned}
$$