Mathematisches Institut
der Universität München
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SS 2002
Set 9

Problem set for Quantum Groups and Noncommutative Geometry

(33) (Linear Algebra) For $U \subseteq V$ define $U^\perp := \{f \in V^* | f(U) = 0\}$. For $Z \subseteq V^*$ define $Z^\perp := \{v \in V | Z(v) = 0\}$. Show that the following hold:
 (a) $U \subseteq V \Longrightarrow U = U^{\perp\perp}$;
 (b) $Z \subseteq V^*$ and $\dim Z < \infty \Longrightarrow Z = Z^{\perp\perp}$;
 (c) $\{U \subseteq V | \dim V/U < \infty\} \cong \{Z \subseteq V^* | \dim Z < \infty\}$ under the maps $U \mapsto U^\perp$ and $Z \mapsto Z^\perp$.

(34) Let $V = \bigoplus_{i=1}^\infty \mathbb{K}x_i$ be an infinite-dimensional vector space. Find an element $g \in (V \otimes V)^*$ that is not in $V^* \otimes V^* \quad (\subseteq (V \otimes V)^*)$.

(35) For morphisms $f : I \longrightarrow M$ and $g : I \longrightarrow N$ in a monoidal category we define $(f \otimes 1 : N \longrightarrow M \otimes N) := (f \otimes 1_I)\rho(I)^{-1}$ and $(1 \otimes g : M \longrightarrow M \otimes N) := (1 \otimes g)\lambda(I)^{-1}$. Show that the diagram

$$\begin{array}{ccc} I & \xrightarrow{f} & M \\ {\scriptstyle g}\downarrow & & \downarrow{\scriptstyle 1\otimes g} \\ N & \xrightarrow[f\otimes 1]{} & M \otimes N \end{array}$$

commutes.

(36) Let G be a finite group and $\mathbb{K}^G := \mathbb{K}[G]^*$ the dual of the group algebra. Show that $\mathbb{K}^G$ is a Hopf algebra and that each module structure $\mathbb{K}[G] \otimes M \longrightarrow M$ translates to the structure of a comodule $M \longrightarrow \mathbb{K}^G \otimes M$ and conversely. Show that this defines a monoidal equivalence of categories.
Describe the group valued functor $\mathbb{K}\text{-}c\mathcal{A}lg(\mathbb{K}^G, -)$ in terms of sets and their group structure.

Due date: Tuesday, 25.06.2002, 16:15 in Lecture Hall E41