**Mathematisches Institut**
**der Universität München**
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**SS 2002**
**Set 4**

# Problem set for
# Quantum Groups and Noncommutative Geometry

(13) Determine explicitly the dual coalgebra $A^*$ of $A := \mathbb{K}\langle x\rangle/(x^3)$.

(14) Determine and describe the coendomorphism bialgebra of $A$ from problem 12. (Hint: Determine first a set of algebra generators of $M(A)$. Then describe the relations.)

(15) Let $A$ be a finite dimensional $\mathbb{K}$-algebra with universal bialgebra $A \longrightarrow B \otimes A$. Show

i) that $A^{op} \longrightarrow B^{op} \otimes A^{op}$ is universal (where $A^{op}$ has the multiplication $\nabla\tau : A \otimes A \longrightarrow A \otimes A \longrightarrow A$);

ii) that $A \cong A^{op}$ implies $B \cong B^{op}$ (as bialgebras);

iii) that for commutative algebras $A$ the algebra $B$ satisfies $B \cong B^{op}$ but that $B$ need not be commutative.

iv) Find an isomorphism $B \cong B^{op}$ for the bialgebra $B = \mathbb{K}\langle a, b\rangle/(a^2, ab+ba)$. (compare problem 14).

(16) Consider the algebra $K[\epsilon]/(\epsilon^2)$ the so called algebra of dual numbers over a field $K$. Consider the algebra $B$ with (noncommuting) generators $a, b, c, d$ and relations:

$$ac = acac \quad ad = acad + adac \quad bc = acbc = bcac$$

$$bd = acbd + adbc = bcad + bdac \quad bcbc = bcbd + bdbc = 0$$

$$ac = 1 \quad ad = 0$$

Show that $B$ together with the cooperation

$$\delta : A \to B \otimes A$$

with $\delta(\epsilon) = bc \otimes 1 + bd \otimes \epsilon$ is the coendomorphism bialgebra of $A$.

Due date: Tuesday, 14.05.2002, 16:15 in Lecture Hall E41