Mathematisches Institut
der Universität München
Prof. Dr. B. Pareigis

WS 2001/02
Set 9

Problem set for Advanced Algebra

(33) Show that there is a natural isomorphism $\mathrm{Map}(A \times B, C) \cong \mathrm{Map}(B, \mathrm{Map}(A, C))$. Determine the associated unit and counit.

(34) Show that the underlying functor $U : \mathrm{Ab} \longrightarrow \mathrm{Gr}$, that forgets the commutativity of Abelian groups, has a left adjoint functor $\mathcal{F} : \mathrm{Gr} \longrightarrow \mathrm{Ab}$. Determine the associated unit and counit. (Hint: Consider the construction of the commutator factor group.)

(35) Show that the underlying functor $U : \mathrm{Ab} \longrightarrow \mathrm{comMon}$ from the category of Abelian groups to the category of commutative monoids, that forgets the existence of inverses in groups, has a left adjoint functor $\mathcal{F} : \mathrm{comMon} \longrightarrow \mathrm{Ab}$. Determine the associated unit and counit. (Hint: Consider the construction of fractions for rational numbers.)

(36) Let $f : R \longrightarrow S$ be a ring homomorphism. Show that the underlying functor $U : S\text{-}\mathrm{Mod} \longrightarrow R\text{-}\mathrm{Mod}$ from the category of S-modules to the category of R-modules, that forgets the S-module structure on modules and replaces it by the R-module structure $r \cdot m := f(r)m$, has a left adjoint functor $\mathcal{F} : R\text{-}\mathrm{Mod} \longrightarrow S\text{-}\mathrm{Mod}$ and a right adjoint functor $\mathcal{G} : R\text{-}\mathrm{Mod} \longrightarrow S\text{-}\mathrm{Mod}$. Determine the associated units and counits.

Due date: Tuesday, 18.12.2001, 16:15 in Lecture Hall 138