

**Problem set for
Advanced Algebra**

- (29) Let $\mathcal{F} : \mathcal{C} \rightarrow \text{Set}$ be a covariant representable functor and let $f : M \rightarrow N$ be a monomorphism in \mathcal{C} . Show that $\mathcal{F}(f) : \mathcal{F}(M) \rightarrow \mathcal{F}(N)$ is a monomorphism, too.
- (30) (a) Let $U : \text{Ab} \rightarrow \text{Set}$ be the underlying functor. Show that U is a representable functor.
(b) Show that $\mathcal{F} : \mathbb{Q}\text{-Vec} \ni V \mapsto U(\mathbb{Q}^n \otimes_{\mathbb{Q}} V) \in \text{Set}$ is a representable functor.
- (31) Show that the functor $\mathcal{F} : \text{Ab} \ni M \mapsto U(\mathbb{Z}/(n) \otimes_{\mathbb{Z}} M) \in \text{Set}$ is not representable (for $n \geq 2$).
- (32) For a \mathbb{K} -algebra A let A^* denote the multiplicative group of elements that are invertible under the multiplication of A . Let $U : \text{Gr} \rightarrow \text{Set}$ be the underlying functor. Show that the functor $\mathcal{F} : \mathbb{K}\text{-Alg} \ni A \mapsto U(A^*) \in \text{Set}$ is a representable functor.

Due date: Tuesday, 11.12.2001, 16:15 in Lecture Hall 138