Mathematisches Institut
der Universität München
Prof. Dr. B. Pareigis

WS 2001/02
Set 6

Problem set for Advanced Algebra

(21) Let $R := K \times K$ for K a field.
  (a) Show that $P := \{(a,0)|a \in K\}$ is a finitely generated projective R-module.
  (b) Decide if the R-modules P and $Q := \{(0,a)|a \in K\}$ are isomorphic?
  (c) Find a dual basis for P.

(22) (a) Let R be a ring and P_R be an R-module. Show that P is a finitely generated projective module if and only if P is a direct summand of the R-module R^n.
  (b) Let P_R be a finitely generated projective right R-module. Show that $P^* = \mathrm{Hom}_R(P., R.)$ is a finitely generated projective left R-module.

(23) Let R be a ring. Show that for each projective R-module P there is a free R-module F such that $P \oplus F \cong F$.

(24) Let $\mathcal{C}$ be a category with finite products. For each object A in $\mathcal{C}$ show that there exists a morphism $\Delta_A : A \longrightarrow A \times A$ satisfying $p_1\Delta_A = 1_A = p_2\Delta_A$. Show that this defines a natural transformation. What are the functors?

Due date: Tuesday, 27.11.2001, 16:15 in Lecture Hall 138