Mathematisches Institut
der Universität München
Prof. Dr. B. Pareigis

WS 2001/02
Set 5

Problem set for Advanced Algebra

(17) Show that $V \otimes V^*$ is a coalgebra for every finite dimensional vector space V over a field $\mathbb{K}$ if the comultiplication is defined by $\Delta(v \otimes v^*) := \sum_{i=1}^n v \otimes v_i^* \otimes v_i \otimes v^*$ where $\{v_i\}$ and $\{v_i^*\}$ are dual bases of V resp. V^*.

(18) Show that a finite dimensional vector space V is a comodule over the coalgebra $V \otimes V^*$ as defined in problem 17 with the coaction $\delta(v) := \sum v \otimes v_i^* \otimes v_i \in (V \otimes V^*) \otimes V$ where $\sum v_i^* \otimes v_i$ is the dual basis of V in $V^* \otimes V$.

(19) Show that the free $\mathbb{K}$-modules $\mathbb{K}X$ with the basis X and the comultiplication $\Delta(x) = x \otimes x$ is a coalgebra. What is the counit? Is the counit unique?

(20) Show that the tensor product of two commutative $\mathbb{K}$-algebras is a coproduct.

Due date: Tuesday, 20.11.2001, 16:15 in Lecture Hall 138